



# Investment Portfolio Optimization Using Black-Litterman Model in Smart Carbon Economy Transition

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## Abstract

An optimal investment portfolio needs to be formed before an investor invests because it can help investors determine which financial instruments are suitable to choose in order to get the maximum return or profit and the minimum level of risk. In the current situation, where there is an economic transition to a smart carbon economy or low carbon economy, it is necessary to form the optimal portfolio of stocks to facilitate investors in making investments. The purpose of this study is to form the optimal investment portfolio using the Black-Litterman model in a smart carbon economy. The data used is stock data from 24 companies listed on the LQ45 Low Carbon Leaders index for the period 2022-2023. Based on the research results, the Black-Litterman model generates the optimal portfolio with a 0.1% expected return. Thus, the optimal portfolio results with the Black-Litterman model are estimated to generate a profit of 0.1% for smart carbon stock data listed on the LQ45 Low Carbon Leaders index for the 2022-2023 period.

*Keywords:* Optimal Portfolio; Smart Carbon Economy; Black-Litterman.

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## 1. Introduction

Global policymakers have been concerned about the effects of rising global temperatures and climate instability since the late 1980s. This is still an issue today because greenhouse gas (GHG) emissions are fast contributing to climate change. Seeing this situation, the economic transition to a low-carbon economy or what will be called smart carbon (Benedetti et al., 2021) is one of the strategies in reducing CO<sub>2</sub> emissions. According to the International Energy Agency analysis report (2021), globally, greenhouse gas emissions are likely to increase by 2023 due to declining investment in low-carbon technologies. This makes the global net-zero emissions plan, which is expected to be achieved by 2050, even further away. Thus, the government calls for a rebuilding of investment in the smart carbon economy for a cleaner future.

Reflecting on the existing issues, the current investment is investing in smart carbon stocks where it can help the movement of the smart carbon economy which aims to reduce greenhouse gas emissions. Investors make investments in the hope of future returns. However, investing in a single asset carries a considerable risk; hence, a portfolio of assets is required. A portfolio is an assortment of assets designed to lower the amount of risk involved (Subekti et al., 2019). In order to generate maximum profit, it is necessary to form an optimal portfolio, namely a portfolio that can provide high profits or returns with the lowest possible risk (Zhang et al., 2020). The Black-Litterman Model is the portfolio model that will be applied in this study. A development of the Capital Asset Pricing Model (CAPM), the Black-Litterman model includes factors related to investor analysis in the computation.

There is relevant previous research conducted by Benedetti et al. (2021) on portfolio construction entitled Climate Change Investment Risk: Optimal Portfolio Construction Ahead of the Transition to A Lower-Carbon Economy. The study examines the construction of smart carbon stock portfolios using the Bayesian and Black-Litterman approaches. Then, there is also previous research with the title The Application of Black-Litterman Bayesian Model for the Portfolio Optimization on The Liquid Index 45 (LQ45) with Information Ratio Assessment conducted by Murtadina et al. (2021). The study discusses the application of Black-Litterman to portfolio optimization by assessing the information ratio with the results stating that Black-Litterman produces a value that is more optimal than CAPM.

Based on the phenomenon that has been explained, this research was conducted to determine the optimization of the smart carbon stock investment portfolio in the smart carbon economic transition using the Black-Litterman model. The Black-Litterman model is a refinement of the Capital Asset Pricing Model (CAPM) by combining equilibrium expected returns and investor views to form new expected returns. Thus, it is expected to form a more profitable portfolio. This research was conducted to facilitate policy makers or investors in determining the investment portfolio of smart carbon stocks. As a result, it can play an active role in the smart carbon economic transition by investing in smart carbon stocks.

## 2. Literature Review

### 2.1. Smart Carbon Economy

In Zhongyu & Zhongxiang (2021), citing Lu and Zhu, said that the smart carbon economy is a form of economy and sustainable development that has a low carbon productivity level and is environmentally friendly to accomplish the worldwide goal of managing greenhouse gas emissions and expanding the social economy. The smart carbon economy has been developed from an idea to a practice by policy makers around the world in recent years.

