



# Determination of Collective Premiums for Seven Benefits of BPJS Employment Insurance JKK Program Using Poisson-Normal Aggregate Distribution

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## Abstract

Work Accident Insurance (JKK) is one of the programs of the Social Security Administering Agency (BPJS) for Employment. Insurance brokers need to make an initial estimate of the premium to determine the collective premium for JKK. Premium calculations can be done using the aggregate distribution method. The total loss of the insurance policy can be owned by the random variable of the aggregate distribution. In calculating the premium using the aggregate distribution, one of the principles that can be used is the standard deviation principle. Based on this principle, the amount of the premium can be calculated by the standard deviation of the aggregate distribution. This study uses the aggregate Poisson-Normal distribution to calculate the collective premium based on the seven benefits of the JKK BPJS Ketenagakerjaan Bojongsong program. The data used are the number of claim events and the number of claims from the seven benefit claims of JKK BPJS Ketenagakerjaan Bojongsong participants for the 2022 period. The principle used in calculating the collective premium with the aggregate distribution is the standard deviation principle. The results of the analysis show that the Poisson distribution is followed by claim frequency data and the Normal distribution is followed by the amount of the claim. This study shows that the amount of collective premium calculated tends to be greater than the amount of collective premium sourced from existing data of BPJS Ketenagakerjaan Bojongsong company. It is expected for insurance brokers and insurance companies to consider this study.

*Keywords:* JKK Insurance; aggregate distribution; Poisson; Normal; expectations; variance.

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## 1. Introduction

National development accelerates the nation's ability to prosper with other nations that are much more advanced so that Indonesian society becomes prosperous and prosperous (Ifurueze, 2013). One important element of national development is the workforce. Workers and their families must receive protection in accordance with human dignity (Ramadhan et al, 2013). Workers face risks, both accidents, losses, and death (Dong & Cui, 2010). Work accidents are unwanted and unexpected events that occur in the workplace and can result in material or life losses. Therefore, companies need to protect their workers so that they can work optimally. Workers in several companies are registered for health insurance to help ease the financial burden of paying for health costs. Companies often use collective health insurance so that the same insurance policy protects their workers. Often, companies face several obstacles in choosing insurance. These obstacles are that most of the benefits provided by insurance companies are almost the same, but the premium prices vary. In addition, the information provided is incomplete, making it even more difficult for companies to determine which health insurance product they want to choose. Therefore, in choosing the right insurance product, a broker service is needed, which is often referred to as a health insurance company customer. A broker is a company that provides services to provide opinions regarding insurance products that are in accordance with the needs and economic capabilities of a company (Sumarni et al, 2010).

The calculation of insurance premiums must provide benefits to both parties, both customers and companies. Aggregate distribution is one of the methods used in calculating insurance premiums. Several previous studies used aggregate distributions, such as those conducted by Kalfin et al., (Kalfin et al, 2023). Kalfin et al., (Kalfin et al, 2023) discussed the use of aggregate distributions for natural disaster insurance. The number of natural disaster events is calculated using the Poisson distribution and the amount of natural disaster losses is calculated using the Weibull distribution. MLE (Maximum Likelihood Estimation) is carried out to estimate the amount of loss and the number of events. The results of the estimation of the number of events and the amount of loss are then used to determine the collective risk and natural disaster insurance premiums. In this study, the standard deviation principle provides an estimate of the efficient premium value compared to the expected value principle. The estimated efficient premium value obtained is IDR150,420,679,458,000.00 with a loading factor of 1%.

