



Application of The Variance Minimization Model with a Certain Return for The Investment Portfolio of Seven Good-Performing Stocks on The IDX30 Index

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Abstract

Nowadays, investing in stocks is in great demand, especially among young people in Indonesia. In investing, investors will be faced with various problems in developing a good portfolio, including determining the assets for investment and determining the amount of capital required. Therefore, this study aims to determine how to choose stocks and the proportion of capital to invest in order to form an optimal portfolio. In this study, the Markowitz model is used to specifically minimize variance by constraining the number of weights and profit targets. An analysis of the closing prices of stocks included in the IDX30 index for the period August 2021 to January 2021 was carried out with data taken from September 11, 2020 to September 9, 2021, sourced from <https://finance.yahoo.com/>. The results of the study show that from 30 stocks, 7 stocks were selected that were categorized as good performers in each sector. Seven shares with the proportion of capital allocated for each share, namely BMRI (8.92051%), BRPT (3.718450%), ANTM (12.80686%), TBIG (23.30450%), MIKA (17.98822%), KLBF (7.96699%), and ASII (25.29447%).

Keywords: Investment, Optimal Portfolio, Stocks, IDX30, Markowitz.

1. Introduction

Rapid economic growth is supported by the increasing public interest in investing. According to Hong et al. (2014), there are two kinds of ways to invest, namely: in the form of real assets such as land, gold, and machinery, and in the form of financial assets such as stocks and bonds. It has been mentioned that investments can be made, one of which is investing in the form of buying shares, but investing is also faced with the problem of the risk of loss. This can be done by forming a portfolio for diversification.

Nowadays, investing in stocks is in great demand, especially by young people in Indonesia. According to Putra & Dana (2020), by investing in the capital market, investors can compile a diversified portfolio of stocks by choosing stocks based on performance and risk, then composing an optimal portfolio of these stocks. Building an investment portfolio includes determining which assets are selected as investments and determining the proportion of the funds to be invested. Investors are faced with the problem of choosing a good portfolio, so it is necessary to know how to choose stocks to form an optimal portfolio. One of the ways to form an optimal portfolio is by using the Markowitz model. This model is a portfolio determination model that emphasizes the relationship between return and investment risk.

Many experts have researched optimal portfolio formation using the Markowitz model. This study intends to apply a variance minimization model with an inevitable return to an investment portfolio of seven stocks that perform well on the IDX30 index. The focus of this research is to find out what stocks can be included in the optimal portfolio and the weight of the capital proportion of each stock, which is formed using the Markowitz model. After getting the model, an analysis of the IDX30 index was carried out. According to idx.co.id, the IDX30 is an index that measures

the price performance of 30 stocks that have high liquidity and large market capitalization and are supported by good company fundamentals. Completing this research was assisted by using Microsoft Excel software.

2. Literature Review

2.1 Investation

An investment is an investment in assets with the hope that the value of the assets will increase to generate profits in the future. Investors generally want big profits with little risk. According to Partono et al. (2017), there are several reasons people make investments, and the specific objectives of investment are as follows:

- a. To have a decent life in the future,
- b. To reduce inflationary pressure,
- c. To save taxes.

2.2 Share

Shares are securities that explain that the owner of the letter is also the owner of the company that issued the letter. Shares can also be defined as a sign of a person's participation or ownership in a company (Partono et al., 2017). Many investors choose stocks as investment instruments because they can provide an attractive profit. One of the goals of people who invest, also called investors, is to receive capital gains and dividends.

2.2.1 Share Types

Shares can be divided into several types according to their classification. According to Yasin (2021) in terms of the ability to claim rights, shares are divided into:

- a. Common stock
- b. Preferred stock

2.2.2 Share Price

The stock price is determined by the supply and demand ratio of the stock itself. The more investors who buy shares, the more the stock price tends to rise. On the contrary, if many investors sell shares, the stock tends to decrease in price (Rahmadewi & Abundanti, 2018).

2.3 Portfolio Theory

The term "portfolio theory" was coined by Harry Max Markowitz in 1952, and it was widely published in The Journal of Finance under the title "Portfolio Selection", also known as the mean-variance theory. This theory is motivated by the risk that the portfolio will be smaller if stocks that have high risk are combined into one portfolio, compared to the risk of individual stocks (Safitri et al., 2020).