Zhong et al. (2022) stated that smart carbon investment is a popular method that can encourage behavior for sustainable development and is known as environmental investment. In Zhong et al. (2022), Zhang et al. stated that today's consumers have environmental awareness which has an impact on low-carbon products. Chen et al. also stated that green marketing provides an increase in market demand and creates a competitive advantage for companies. Thus, the purpose of investing in smart carbon is not only to reduce harmful effects on the environment but also to meet consumer needs (Zhong et al., 2022).

### 2.2. Capital Asset Pricing Model (CAPM)

In 1966, Sharpe developed a Mean-Variance analysis model that eventually led to the creation of the Capital Asset Pricing Model (CAPM). The CAPM establishes a relationship between the risk and the expected return of high-risk assets in an equilibrium market. The expected return equation and beta value estimation are written as follows (Murtadina et al., 2021):

$$E(R_i)_{CAPM} = R_f + \hat{\beta}_i [E(R_M) - R_f] \quad (1)$$

$$\hat{\beta}_i = \frac{Cov(R_i, R_M)}{\hat{\sigma}_M^2} \quad (2)$$

$$Cov(R_i, R_M) = \frac{\sum_{t=1}^m (R_{it} - E(R_i))(R_{Mt} - E(R_M))}{m - 1} \quad (3)$$

$$\hat{\sigma}_M^2 = \frac{\sum_{t=1}^m (R_{Mt} - E(R_M))^2}{m - 1}, t = 1, 2, \dots, m \quad (4)$$

- $E(R_i)_{CAPM}$  : expected stock equilibrium return,
- $\hat{\beta}_i$  : estimated beta coefficient value,
- $E(R_M)$  : expected market return,
- $R_f$  : risk free rate,
- $Cov(R_i, R_M)$  : market stock covariance,
- $\hat{\sigma}_M^2$  : market variance,
- $E(R_i)$  : expected stock return,
- $R_{it}$  : stock return,
- $R_{Mt}$  : market return.

### 2.3. Black-Litterman Model

The Black-Litterman concept was first presented in the 1990s by Robert Litterman and Fisher Black. The Black-Litterman model creates a new expected return by combining investor perspectives and equilibrium expected returns, which is an improvement over the Capital Asset Pricing Model (CAPM). Besides historical data results, an investor has credible opinions and thoughts on a portfolio. Thus, including the opinions of investors in a model is expected to form a more profitable portfolio. This investor opinion is also usually included as an external factor in other portfolio

determination models (Subekti et al., 2019). The following is the equation for implementing the Black-Litterman model to create an optimal portfolio (Subekti et al., 2021; Zakamulin, 2010):

1) Views

Views are investor opinions that are determined based on the analysis of the investor, so they are relative depending on the investor who conducts the analysis. The views expressed will produce two matrices, namely, the **Q** matrix, which is a matrix containing expected returns, and the **P** matrix, which is a link matrix as a link between views and weights whose rows total 1 for absolute views and 0 for relative views.

$$Q = \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_q \end{bmatrix} \tag{5}$$

$$P = \begin{bmatrix} P_{11} & \dots & P_{1n} \\ \vdots & \ddots & \vdots \\ P_{q1} & \dots & P_{qn} \end{bmatrix} \tag{6}$$

- Q** : expected return matrix from views,
- P** : link matrix,
- Q* : expected return from views,
- P* : stock,
- q* : number of views.

2) Error

In the resulting views, there is a possibility that investors have doubts. This doubt is the value of the coefficient ( $\tau$ ) determined by the investor. In Seimertz (2015), Black-Litterman states that the value of  $\tau$  should be close to zero. In addition, risk aversion ( $\delta$ ) is determined to explain the ratio of profit and risk. Then, errors can appear in the views due to doubts, which are denoted by *v* and can be obtained through the following equations:

$$v = P' \tau \Sigma P \tag{7}$$

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{1,2} & \dots & \sigma_{1,n} \\ \sigma_{1,2} & \sigma_2^2 & \dots & \sigma_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1,n} & \sigma_{n,2} & \dots & \sigma_n^2 \end{bmatrix} \tag{8}$$

- $\tau$  : coefficient value of the investor,
- $\Sigma$  : covariance matrix.