In the study of Sumarni et al., (Sumarni et al, 2010) determined insurance premiums with the Poisson-Rayleigh aggregate distribution at the BPJS Ketenagakerjaan Langsa branch. The data used were monthly Work Accident Insurance (JKK) claim data at the BPJS Ketenagakerjaan Langsa branch for the period January to December 2019. The results of the study showed that the number of Work Accident Insurance (JKK) claims was 306 participants with a discrete distribution (Poisson) and the number of continuous distributions (Rayleigh) was IDR7,803,703 per participant. From the two studies above, it is necessary to pay attention to the number and magnitude of risks. For the number of claims, the model generally uses discrete distributions such as the Binomial distribution, Negative Binomial distribution, Geometric distribution, and Poisson distribution. Meanwhile, for the amount of claims, the model generally adopts continuous distributions such as Exponential distribution, Weibull distribution, Normal distribution, and Rayleigh distribution. In addition, Nugraha's research (Nugraha et al, 2021) conducted a study on the analysis of the amount of flood disaster management reserve fund contributions. In the study, the amount of collective risk losses and flood disaster risk management reserve fund contributions were calculated based on the distribution of the number of incidents and the amount of losses in each loss level group. The total incidents were calculated using a discrete distribution (Poisson) and the total losses were calculated using a continuous distribution (Lognormal & Gamma), the premium calculation was also calculated using the expectation principle. Inayah (Inayah, 2023) conducted a study on the estimation of gross premiums for dental care for group health insurance using the Poisson-Gamma aggregate distribution. The study discusses the application of the concept of sick health insurance in interval form for premium estimation. Yohandoko et al., (Yohandoko & Prabowo, 2023) conducted a study on gross premiums of life insurance stating that the results were different when calculated using the principles of expectation, variance and standard deviation. Calculation of gross premiums based on these three principles with a risk level ranging from 1% to 10% gave the highest gross premium calculation results which was 516,584 trillion rupiah. On the other hand, the lowest gross premium is 295,470 trillion rupiah. Based on the description above, in this study the author uses the aggregate distribution method for each of the seven benefits of the BPJS Employment JKK health service program. The initial stage is that the claim frequency data is modeled with a Poisson distribution, while for large claim data it is modeled with a Normal distribution. Furthermore, the parameters of each claim distribution are calculated. Based on the parameters of each claim distribution, the expectation and variance are calculated for each claim distribution. After that, the expectation and variance of each claim are used to calculate the premium using an aggregate distribution with the standard deviation principle. Research with this approach is expected to be more precise because the premium calculation considers the frequency and amount of claims from each incident when realizing the benefits provided.

## 2. Literature Review

### 2.1 Insurance

According to Law of the Republic of Indonesia Number 40 of 2014 concerning Insurance, insurance is an agreement between two parties, namely the policyholder and the insurance company, which is the basis for receiving a number of premiums from the insurance company as compensation. The benefits are intended to compensate the insured or policyholder for losses, damages, costs incurred, and so on. In addition, the compensation is used to pay if the insured dies.

#### 2.1.1 Individual insurance and collective insurance

Health insurance is divided into two types, namely individual health insurance and collective health insurance. Individual health insurance is used for families with a maximum of five family members. The premium paid is also relatively larger than collective insurance. Meanwhile, in collective health insurance, the number of participating individuals is greater, and the premium paid is smaller because the risk of claims is divided equally among all individuals and groups. The more members in a group, the smaller the premium paid. Income Protection (IP) is a number of assistance that will be distributed to the insured periodically, either weekly or monthly if the insured cannot or is not allowed to work due to illness or injury. If the client is sick and the illness is included in the policy, it can be paid for by Critical Illness Insurance (CII). Insurance products that deal with chronic illnesses but are temporary or long-term can be handled by Long-Term Care Insurance (LTCI).

### 2.1.2 Health insurance

Health insurance provides benefits in the event of an accident or illness that causes a person to spend a significant amount of money on health care needs. The premium paid to the company will be used as compensation for the benefits to be distributed. Generally, the premium is small and is paid regularly according to the agreement of both parties. There are three roles in health insurance, namely the insured or insured party (policyholder), the insurer or guarantor (insurance company), and the health facility of the insurer's partner.

### 2.2 Insurance Brokers

Most people assume that insurance brokers and insurance agents have the same duties. However, as regulated in Law Number 40 of 2014 and Government Regulation Number 63 of 1999, there are differences between the two. Insurance brokers are consultants for insurance customers (insured) who handle claims as a legal entity or company representing the insured. In contrast, insurance agents represent the company and are licensed to offer insurance products.

According to insurance brokers, on behalf of and for the insured, they look for insurance companies that can provide the most appropriate protection for their needs (Sumarni et al, 2010). Generally, this type of insurance is related to commercial activities, the protection is specific, and the coverage value is large, including collective health insurance.

In their duties, brokers look for the right insurance company for their clients according to their needs and financial conditions. The insurance they handle is generally commercial and has a large premium value, one of which is collective health insurance.

The premium rate is given as a gross premium when using a broker. This is because the gross premium rate includes some broker commission fees.