2.4 Optimal Portfolio

Portfolio optimization is selecting the proportion of assets in a portfolio that makes a portfolio better than others based on several criteria, such as minimizing risk and maximizing returns (Chin et al., 2017). One of the criteria for determining a good stock category or measuring performance is the Sharpe ratio. Sharpe's ratio, developed by William F. Sharpe, has been tried and tested. This ratio is one of the standards for measuring mutual fund performance internationally and can be applied to all types of mutual funds. The higher the value of the Sharpe ratio, the better the performance of the stock. The Sharpe ratio formula is given in the following equation:

$$Sp_i = \frac{\mu_i - \mu_f}{\sigma_i} \quad (1)$$

with

Sp_i : Sharpe Ratio of i -th shares

μ_i : Average return of i -th shares

μ_f : Risk-free average return

σ_i : Standard deviation of i -th shares

In stock investment, the return can be in the form of a capital gain or loss. The similarities is (Hendrawan & Salim, 2017).

$$r_{ij} = \frac{P_j - P_{j-1}}{P_{j-1}} \quad (2)$$

with

r_{ij} : Return of the i -th shares in the j -th period

P_j : Share price in period to- j

P_{j-1} : Share price in period to- $(j - 1)$

Furthermore, to calculate the average stock return, you can use the following formula (Hendrawan & Salim, 2017).

$$\mu_i = \frac{\sum_{j=1}^l r_{ij}}{l} \quad (3)$$

with

μ_i : Average return of i -th shares

r_{ij} : Return of the i -th shares in the j -th period

l : Number of observation data periods

To calculate the value of variance, one can use the following formula (Agustinus, 2021).

$$\sigma_i^2 = \frac{\sum (r_{ij} - \mu_i)^2}{l-1} \quad (4)$$

with

σ_i^2 : Variance of i -th shares

r_{ij} : Return of the i -th shares in the j -th period

μ_i : Average return of i -th shares

$l - 1$: Number of observation data periods minus one

Next, calculate the standard deviation, which is the square root of the variance, formulated as follows (Agustinus, 2021).

$$\sigma_i = \sqrt{\frac{\sum (r_{ij} - \mu_i)^2}{l-1}} \quad (5)$$

According to Alexander & Dakos (2022), there are two statistical concepts that can be used to see the relationship between two stocks, namely covariance and correlation. The formula for the covariance between stocks X and Y can be formulated as follows:

$$\sigma_{XY} = \frac{\sum_{i=1}^n [(r_{X,ij} - \mu_X)(r_{Y,ij} - \mu_Y)]}{n-1}, \quad i = 1, 2, 3, \dots, n \quad (6)$$

with

σ_{XY} : Covariance between stock X and stock Y

$r_{X,ij}$: Stock return X based on sample observation

μ_X : Average stock return X

$r_{Y,ij}$: Stock return Y based on sample observation

μ_Y : Average stock return Y

n : The number of stocks in the formation of an investment portfolio

While the correlation between stocks X and Y can be formulated as follows:

$$\rho_{X,Y} = \frac{\sigma_{XY}}{(\sigma_X)(\sigma_Y)} \quad (7)$$

with

$\rho_{X,Y}$: Correlation between stock X and stock Y

σ_X : Standar deviation stock X

σ_Y : Standar deviation stock Y

2.5 Quadratic Programming

According to Mencarelli & d'Ambrosio (2019), quadratic programming solves non-linear optimization problems where the constraint is a linear function, and the objective function is the square of the decision variable or the product of the two decision variables. The general form of quadratic programming is as follows:

$$\text{Minimize } F(\mathbf{x}) = \mathbf{c}^T \mathbf{x} + \frac{1}{2} \mathbf{x}^T \mathbf{Q} \mathbf{x} \quad (8)$$

with constraints

$$\mathbf{A} \mathbf{x} \leq \mathbf{b}$$

$$\mathbf{x} \geq 0$$

with

\mathbf{c} : Vector of objective function coefficient $n \times 1$

\mathbf{x} : Vector of decision variable $n \times 1$

\mathbf{Q} : The objective function coefficient matrix, forming an $n \times n$

\mathbf{A} : Matrix the coefficient matrix of the constraint function, forming an $m \times n$