Views expressed on the stock are assumed to be independent so as to cause the error value to be zero except on its diagonal, and the following matrix  $\Omega$  is obtained:

$$\Omega = \begin{bmatrix} v_1 & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & v_q \end{bmatrix} \tag{9}$$

- $\Omega$  : diagonal covariance matrix of views.

## 3) Expected Return and Weight of the Black-Litterman Model

$$E(R_{BL}) = [(\tau\Sigma)^{-1} + \mathbf{P}'\Omega^{-1}\mathbf{P}]^{-1}[(\tau\Sigma)^{-1}\Pi + \mathbf{P}'\Omega^{-1}\mathbf{Q}] \quad (10)$$

$$\text{Var}(R_{BL}) = [(\tau\Sigma)^{-1} + \mathbf{P}'\Omega^{-1}\mathbf{P}]^{-1} \quad (11)$$

$$w_{BL} = (\delta\Sigma)^{-1}E(R)_{BL} \quad (12)$$

$E(R_{BL})$  : Black-Litterman expected return model,

$\text{Var}(R_{BL})$  : Black-Litterman model variance,

$w_{BL}$  : Black-Litterman model weights,

$\delta$  : risk aversion coefficient.

## 4) Optimal Portfolio

$$E(R)_{P_{BL}} = \sum_{i=1}^n w_{BL_i} E(R)_{BL_i} \quad (13)$$

$$\text{Var}(R)_{P_{BL}} = (w_{BL})' \Sigma w_{BL} \quad (14)$$

### 3. Materials and Methods

#### 3.1. Materials

The research data used in this study is daily closing stock price data from companies listed on the IDX LQ45 Low Carbon Leaders for the period December 2022 - September 2023 obtained through yahoo finance. The IDX LQ45 Low Carbon Leaders Index strives to lower the portfolio's exposure to carbon emission intensity by at least 50% when compared to the LQ45 Index. The selection of this data is based on the research objectives in forming an optimal investment portfolio in the smart carbon economy, so the stocks selected are smart carbon stocks. Data processing is done using Microsoft Excel and Python software to simplify and speed up calculations.

#### 3.2. Methods

##### 1) Equilibrium Return with CAPM

The equilibrium return is ascertained using equation (1) for CAPM estimation as the first step in the Black-Litterman model.

##### 2) Views Expected Return

Views are investor views that are determined based on analysis and are relative depending on the investor conducting the analysis. The views expressed will produce two matrices, namely, the  $\mathbf{Q}$  matrix containing the expected return and the  $\mathbf{P}$  matrix containing the link matrix, such as equations (5) and (6).

##### 3) Coefficient Value and Risk Aversion

In the resulting views, there is a possibility that investors have doubts. This doubt is the coefficient value ( $\tau$ ) determined by the investor. Then, risk aversion ( $\delta$ ) is determined to explain the comparison of profit and risk.

##### 4) Expected Return and Weight of the Black-Litterman Model

Equations (10), (11), and (12) are used to calculate the expected return, variance, and weight of the Black-Litterman model.

##### 5) Expected return and Optimal Portfolio Variance

Next, find the portfolio's expected return and variance using equations (13) and (14).

### 4. Results and Discussion

#### 1) Equilibrium Return CAPM

The first step in Black-Litterman is to calculate the CAPM equilibrium return. In this study, the equilibrium return is obtained by CAPM estimation using equation (1). The calculation results using Microsoft Excel are shown in Table 1.

**Table 1:** Table of equilibrium return values

Stocks	Equilibrium Return
BRPT	0.00025039
BMRI	0.00024937
MEDC	0.00026568
EXCL	0.00024723
TPIA	0.00024542

The equilibrium return value serves as the foundation for the expected return in the Black-Litterman model before the investor's perspective is added.

## 2) Views Expected Return

The value of views used in this study is obtained from the analysis of historical data obtained as follows:

- BRPT is predicted to give a return of 0.021
- BMRI is predicted to give a return of 0.012
- MEDC is predicted to provide a return of 0.090
- EXCL is predicted to provide a return of 0.029
- TPIA is predicted to give a return of 0.056

Based on this data, the expected return matrix **Q** and link matrix **P** are formed as follows:

$$\mathbf{Q} = \begin{bmatrix} 0.021 \\ 0.012 \\ 0.090 \\ 0.029 \\ 0.056 \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The **Q** matrix consists of views expected returns and the **P** matrix is a link matrix between views and weights whose row count is 1 for absolute views.