### 2.3 Basic Benefits of Health Insurance

BPJS Employment has several benefit categories, one of which is the Work Safety Guarantee (JKK) program. The JKK program, according to Government Regulation Number 82 of 2019, consists of three programs, namely the health service program, compensation in the form of money, and the return to work program. There are seven benefits of the JKK program, namely:

- a. STMB (Temporary Unable to Work) Benefits
- b. Partial disability benefits
- c. Death benefits
- d. Funeral benefits
- f. Transportation benefits
- g. Drug and treatment benefits
- h. Rehabilitation benefits

### 2.4 Distribution of Discrete Random Variables

Ibe stated that the number of values that can be calculated and accommodated in a variable is called a discrete variable (Ibe, 2012). The following is an explanation of the Poisson distribution

Kissel, et al., stated that certain events in a certain period of time can be calculated using the Poisson distribution (Kissel, 2016). The probability of an event  $p$  in the Poisson distribution interval is (Yates et al, 2014):

$$P(N = p) = e^{-\lambda} \frac{\lambda^p}{p!}, \quad (1)$$

for . The moment generating function of equation (1) is expressed as equation (2).  $p = 0, 1, 2, \dots$

$$M_N(t) = \sum_{p=0}^{\infty} e^{tp} e^{-\lambda} \frac{\lambda^p}{p!} = e^{-\lambda} \sum_{p=0}^{\infty} \frac{(\lambda e^t)^p}{p!} = \exp\{\lambda(e^t - 1)\}, \quad (2)$$

with  $e^t = r$  and the probability generating function of equation (1) is expressed as equation (3).

$$P(r) = \sum_{p=0}^{\infty} r^p e^{-\lambda} \frac{\lambda^p}{p!} = \exp\{\lambda(r - 1)\}. \quad (3)$$

The expectation and variance can be obtained by differentiating equation (2) with respect to  $t$ , and the results are expressed as equations (4) and (5).

$$M'_N(t) = \frac{d}{dt} \exp\{\lambda(e^t - 1)\},$$

$$M'_N(t) = \lambda e^t e^{\lambda(e^t - 1)}, \quad (4)$$

and if , then  $T = 0$

$$M'_N(0) = \lambda,$$

$$E(N) = \lambda \quad (5)$$

The second derivative of equation (2) for t can be written as equations (6) and (7).

$$M''_N(t) = \frac{d}{dt} \lambda e^t e^{\lambda(e^t - 1)},$$

$$M''_N(t) = (\lambda e^t)^2 e^{\lambda(e^t - 1)} + \lambda e^t e^{\lambda(e^t - 1)}, \quad (6)$$

and if , then  $T = 0$

$$M''_N(0) = \lambda^2 + \lambda,$$

$$E(N^2) = \lambda^2 + \lambda. \quad (7)$$

The values  $E(N) = \lambda$  and are obtained to obtain equation (8).  $E(N^2) = \lambda + \lambda^2$

$$Var(N) = E(N^2) - \{E(N)\}^2$$

$$= \lambda + \lambda^2 - \lambda^2$$

$$Var(N) = \lambda. \quad (8)$$

This is denoted by  $P \sim (\lambda)$  to describe the Poisson distribution with parameter  $\lambda$  (Dickson, 2005). The claim frequency can be modeled with a distribution that only requires the value of a non-negative discrete random variable (Oma). The Poisson distribution is suitable for a large number of observations but has a small probability of occurrence, such as car accidents, the number of factory machine errors in printing product images, the number of carbon dioxide particles in a certain time (Sumarni et al, 2010). The JKK claim frequency is data that has many observations and a small probability of occurrence. Therefore, the Poisson distribution is used to simulate the claim frequency in this study.

## 2.5 Distribution of Continuous Random Variables

One of the distributions of continuous random variables is the Normal distribution. The Normal distribution is one of the most widely used continuous probability distributions in various fields, such as statistics, economics, and engineering. This distribution plays an important role in data analysis and probability-based decision making. Many classical statistical methods assume that the data being analyzed comes from a normal distribution (Wackerly, Mendenhall, & Scheaffer, 2014).

The normal distribution has a probability density function pdf given by:

$$f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \times e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)} \quad (9)$$

Where (Montgomery & Runger, 2014):

- $\mu$  : mean of distribution,
- $\mu$  : mean of distribution,
- $\sigma^2$  : variance,
- $\sigma$  : standard deviation.