\mathbf{b} : Matrix the vector of values to the right of the constraint, forming a vector $m \times 1$
 n : Number of decision variable
 m : The number of obstacles

2.5.1 Quadratic Programming Problem Solving

Solving quadratic programming problems can be solved using the Lagrange multiplier technique approach. The standard form of the quadratic problem, which has added a variable to the constraint function with the Lagrange function for the minimization problem, is (Chen & Chen, 2013).

$$L(\mathbf{x}, \lambda_1, \lambda_2, \dots, \lambda_m) = F(\mathbf{x}) + \sum_{i=1}^m \lambda_i (\mathbf{a}_i \mathbf{x} - b_i) \quad (9)$$

with

\mathbf{a}_i : The i -th row vector of the constraint coefficient matrix \mathbf{A}
 b_i : The i -th line entry of vector \mathbf{b}
 λ_i : Lagrange multiplier of the i -th constraint

2.6 Markowitz Model

The Markowitz model is the first model that provides a way to select various stocks and form them into one portfolio (Safitri et al., 2020). One of the goals of the Markowitz portfolio optimization models is to minimize risk by constraining the number of weights and profit targets. The model can be described as follows:

Objective function:

$$\text{Minimize} \quad \left\{ \frac{\rho}{2} \sigma_p^2 = \frac{\rho}{2} \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij} \right\} \quad (10)$$

$$\text{with obstacles} \quad \sum_{i=1}^n w_i = 1 \quad (11)$$

$$\sum_{i=1}^n \mu_i w_i = K \quad (12)$$

with

μ_i : Average return of i -th shares
 ρ : Coefficient of risk aversion
 σ_p^2 : Variance (as a measure of investment portfolio risk p)
 w_i : Weight (proportion) of funds invested in the i -th stock; $i = 1, 2, \dots, n$
 n : The number of shares in the formation of an investment portfolio
 K : Profit target (certain constant)

2.6.1 Weight Vector Search (\mathbf{w})

The problem in this research is a quadratic programming problem, so the minimum solution is determined using a quadratic optimization approach. The following is the Lagrange function of the problem in this study, by referring to equation (9):

$$L(\mathbf{w}, \lambda_1, \lambda_2) = \frac{\rho}{2} \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w} + \lambda_1 (\mathbf{e}^T \mathbf{w} - 1) + \lambda_2 (\boldsymbol{\mu}^T \mathbf{w} - K) \quad (13)$$

Based on the results of the derivation of equation (13), the formula for finding the weight vector is as follows:

$$\mathbf{w} = \frac{\boldsymbol{\Sigma}^{-1}}{b^2 - ac} [(bK - c)\mathbf{e} + (b - aK)\boldsymbol{\mu}] \quad (14)$$

Where $a = \mathbf{e}^T \boldsymbol{\Sigma}^{-1} \mathbf{e}$, $b = \mathbf{e}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} = \boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1} \mathbf{e}$, and $c = \boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}$ with $\boldsymbol{\Sigma}^{-1}$ is the inverse of the covariance matrix.

3. Materials and Methods

3.1. Materials

The object of this study is the closing price data for the IDX30 index for the period August 2021 to January 2022, taken from September 11, 2020 to September 9, 2021. This data was taken from <https://finance.yahoo.com/> on September 27, 2021. One example is the closing price of Aneka Tambang Tbk's shares. (ANTM) is given in chart form as shown in Figure 1, and the closing price of Adaro Energi Tbk's shares (ADRO) is given in chart form as shown in Figure 2.

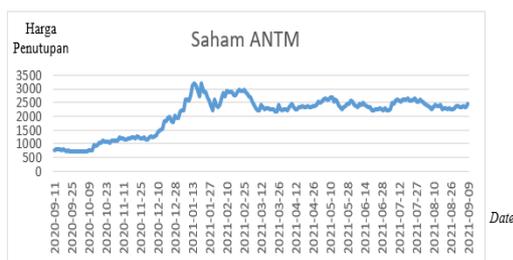


Figure 1: ANTM Closing Price Chart



Figure 2: ADRO Closing Price Chart

3.2. Methods

This study used descriptive research methods with a quantitative approach. Data analysis in this study is quantitative because the data used involves numbers, namely the list of closing stock prices for the IDX30 index. The research sample was taken using a purposive sampling technique because the sampling was not random, namely selecting sample members in the form of stocks included in the IDX30 index for the period August 2021 to January 2022. This research was carried out in detail in the form of certain objects within a certain time frame, and the results obtained were only applicable to the object under study, which can be referred to as case study research.

