



# Determination of Optimal Stock Portfolio Return by Single Index Model (Case Study on Banking Sector Stocks in Indonesia)

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## Abstract

The optimal portfolio is a portfolio chosen by investors from the many options available in the collection of efficient portfolios. To get the optimal proportion, which is the maximum return and minimum risk, it is necessary to analyze the stocks to be selected in the investment model. The research objective is to determine the optimal return, risk, and proportion for each banking stock portfolio in Indonesia in the period February - July 2023. The method used is the Single Index Model. The process of determining the optimal proportion of stocks with the Single Index Model requires stock and market return data as the main basis for applying this method. This study involves the formation of an optimal portfolio of daily closing prices of 46 banking stocks. As a result of this research, there are 5 optimal stocks that meet the criteria for optimal portfolio formation with each fund proportion of 21.43% (BNII), 13.52% (BDMN), 35.02% (BBRI), 23.69% (BTPN), and 6.34% (BBCA). Expected return from optimal stocks is 0.152% and the risk that will be borne by investors is 0.0011% per day.

*Keywords:* Optimal Portfolio; Single Index Model; Banking Stocks.

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## 1. Introduction

Stock are a sign of participation or ownership of a person or entity in a company or limited liability company (Darmadji and Fakhrudin, 2012). stocks is one of the choices a company has to support its operations. On the other hand, stocks are the investment vehicle of choice for many investors because of their attractive returns. Stock investments can get dividends and capital gains. Cash dividends are profit distributions given to shareholders for making investments and are paid periodically in cash (Holland, 2003). Capital gain is the profit obtained from the sale of shares. the amount of capital gain can be calculated by calculating the historical return that occurred in the previous period, so that we can determine the expected rate of return (Baye et al., 2014).

According to Tandelilin (2010), the current stock market index is an indicator that reflects stock performance in the market. Stocks are traded on the capital market or stock exchange. When someone buys stocks in a company, they become a shareholder in that company. Stock prices rise and fall according to the forces of supply and demand. Demand, in economics, is defined as a person's desire for the desired item (Sharpley, 2018).

Investment is an effort to invest capital or postpone consumption to obtain greater profits in the future (Tandelilin, 2010). According to Goetzmann et al. (2014), wise investors will choose investments that produce maximum returns and low risks, but investments have a high risk-high return concept, which means that the opportunity to gain large profits is often directly proportional to the level of risk faced. This shows that the higher the risk faced by investors, the greater the possibility of profit.

An optimal portfolio is a portfolio chosen by an investor from among the many choices available in a collection of efficient portfolios (Tandelilin, 2010). The goal of forming an optimal stock portfolio is to achieve diversification, manage risk, and generate returns. In the field of portfolio analysis, there are various models that can be used to analyze portfolios effectively, including the single index model, Markowitz model, and Capital Asset Pricing Model (CAPM).

Based on the explanation above, this research uses data on banking stocks listed on the Indonesia Stock Exchange to determine the optimal proportion, return and risk of an optimal stock portfolio. The method used is a single index model. The results of the research can be used by investors to determine the optimal portfolio of banking stocks.

## 2. Literature Review

### 2.1 Single Index Model

The single index model is a method developed by Sharpe in 1963, this method is used to simplify the calculation of the Markowitz method because it calculates risk with covariance through the use of a variance-covariance relationship matrix, which requires complex calculations (Goetzmann et al., 2014). In the single index model, risk is simplified into two components, namely market risk and company-specific risk, so that based on this simplification, the calculation of complex Markowitz portfolio risk becomes a simple calculation (Chong, 2004).

There are several assumptions of the single index model according to Goetzmann et al. (2014) including: Stocks will be correlated if they have the same response to the market price index return, the distribution of stock returns is normally distributed, and short sales are not allowed, which is a transaction by an investor by borrowing funds from a broker first, then selling the stocks at a high price with a note that the investor has made a prediction that the stock will go down. There are several characteristics of the single index model, namely alpha and beta portfolios, as equations (1) and (2).

$$\alpha_p = \sum_{i=1}^n w_i \cdot \alpha_i, \quad (1)$$

$\alpha_p$  : portfolio alpha,  
 $w_i$  : proportion in the i-th stock,  
 $\alpha_i$  : alpha of the i-th stock.

$$\beta_p = \sum_{i=1}^n w_i \cdot \beta_i, \quad (2)$$

$\beta_p$  : beta of the portfolio,  
 $w_i$  : proportion in the i-th stock,  
 $\beta_i$  : beta of the i-th stock.