The expectation or expected value of the normal distribution is given by (Wackerly, Mendenhall, & Scheaffer, 2014):

$$E(x) = \mu \tag{10}$$

The derivation of this formula is obtained from the definition of expectation in probability (Montgomery, 2020):

$$E(X) = \int xf(x) dx \tag{11}$$

Since the normal distribution is symmetric about  $\mu$ , the result is  $\mu$ . The standard deviation of the normal distribution is given by (Montgomery, 2020):

$$\sigma = \sqrt{E[(X - \mu)^2]} \tag{12}$$

The derivation of this formula comes from the variance:

$$Var(X) = E[(X - \mu)^2] = \sigma^2 \tag{13}$$

So the standard deviation is the square root of the variance:

$$\sigma = \sqrt{\sigma^2} \tag{14}$$

The main characteristics of the normal distribution are:

- a. Symmetrical in nature : The normal distribution is bell-shaped and symmetrical about the mean.
- b. Four Main Statistical Parameters : Mean, median, and mode have the same value.
- c. Empirical Law : Approximately 68% of the data is within one standard deviation of the mean, 95% within two standard deviations, and 99.7% within three standard deviations (Devore, 2011).

### 2.6 Parameter Estimation with Maximum Likelihood Estimation (MLE)

According to Pratiwi et al. (2020) the probability density function with random variables  $X_1, X_2, \dots, X_n$  calculated on  $x_1, x_2, \dots, x_n$  is  $f(x_1, x_2, \dots, x_n | \mu)$  and this is referred to by the likelihood function for fixed  $x_1, x_2, \dots, x_n$ , the likelihood function is a function of  $\mu$  denoted by  $L(\mu)$ . If  $X_1, X_2, \dots, X_n$  is a random sample of  $f(x_i; \mu)$  then:

$$L(\mu) = \prod_{i=1}^n f(x_i; \mu). \tag{15}$$

The steps in determining the maximum are:

- a. Determine the likelihood function  $L(\mu) = f(x_1, x_2, \dots, x_n; \mu)$  with  $X_1, X_2, \dots, X_n$  mutual independence.
- b. The natural logarithm form of the likelihood function can be written as  $\ln L(\mu)$ .
- c. Maximize the previously  $\ln L(\mu)$  obtained function by finding the first derivative of the function and equating the resulting derivative to zero as follows.

$$\frac{\partial \ln L(\mu)}{\partial \mu} = 0. \tag{16}$$

- d. The solution to the equation in step 3, namely  $\hat{\mu}_{MLE}$ , is a candidate for the maximum likelihood estimator for  $\mu$ .

- f. Determine the second derivative of the natural logarithm of the likelihood function with respect to  $\mu$ . If  $\frac{\partial^2 \ln L(\mu)}{\partial^2 \mu} < 0$ , then proves  $\hat{\mu}_{MLE}$  that will actually maximize the likelihood function  $L(\mu)$ , so  $\hat{\mu}_{MLE}$  it is a maximum likelihood estimator for  $\mu$ .

### 2.7 Kolmogorov-Smirnov Test

The distribution suitability test is conducted to determine whether the assumed distribution pattern can reflect its population (Lubis, 2016). The Kolmogorov-Smirnov test is used to test the suitability of the observed sample distribution with a certain theoretical distribution.

The Kolmogorov-Smirnov test is the largest absolute difference between  $F_0(x)$  the cumulative distribution function of the population and  $F_k(x)$  the empirical distribution function of the sample, then

$$D = \max|F_k(x) - F_0(x)|. \tag{17}$$

The hypothesis to be tested is:

$H_0$  : data is distributed the same as the theoretical distribution

$H_1$  : data is not distributed the same as the theoretical distribution

with the decision criteria, namely if  $D < D_{tabel}$  then  $H_0$  is accepted, which means that the observed sample distribution has the same distribution as the theoretical distribution (Putri, 2020).

### 2.8 Aggregate Distribution

Aggregate distribution is one of the important concepts in insurance mathematics that is used to model the total losses experienced by an insurance company in a certain period. This modeling plays a crucial role in determining the amount of premium that must be charged to the insured and in the risk management of the insurance company (Sarabia et al., 2017).

The aggregate claim model consists of two main components, namely the number of claims and the claim size. The number ( $N$ ) of claims is the number of claim events in a period, while the claim size ( $X$ ) is the amount of loss or claim value per event. The total loss ( $S$ ) can be expressed as:

$$S = \sum_{i=1}^N X_i \tag{18}$$

where  $X_i$  is the magnitude of the  $i$ -th claim. In practice, the number of claims is often modeled using a Poisson distribution, which is appropriate for cases where claim events are assumed to occur randomly over a period of time (Sarabia et al., 2017).