4. Results and Discussion

The stocks used in the formation of the investment portfolio, namely stocks that have the best performance in each sector, are included in the IDX30 index. Based on idnfinancial.com/id, the IDX30 index shares for the period August 2021 to January 2022 are divided into 7 sectors. To see stock performance, the highest Sharpe ratio value is calculated using equation (1). If in this study the risk-free asset is a bank deposit, the risk-free asset return is $\mu_f = 0.01\%$ with the assumption that trading days in a year are 255 days. Stocks categorized as stocks with good performance in each sector based on the Sharpe ratio for portfolio formation can be seen in Table 1.

Table 1: Shares Included in the Formation of the Portfolio

Stock	Sharpe Ratio
BMRI	3.54%
BRPT	5.19%
ANTM	12.45%
TBIG	12.34%
MIKA	1.54%
KLBF	-0.32%
ASII	3.89%

The stocks that are included in the formation of the investment portfolio are then calculated to determine the level of correlation of returns between stocks using equation (7), as given in Table 2.

Table 2: The Level of Correlation Between the Seven Stocks

	BMRI	BRPT	ANTM	TBIG	MIKA	KLBF	ASII
BMRI	100%	27.87%	32.37%	21.68%	2.66%	25.36%	45.74%
BRPT	27.87%	100%	23.13%	17.07%	19.47%	9.69%	21.38%
ANTM	32.37%	23.13%	100%	24.85%	5.18%	10.93%	21.76%
TBIG	21.68%	17.07%	24.85%	100%	8.58%	6.21%	13.91%
MIKA	2.66%	19.47%	5.18%	8.58%	100%	26.55%	4.76%
KLBF	25.36%	9.69%	10.93%	6.21%	26.55%	100%	20.30%
ASII	45.74%	21.38%	21.76%	13.91%	4.76%	20.30%	100%

Based on the results in Table 2, the level of correlation of returns between stocks is relatively small, close to 0, so that they do not have much effect on each other. To determine the proportion of each stock being studied, a weight vector calculation is carried out with a K value starting from 0.10% to 0.30% using equation (14). This is because the value of $K < 0.10\%$ has a negative weight, as well as the value of $K > 0.30\%$ has a negative weight value, while the results of the calculation can be seen in Table 3.

Table 3: Calculation Result of Weight Vector w with and K Determined

K (%)	w_1 (%)	w_2 (%)	w_3 (%)	w_4 (%)	w_5 (%)	w_6 (%)	w_7 (%)	Σw_i	μ_p (%)	σ_p^2 (%)	$\frac{\mu_p}{\sigma_p^2}$ (%)
0.08	22.99693	1.39044	-1.63833	7.18491	23.49935	18.13994	28.42676	1	0.08	0.01847	433.11267
0.10	20.98601	1.72301	0.42527	9.48771	22.71205	16.68666	27.97929	1	0.10	0.01845	542.12997
0.12	18.9751	2.05559	2.48887	11.7905	21.92474	15.23338	27.53182	1	0.12	0.01878	638.96436
0.14	16.96418	2.38816	4.55247	14.0933	21.13744	13.7801	27.08435	1	0.14	0.01947	718.8766
0.16	14.95326	2.72073	6.61606	16.3961	20.35013	12.32683	26.63688	1	0.16	0.02053	779.38211
0.18	12.94234	3.05331	8.67966	18.6989	19.56283	10.87355	26.18941	1	0.18	0.02194	820.30165
0.20	10.93143	3.38588	10.74326	21.0017	18.77553	9.42027	25.74194	1	0.20	0.02372	843.27614
0.22	8.92051	3.71845	12.80686	23.3045	17.98822	7.96699	25.29447	1	0.22	0.02585	851.04041
0.24	6.90959	4.05103	14.87045	25.6073	17.20092	6.51372	24.847	1	0.24	0.02834	846.73374
0.26	4.89868	4.3836	16.93405	27.91009	16.41361	5.06044	24.39953	1	0.26	0.0312	833.39951
0.28	2.88776	4.71617	18.99765	30.21289	15.62631	3.60716	23.95206	1	0.28	0.03441	813.70166
0.30	0.87684	5.04875	21.06125	32.51569	14.839	2.15388	23.50459	1	0.30	0.03798	789.81494
0.32	-1.13408	5.38132	23.12485	34.81849	14.0517	0.7006	23.05712	1	0.32	0.04192	763.42564
0.34	-3.14499	5.7139	25.18844	37.12129	13.2644	-0.75267	22.60965	1	0.34	0.04621	735.78919