### 2.2 Stock Return and Risk

#### a). Stock Return

Return is a reward for the investor's courage in bearing the risk of the investment made. According to Goetzmann et al. (2014) return can also be referred to as the return obtained from investing in stocks that can be calculated. Return is the result obtained from investment which consists of two types, namely realized return and expected return. Calculating return can be calculated using equation (3).

$$R_{i,t} = \frac{P_t - P_{t-1}}{P_{t-1}}, \quad (3)$$

$R_{i,t}$  : i-th stock return period t,  
 $P_t$  : closing price of the i-th stock in period t,  
 $P_{t-1}$  : closing price of the i-th stock in period t-1,  
t : discrete time period.

According to Francis & Kim (2013) expected shares can be found with equation (4).

$$E(R_i) = \frac{\sum_{t=1}^n R_{i,t}}{n}, \quad (4)$$

$E(R_i)$ : expected return of the i-th stock,  
 $R_{i,t}$  : i-th stock return at time t,  
n : many stock data.

#### b). Stock Return

The risk of a stock market index can be found by using variance as a measure of its risk with the equation (5).

$$\sigma_i^2 = \frac{1}{n-1} \sum_{t=1}^n [R_{i,t} - E(R_i)]^2, \quad (5)$$

n : many stock data,

$\sigma_i^2$  : stock variance value,  
 $R_{i,t}$  : realized stock return in period t,  
 $E(R_i)$ : expected return of the stock.

### 2.3 Optimal Portfolio with Single Index Model

The purpose of finding the optimal proportion is to achieve a balance between the potential profit and risk desired by the investor. In the formation of the optimal portfolio, the stock weight will affect the return and risk of the portfolio.

$$Z_i = \frac{\beta_i}{\sigma_{ei}^2} [(ERB)_i - C^*], \quad (6)$$

The formula for the proportion can be used with equation (7).

$$w_i = \frac{Z_i}{\sum_{i=1}^n Z_i}, \quad (7)$$

$\beta_i$  : beta of the i-th stock,  
 $\sigma_{ei}^2$  : the residual variance of the i-th stock,  
 $(ERB)_i$ : Excess Return to Beta of the i-th stock,  
 $C^*$  : Cut-off Rate of the largest value of  $C_i$ ,  
 $w_i$  : the proportion of the i-th security,  
 $Z_i$  : scale weighted.

In calculating the expected return of a portfolio of risky assets, the equation (8) is used.

$$E(R_p) = \alpha_p + \beta_p \cdot E(R_m) \quad (8)$$

$E(R_p)$  : expected return of the portfolio,  
 $\alpha_p$  : portfolio alpha,  
 $\beta_p$  : beta of the portfolio,  
 $E(R_m)$  : average market return.

The variance of market stock returns is a statistical measure of how far the value of market stock returns differs from its average value. The variance can be calculated by equation (9).

$$\sigma_m^2 = \frac{1}{n-1} \sum_{t=1}^n (R_{m,t} - E(R_m))^2, \quad (9)$$

$\sigma_m^2$  : market return variance,  
 $R_{m,t}$  : market return at time t,  
 $E(R_m)$  : average market return.

Meanwhile, the risk value of the stock portfolio can be calculated in the equation (10).

$$\sigma_p^2 = \beta_p^2 \cdot \sigma_m^2 + \left( \sum_{i=1}^n w_i \cdot \sigma_{ei}^2 \right)^2, \quad (10)$$

$\sigma_p^2$  : portfolio variance,  
 $\beta_p^2$  : quadratic portfolio beta,  
 $\sigma_m^2$  : market price variance,  
 $w_i$  : proportion of the i-th stock,  
 $\sigma_{ei}^2$  : the residual variance of the i-th stock.

## 3. Materials and Methods

### 3.1. Materials

The object of this research is the analysis of banking sector stocks in Indonesia using the single index model method. The data used is secondary data, namely stock closing data every day during February - July 2023. The stock

data used was obtained from the site yahoo finance which was accessed in October 2023. The applications used are Microsoft Excel and SPSS to facilitate the calculation process. In this study, 46 bank stocks were taken from private and public banks.

### 3.2. Methods

- Collect historical daily closing price data on banking stocks during February - July 2023 and JKSE stocks. Calculate the return in equation (3) and the expected return of stocks using the single index model method in equation (4). negative expected returns will be excluded from the calculation.
- Obtain stock risk using the single index model method in equation (5).
- Find the percentage proportion of stocks in equation (6) and in equation (7).
- Determine the portfolios  $\alpha_p$  and  $\beta_p$  in equations (1) and (2).
- Determine the expected return and risk of the portfolio in equations (8) and (10) by calculating the market return in equation (9).