For claim size  $X$ , various continuous distributions can be used, depending on the characteristics of the data. One commonly used distribution is the Normal distribution. In some insurance cases, the Normal distribution can provide a good approximation to claim size if the data is symmetrical and does not have too many extreme outliers.

Insurance premiums are determined based on the expected total loss ( $E(S)$ ) and added with a loading factor to anticipate the risk. The expected total loss is calculated as follows:

$$E(S) = E(N) \times E(X) \tag{19}$$

If the claim amount ( $X$ ) is normally distributed with mean and variance, then the expected claim amount is:  $\mu\sigma^2$

$$E(X) = \mu \tag{20}$$

Meanwhile, if the number of claims ( $N$ ) is distributed Poisson with parameter  $\lambda$ , then the expected number of claims is:

$$E(N) = \lambda \tag{21}$$

Thus, the total expected loss can be written as:

$$E(S) = \lambda \times \mu \tag{22}$$

The convolution method is used to determine the distribution of total losses ( $S$ ) based on the distribution of the number of claims ( $N$ ) and the claim size ( $X$ ). In the context of the Normal distribution, if the number of claims ( $N$ ) is distributed Poisson and the claim size ( $X$ ) is distributed Normally, then the distribution of total losses can be approximated by the Normal distribution as well, with appropriate parameters.

Considering these aspects, modeling aggregate distribution in insurance mathematics is very important in determining the amount of premium and in managing risk. Choosing the right distribution for the number of claims and the amount of claims, as well as the use of the convolution method, can help insurance companies in predicting total losses and setting appropriate premiums (Sarabia et al., 2017).

### 2.9 Determining Premiums Using the Standard Deviation Principle

Premium is a payment paid to the insurance party as compensation for losses incurred by the insured (Syavitri, 2018). There are several principles in calculating premiums. In this study, the principle of standard deviation value is used to calculate the premium value.

The principle of standard deviation is established as equation (22).

$$\Pi_x = E(S) + \theta \{V(S)\}^{\frac{1}{2}}, \theta > 0, \tag{23}$$

where  $\theta$  is the loading factor.

## 3. Methodology

### 3.1 Research Object

The object of this research is data taken from the BPJS Ketenagakerjaan office, Bojongsoang Branch, Bandung Regency, from January to December 2022. This office is located on Jalan Raya Bojongsoang Buah Batu Square, Bandung Regency. This BPJS Ketenagakerjaan office is a branch office in Bandung. In addition, the BPJS Ketenagakerjaan branch office in Bandung City is also located on Jalan Suci.

The data used in this study are data on the frequency of claims and the amount of claims of JKK BPJS Ketenagakerjaan participants. Complete data can be seen in Appendix 1. In this study, calculations were carried out using the Poisson-Normal aggregate distribution method with the help of Microsoft Excel software and Easyfit software.

### 3.2 Methods

This study uses an insurance premium estimation method based on the Poisson-Normal aggregate distribution. The main steps in this process include:

a. Distribution Modeling

The frequency of JKK claims is assumed to follow a Poisson distribution and the magnitude of large JKK claims is assumed to follow a Normal distribution.

b. Parameter Estimation

The distribution parameters are estimated using the Maximum Likelihood Estimation (MLE) method.

c. Distribution Suitability Test

Model fit testing was performed using the Kolmogorov-Smirnov method to ensure that the data conformed to the assumed distribution.

d. Calculation of Expectations and Variance

The expectation and variance of claim frequency are calculated using a predetermined formula. The expectation and variance of claim magnitude are also calculated according to the Poisson-Normal aggregate distribution model.

f. Insurance Premium Calculation

The total premium is calculated using a predetermined equation. Additional factors such as loading factors (10% for groups and 50% for individuals) also affect the premium calculation.

g. Analysis and Conclusion

The results of the estimation and data analysis are visualized in a flowchart to show the process systematically. This study aims to provide a more accurate approach in calculating insurance premiums by considering the distribution of claims and relevant risk factors.

## 4. Results and Discussion

### 4.1. Research data

The data used in this study is the 2022 BPJS Ketenagakerjaan JKK claim data for the Bojongsoang Branch. The data used has been approved by the management of the company concerned. The data content includes details of the frequency of claims and the amount of JKK benefit claims for Participants Who Are Temporarily Unable to Work (STMB), Functional Disability, Partial Disability, Death Benefits, Burials, Transportation Benefits, Medicine and Care, Rehabilitation, Scholarships, and Periodic Benefits. However, the data used in this study has a small value of 0, so the data can be easily processed. The data includes claims for STMB, Partial Disability, Death Benefits, Burials, Transportation, Medicine, and Care and Rehabilitation. Claim Frequency Data is the number of claim events with discrete variables. Claim Amount Data is the amount of money paid to participants with continuous random variables so that the data follows a continuous distribution.