Based on Table 3, the highest value of $\frac{\mu_p}{\sigma_p^2}$ is 851.04041% at $K = 0.22\%$. Therefore, at $K = 0.22\%$, this is the optimal portfolio. In the optimal portfolio, the respective proportions are BMRI of 8.92051%, BRPT of 3.718450%, ANTM of 12.80686%, TBIG of 23.30450%, MIKA of 17.98822%, KLBF of 7.96699%, and ASII of 25.29447%. The average optimal profit from this portfolio return is 0.22% per day, and the variance of the portfolio return is 0.02585%.

A series of efficient portfolios is on the efficient frontier. The efficient frontier is an efficient surface on which there are portfolios whose returns are commensurate with their risks. Based on the results of the calculations in Table 3, it is found that the efficient portfolios lie in the $0.10\% \leq K \leq 0.30\%$ interval, as can be seen in Figure 3.

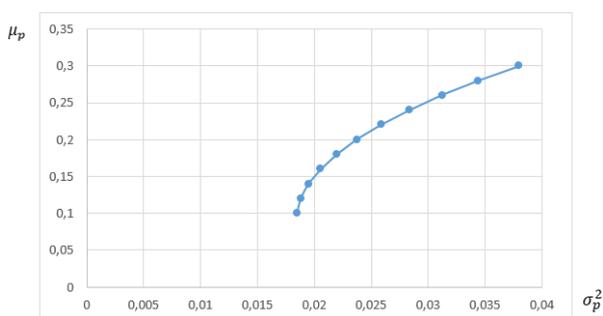


Figure 3: Efficient Frontier Chart

Based on Figure 3, the relationship between variance and the average return of the portfolio looks perfectly positive, where the greater the variance, the greater the return value of the portfolio and vice versa. It can be interpreted that if an investor is going to invest in stocks, it must be along the lines of having an efficient portfolio. Every investor wants a high return value and a low risk. It is assumed that investor preferences are only based on the ratio and risk of the portfolio, so that the optimal portfolio selection can be determined based on the composition of the efficient portfolio that produces the largest ratio. The chart between ratio and variance can be seen in Figure 4.

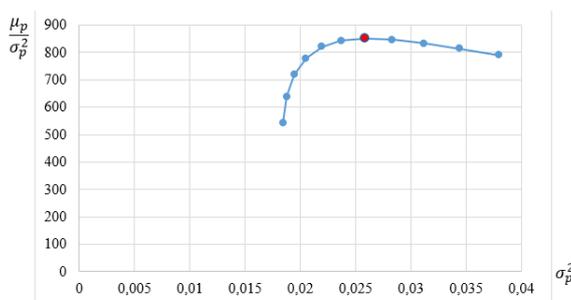


Figure 4: Ratio Chart

Based on Figure 4, the largest ratio which is the optimal portfolio, is located at the red dot with a value of 851.04041% at $K = 0.22\%$. The ratio between the mean and variance of portfolio returns increased at the profit target interval of $0.10\% \leq K \leq 0.22\%$ and decreased at the profit target interval of $0.22\% < K \leq 0.30\%$.

5. Conclusion

The IDX30 index stocks that fall into the category of stocks with good performance in each sector based on the Sharpe ratio criteria include BMRI (Bank Mandiri (Persero) Tbk.), BRPT (Barito Pacific Tbk.) shares, ANTM (Aneka Tambang Tbk.) shares, shares of TBIG (Tower Bersama Infrastructure Tbk.), shares of MIKA (Mitra Keluarga Karyasehat Tbk.), shares of KLBF (Kalbe Farma Tbk.), and shares of ASII (Astra International Tbk.). The proportion of capital in each share sequentially is 8.92051%; 3.718450%; 12,80686%; 23.30450%; 17.98822%; 7.96699%; 25.29447%.

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