### 4. Results and Discussion

- Calculating expected return

The first step is to calculate the expected return. expected return is obtained from the average return. The calculation of the expected return of daily stock closings is carried out using the Microsoft Excel application. the following results are shown in Table 1.

**Table 1:** Table of expected return

| Stock | $E(R_i)$ |
|-------|----------|
| BBCA  | 0.000684 |
| BBRI  | 0.001758 |
| BDMN  | 0.001349 |
| BNII  | 0.002002 |
| BTPN  | 0.00105  |

Expected return is used to get an idea of how profitable a stock is. Table 1 results show BBCA stocks will provide a profit of 0.000684, BBRI 0.001758, BDMN 0.001349, BNII 0.002002 and BTPN 0.00105.

- Stock risk

Finding stock risk can use the variance formula. To obtain stock risk using the single index model method, equation (5) is used. The results of the stock risk can be seen in Table 2.

**Table 2:** Table of stock risk

| Stock | $\sigma_i^2$ |
|-------|--------------|
| BBCA  | 0.000102     |
| BBRI  | 0.000151     |
| BDMN  | 0.000268     |
| BNII  | 0.0003       |
| BTPN  | 0.000094     |

Stock risk is used to determine how much risk investors will bear. The results from Table 2 show that BBCA stocks have a risk of 0.000102, BBRI 0.000151, BDMN 0.000268, BNII 0.0003 and BTPN 0.000094.

- Proportion and scale weighted stocks

The value of  $W_i = 1$  is obtained after finding the value of  $Z_i$ . The calculation of  $Z_i$  is done by the formula equation (6). after getting the value of  $Z_i$ , proceed to find the value of  $W_i$  in the formula equation (7), and the results are in Table 3.

**Table 3:** Table of proportions and weighted scales stocks

| Stock | $Z_i$  | $W_i$  |
|-------|--------|--------|
| BBCA  | 1.4716 | 0.0634 |
| BBRI  | 8.1263 | 0.3502 |
| BDMN  | 3.1380 | 0.1352 |
| BNII  | 4.9719 | 0.2143 |
| BTPN  | 5.4976 | 0.2369 |
| Total |        | 1      |

$W_i$  is a parameter that shows the proportion of allocation of a financial instrument in a portfolio, and the value of  $w$  is equal to 1, then it can be interpreted that all investment funds from investors have been fully allocated to the selected stocks and reached a full allocation of 100%.

d). Portfolios  $\alpha_p$  and  $\beta_p$

There are  $\alpha_p$  and  $\beta_p$  which are characteristics in the single index model. There are  $\alpha_p$  and  $\beta_p$  are used to find the expected return of the optimal stock portfolio. to find  $\alpha_p$  and  $\beta_p$  use equations 1 and 2. The following are the  $\alpha_p$  and  $\beta_p$  results in Table 4.

**Table 4:** Table of  $\alpha_p$  and  $\beta_p$ 

| Stock | $\alpha_p$ | $\beta_p$ |
|-------|------------|-----------|
| BBCA  | 0.000038   | 0.054302  |
| BBRI  | 0.000588   | 0.265768  |
| BDMN  | 0.000179   | 0.033618  |
| BNII  | 0.000426   | 0.031422  |
| BTPN  | 0.000229   | 0.190316  |

e). Portfolio expected return and risk

The portfolio expected return is a weighted average of the expected return of each security in the portfolio. The realized return and portfolio expected return is a weighted average return of all single securities. The results of the expected return are shown in Table 5.

**Table 5:** Table of portofolio expected return

| Stock | $\alpha_p$             | $\beta_p$ | $E(R_m)$  |
|-------|------------------------|-----------|-----------|
| BBCA  | 0.000038               | 0.054302  |           |
| BBRI  | 0.000588               | 0.265768  |           |
| BDMN  | 0.000179               | 0.033618  | 0.000105  |
| BNII  | 0.000426               | 0.031422  |           |
| BTPN  | 0.000229               | 0.190316  |           |
| Total | 0.001458               | 0.575426  |           |
|       | $\beta_p \cdot E(R_m)$ |           | 0.0000604 |
|       | $E(R_p)$               |           | 0.0015187 |

The expected return of the portfolio is 0.0015187. This means that the return that will be generated by investors if they invest with the portfolio is 0.151% per day. After obtaining the expected return results, find the risk portfolio to find out the risk that investors will bear with equation (10), and the results are in Table 6.