The following is one of the claim frequency data and number of claims used in this study which is presented in Table 1.

**Table 1:** Table of claim frequency and death benefit claim size

No	Month	Death Benefits	
		Claim Frequency	Claim Size (IDR)
1	January	1	150,742,800
2	February	2	244.263.120
3	Line up	0	0
4	April	3	2,016,000,000
5	Possible	3	476,265,840
6	June	1	113,442,720
7	July	1	150,685,200
8	August	5	738,666,650
9	September	0	0
10	October	1	181.193.330
11	November	2	331,878,530
12	December	1	179,629,250
Total		20	4,582,767,440

The first thing to do is to declare the data as variables, as in Table 2.

**Table 2:**Benefit variable declaration table

Profit	Benefit Variables	Frequency of Benefit Claims	Claim Size Benefits
Compensation for death	$a$	$N_a$	$X_a$
STMB	$b$	$N_b$	$X_b$
Partial Disability	$c$	$N_c$	$X_c$
Burial	$d$	$N_d$	$X_d$
Transport	$e$	$N_e$	$X_d$
Treatment and Care	$f$	$N_f$	$X_e$
Rehabilitation	$g$	$N_f$	$X_f$

## 4.2 Data processing

In calculating the premium amount in this study, the expectation and variance of the aggregate distribution are estimated. Furthermore, the estimate is used to formulate the premium amount using the standard deviation principle premium model.

### 4.2.1 Frequency of model claims

The claim frequency can be calculated with discrete probability over several random events.

#### 4.2.1.1 Identifying claim frequency models

The distribution model was identified by creating a Probability Density Function (PDF) histogram of the claim frequency data, assisted by Excel software.

#### 4.2.1.2 Estimation of claim frequency parameters

Parameter estimation is done using the Maximum Likelihood Estimation (MLE) method. The Likelihood function of the Poisson distribution refers to equation (25), namely:

$$\frac{d \ln L(\lambda)}{d\lambda} = 0$$

$$\frac{d(-12\lambda \ln e + \sum_{i=1}^{12} n_i \ln \lambda - \ln \prod_{i=1}^{12} n_i!)}{d\lambda} = 0$$

$$-12 + \frac{\sum_{i=1}^{12} n_i}{\lambda} = 0$$

$$\hat{\lambda} = \frac{\sum_{i=1}^{12} n_i}{12}$$

Monthly periods containing many frequencies are not included in the calculation. Parameter calculations are performed to collect data on the frequency of death benefit claims.

$$\hat{\lambda}_a = \frac{n_1 + n_2 + n_4 + n_5 + n_6 + n_7 + n_{10} + n_{11} + n_{12}}{10}$$

$$\hat{\lambda}_a = \frac{1 + 2 + 3 + 3 + 1 + 1 + 5 + 1 + 2 + 1}{10}$$

$$\hat{\lambda}_a = \frac{20}{10}$$

$$\hat{\lambda}_a = 2.$$

Easyfit software calculates data parameters for the frequency of claims for other JKK benefits. Table 3 describes the parameters for each benefit.



**Table 3:**parameters for each benefit

Benefits Details	Parameter $\hat{\lambda}$
Compensation for death	2.0000
STMB	10.7500
Partial Disability	2.5000
Burial	2.0000
Transport	3.7300
Treatment and Care	120.7500
Rehabilitation	6.3000

#### 4.2.1.3 Test of suitability of claim frequency distribution

In testing the correct model distribution, a statistical test analysis is required using the Kolmogorov-Smirnov test assisted by Easyfit software with the following hypothesis:

$H_0$ : normally distributed data

$H_1$ : data is not normally distributed

The results of the Kolmogorov-Smirnov test at a 95% confidence level for death benefits are given in Table 4. Based on Table 4, the claim frequency data for death benefits follows a Poisson distribution. The results of the Poisson distribution goodness-of-fit test for each other benefit were also calculated and the results showed that all data showed that the claim frequency follows a Poisson distribution.