## 3) Coefficient Value and Risk Aversion

The confidence level value used in this study is 0.05. In (Seimertz, 2015), Lee stated that in real life the value of  $\tau$  is usually between 0.01 and 0.05. Then, the value of the risk aversion coefficient ( $\delta$ ) to be used is 2.5 where this number is considered the standard number for risk aversion (Olsson & Trollsten, 2018). Thus, the error value is obtained as follows:

$$v = \begin{bmatrix} 0.0000324 & 0.0000033 & 0.0000099 & 0.0000007 & 0.0000083 \\ 0.0000033 & 0.0003310 & 0.0000037 & 0.0000027 & -0.000006 \\ 0.0000099 & 0.0000037 & 0.0000615 & 0.0000004 & 0.0000010 \\ 0.0000007 & 0.0000027 & 0.0000043 & 0.0000237 & -0.000001 \\ 0.0000083 & -0.000006 & 0.0000010 & -0.000001 & 0.0000187 \end{bmatrix}$$

It is known that the views on the stocks are independent, so the covariance values are made to be zero except on the main diagonal.

$$\Omega = \begin{bmatrix} 0.0000324 & 0 & 0 & 0 & 0 \\ 0 & 0.0003310 & 0 & 0 & 0 \\ 0 & 0 & 0.0000615 & 0 & 0 \\ 0 & 0 & 0 & 0.0000237 & 0 \\ 0 & 0 & 0 & 0 & 0.0000187 \end{bmatrix}$$

## 4) Expected Return and Weight of the Black-Litterman Model

By applying equation (10) to derive the Black-Litterman model's expected return value from the expected return view and the expected equilibrium return, the resulting Black-Litterman expected return value using Microsoft Excel is shown in Table 2 below:

**Table 2:** Table of Black-Litterman expected return

Stocks	Expected Return
BRPT	0.00061
BMRI	0.00111
MEDC	0.00138
EXCL	0.00044
TPIA	0.00049

Based on the results of the expected return value, MEDC stock has the highest expected return value, so it is expected to provide the highest profit. Then, using equation (12) and Microsoft Excel to calculate the Black-Litterman weight and obtain the value as in Table 3.

**Table 3:** Table of Black-Litterman weight

Stocks	Weight
BRPT	0.11408
BMRI	0.06902
MEDC	0.39541
EXCL	0.32924
TPIA	0.50773
Total	1

#### 5) Expected Return and Optimal Portfolio Variance

At this stage, using Python, the expected return and variance of the optimal portfolio of the Black-Litterman model yielded the following calculation results:

$$E(R)_{P_{BL}} = 0.001092429$$

$$Var(R)_{P_{BL}} = 0.000436972$$

## 5. Discussion

This research discusses the steps that can be taken to support a low-carbon economy. The steps taken are to invest in shares of companies that actively contribute to the movement of the smart carbon economy to support the movement. In investing, an optimal portfolio is needed in order to get the maximum return. Optimal portfolios in this smart carbon economy transition situation need to be formed to help investors invest. The portfolio that has been formed using the Black-Litterman model is considered to provide more optimal returns compared to previous models.

Based on the calculation results, the optimal portfolio is obtained with an expected return value of 0.001092429 and a variance value of 0.000436972. This means that the portfolio can provide a profit of 0.1% with a fairly low level of risk because the variance value is close to 0. This portfolio can be used by the investor to invest in the transition to a smart carbon economy, and the Black-Litterman model can also be one of the methods used in the formation of optimal portfolios.

## 6. Conclusion

In this article, the ideal investment portfolio for the transition state of the smart carbon economy has been analyzed and created using only stocks listed on the LQ45 Low Carbon Leaders. The research results indicate that the Black-Litterman model developed an optimal portfolio with an expected return of 0.001092429, which means that the portfolio is expected to provide a profit of 0.1% of the portfolio that has been formed. In addition, the variant value of the portfolio is 0.000436972 which can be interpreted that the level of risk of the portfolio that has been formed is fairly low.

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