**Table 6:** Table of portofolio risk

| Stock | $W_i \cdot \sigma_{ei}^2$ | $\beta_p^2$ | $\sigma_m^2$ |
|-------|---------------------------|-------------|--------------|
| BBCA  | 0.000006                  |             |              |
| BBRI  | 0.000053                  |             |              |
| BDMN  | 0.000036                  | 0.331115    | 0.000032     |
| BNII  | 0.000064                  |             |              |
| BTPN  | 0.000022                  |             |              |
|       | $\sigma_p^2$              | 0.000011    |              |

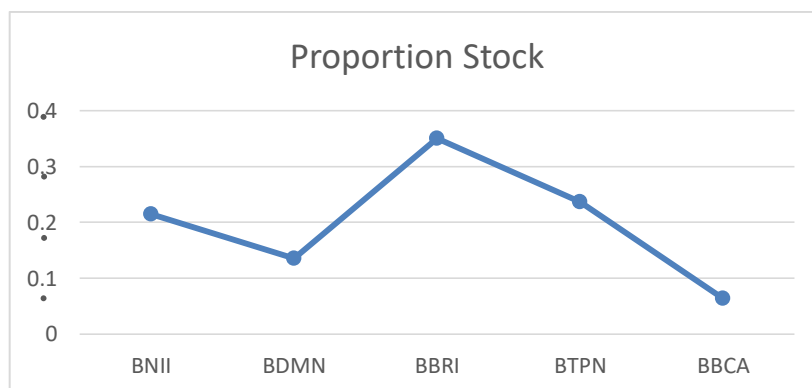
Based on Table 5, the portfolio risk that will be borne by investors is 0.0011% per day. This figure can be interpreted as a percentage of the total investment value that represents fluctuations in portfolio value. With such a small risk, investors can expect a high level of stability in their portfolio performance, as the anticipated change in value is relatively small.

## 5. Discussion

This study discusses determining the optimal portfolio of banking sector stocks in Indonesia. many bank stocks in Indonesia have IPO (Initial Public Offering) and bank stocks are interesting to discuss because their performance reflects general macroeconomic conditions. The research was conducted with the single index model method on the grounds that this method is considered easy with the simplification of the Markowitz model, the single index model only requires fewer calculations, taking into account the market conditions of the expected return and risk of a stock and its portfolio depending on market conditions.

Fund proportion ( $W_i$ ) measures the size distribution for the optimal stock portfolio. The value of  $W_i$  depends on  $Z_i$ , the bigger  $z_i$  the bigger  $W_i$ .  $W_i = 1$  means that the entire fund or 100% of the portfolio is directed to 5 bank stocks. The stock proportions can be seen in figure 1 with 0.0634 BBKA, 0.3502 BBRI, 0.1352 BDMN, 0.2143 BNII, and 0.2369 BTPN. The largest proportion is BBRI stock and the smallest proportion is BBKA stock.

Based on the calculation results, the expected return of the portfolio is 0.00165 and the portfolio risk that will be borne by investors is 0.0046%. With such a small risk, investors can expect a high degree of stability in their portfolio performance, as the anticipated change in value is relatively small. Although this number is small in percentage terms, understanding this risk is important for better investment decision-making. A visual interpretation of the portfolio results can be seen in Figure 1.



**Figure 1:** Proportion Stock

## 6. Conclusion

The results of this study obtained 5 optimal stocks that meet the criteria for optimal portfolio formation with each proportion of funds 21.43% (BNII), 13.52% (BDMN), 35.02% (BBRI), 23.69% (BTPN), and 6.34% (BBKA). The expected return of the optimal stocks is 0.152% and the portfolio risk that will be borne by investors is 0.0011% per day.

## References

- Baye, F. M., Douanla, J., & Fonkem, N. M. (2014). Impact of intellectual capital efficiency on the financial performance of financial institutions in Yaounde, Cameroon. *International Journal of Arts and Commerce*, 3(4), 166-187.
- Chong, Y. Y. (2004). *Investment risk management*. John Wiley & Sons.
- Francis, J. C., & Kim, D. (2013). *Modern portfolio theory: Foundations, analysis, and new developments*. John Wiley & Sons.
- Goetzmann, W. N., Brown, S. J., Gruber, M. J., & Elton, E. J. (2014). Modern portfolio theory and investment analysis. *John Wiley & Sons*, 237.
- Holland, J. (2003). Intellectual capital and the capital market—organisation and competence. *Accounting, Auditing & Accountability Journal*, 16(1), 39-48.
- Sharpley, R. (2018). *Tourism, tourists and society*. Routledge.
- Tandelilin, E. (2010). *Portfolio and investment theory and application*. Yogyakarta: Kanisius.