**Table 4:** Kolmogorov-Smirnov Goodness-of-Fit Test Results for Death Benefit Claim Frequency Data

Kolmogorov-Smirnov Goodness-of-Fit Test Results for Poisson Distribution		
Compensation for death	Test Statistic (D)	0.0797 years
	$D_{tabel}$	0.3754
	Results	$D < D_{tabel}$ or $0.0797 < 0.3754$
	Conclusion	$H_0$ accepted, claim frequency data has a Poisson distribution

#### 4.2.2 Model claim size

Next, the distribution of large data on BPJS Employment JKK benefit claims for the Bojongsong Branch will be identified.

##### 4.2.2.1 Identifying claim sizing models

The distribution of claims for each benefit was identified by looking at the histogram fit of the distribution function, which was processed using Excel software.

##### 4.2.2.2 Estimation of claim size parameters

Parameter estimation is done using the Maximum Likelihood Estimation (MLE) method. The Likelihood function for the claim size for each benefit from the Normal distribution refers to equation (28), namely:

$$\frac{1}{\sigma^2} \sum_{i=1}^{12} (\ln x_i - \mu)^2 = 0$$

$$\sum_{i=1}^{12} (\ln x_i - \mu)^2 = 0$$

$$\sum_{i=1}^{12} \ln x_i - \sum_{i=1}^{12} \mu = 0$$

$$\sum_{i=1}^{12} \ln x_i - 12\mu = 0$$

$$\hat{\mu} = \frac{\sum_{i=1}^{12} \ln x_i}{12},$$

and equation (29) is

$$\hat{\sigma}^2 = \frac{1}{12} \left( \sum_{i=1}^{12} \ln x_i^2 - 12 \left( \frac{\sum_{i=1}^{12} \ln x_i}{12} \right)^2 \right)$$

$$\hat{\sigma}^2 = \frac{1}{12} \left( \sum_{i=1}^{12} \ln x_i^2 - \frac{(\sum_{i=1}^{12} \ln x_i)^2}{12} \right).$$

Furthermore, because this calculation is complicated if done manually, the calculation of this parameter  $(\mu, \sigma)$  is processed with the help of Microsoft Excel software. The parameters for each benefit can be seen in Table 5

**Table 5:** Claim size distribution parameter values

Benefits Details	Parameter $\mu$	Parameter $\sigma$
Compensation for death	3.3809	0.8435
STMB	-0.0416	0.9492
Partial Disability	0.7199	0.9320
Burial	0.5193	0.5717
Transport	-3.5711	0.6991
Treatment and Care	3.8949	0.3767
Rehabilitation	-1.8819	0.6704

#### 4.2.2.3 Test of suitability of claim size distribution

Estimating the appropriate distribution model for large claim data is not sufficient by simply matching the shape of the curve, therefore statistical test analysis is required, the Kolmogorov-Smirnov test is carried out with the help of Easyfit software with the following hypothesis:

$H_0$ : data is normally distributed

$H_1$ : data is not Normally distributed

The results of the Kolmogorov-Smirnov test at a 95% confidence level for death benefit are given in Table 6.

Table 6: Results of the Kolmogorov-Smirnov Goodness-of-Fit Test for death benefit claim amount data

Kolmogorov-Smirnov Goodness-of-Fit Test Results for Normal Distribution		
Compensation for death	Test Statistic (D)	0.2069
	$D_{tabel}$	0.37543
	Results	$D < D_{tabel}$ or $0.2069 < 0.37543$
	Conclusion	$H_0$ accepted, claim size data is normally distributed

### 4.2.3 Aggregate Distribution Calculation

The first step in calculating the premium is to estimate the expected value and variance, referring to equations (5) and (8) for claim frequency and equations (10) and (12) for claim size. Next, the expected value and variance of the aggregate distribution are calculated by referring to equations (20) and (21).

For death benefits, the claims frequency data has a Poisson distribution with parameters  $\lambda_a = 2$  so that the probability density function has the form

$$P(N = x) = e^{-2} \frac{(2)^x}{x!}$$

based on the equations (5) and (8) are obtained and meanwhile for large claims data with a Lognormal distribution with parameters  $E(N_a) = 2$ ,  $V(N_a) = 2$ ,  $\mu_a = 3,3809$  and  $\sigma_a = 0,8435$ .

Based on equations (10) and (12) we get:

$$\begin{aligned} E(X) &= \mu \\ E(X_a) &= 3.3809 \\ V(X) &= \sigma^2 \\ V(X_a) &= (0.8435)^2 = 0,7115 \end{aligned}$$

For other benefit data, calculations are the same as the previous explanation and can be assisted with Microsoft Excel software. Expectation results and variance from claim frequency data are obtained in Table 7.

**Table 7:** Expected value and variance of claim frequency

Benefits Details	$E(N)$	$Var(N)$
Compensation for death	2.0000	2.0000
STMB	10.7500	10.7500
Partial Disability	2.5000	2.5000
Burial	2.0000	2.0000
Transportation	3.7300	3.4200
Medication and Treatment	120.7500	120.7500
Rehabilitation	6.3000	6.3000

The claim data for each benefit has a Normal distribution based on equations (10) and (12). The expected values and variances are obtained as explained in Table 8.

**Table 8:** Expected value and variance of claim size

Benefits Details	$E(X)$	$Var(X)$
Compensation for death	3.3809	0.7115
STMB	-0.0416	0.9010
Partial Disability	0.7199	0.8686
Burial	0.5193	0.3268
Transportation	-3.5711	0.4887
Medication and Treatment	3.8949	0.1419
Rehabilitation	-1.8819	0.4494

The expected value and variance of the aggregate distribution can be calculated using equations (2.22) and (2.24) to obtain

$$\begin{aligned} \widehat{E(S)} &= \widehat{E(P)} \cdot \widehat{E(X)}, \\ \widehat{E(S_a)} &= \widehat{E(P_a)} \cdot \widehat{E(X_a)}, \\ \widehat{E(S_a)} &= 2 \cdot 3.3809 = 6.7618 \\ \widehat{V(S_a)} &= (\widehat{E(X_a)})^2 \cdot \widehat{V(P_a)} + \widehat{E(P_a)} \cdot \widehat{V(X_a)}, \\ \widehat{V(S_a)} &= 24.2840 \end{aligned}$$

The estimated value of the aggregate distribution of expectations and variance can be calculated using equations (20) and (21) and assisted using Microsoft Excel software. The calculation results can be seen in Table 9.

**Table 9:** Expected value and variance of aggregate distribution

Benefits Details	$E(S)$	$V(S)$
Compensation for death	6.7618	24.2840
STMB	-0.4472	9.7041
Partial Disability	1.7998	3.4672
Burial	1.0386	1.1930
Transportation	-13.3202	45.4374
Medication and Treatment	470.3092	1848.9420
Rehabilitation	-11.8560	25.1432

**4.2.4 Calculation of Premium Value**

Based on data taken from Open Data Jabar, the number of JKK participants in Bandung City and Regency was 871,742 participants in 2022. It is assumed that the JKK BPJS Employment participants in the Bojongsong branch are 50% of the total JKK BPJS Employment participants in Bandung City and Regency. In that case, the JKK BPJS Employment participants in the Bojongsong branch are 435,871 people. Based on equation (22), the calculation of premium values for seven JKK benefits in the 2022 period is given in Table 10.

Table 10 shows that the total amount of individual premium contributions in a year for the seven JKK BPJS Employment benefits is IDR 150,426.00. The collective premium for each JKK benefit uses the Poisson-Normal aggregate distribution calculation tends to be larger with the existing data held by the BPJS Employment Bojongsong branch. Table 11 explains a comparison of existing data and research data.

**Table 10:** Collective premium value of JKK BPJS Employment Bojongsong Branch using the Poisson-Normal distribution

Benefits Details	Total Collective Contributions (in Rupiah)	Number of JKK participants	Individual Contribution per year (in Rupiah)
Compensation for death	72,545,875		166.44
STMB	1,356,852		3.11
Partial Disability	19,859,542		45.56
Burial	11,478,258	435,871	26.33
Transportation	126,461,301		290.13
Medication and Treatment	4,746,091,075		10,888
Rehabilitation	113,545,401		260.50

**Table 11:** Comparison of existing data and research data

Benefits Details	Collective premium contributions (Existing Data)	Collective premium contributions (Research Data)
Compensation for death	4,582,767,440	72,545,875
STMB	176,020,374	1,356,852
Partial Disability	324,929,590	19,859,542
Burial	200,000,000	11,478,258
Transportation	3,860,000	126,461,301
Medication and Treatment	6,315,445,878	4,746,091,075
Rehabilitation	18,482,500	113,545,401

## 5. Conclusion

This can be seen from the results of the premium calculation for work safety insurance using the Poison-Normal method, namely the premium tends to be larger than the existing data held by the Bojongsoang BPJS Employment branch. Therefore, this study aims to avoid causing a premium reserve deficit.

BPJS Employment is a form of socio-economic protection for workers, and its benefits are not only felt by the workers themselves, but also by their families. The government has established this program to provide socio-economic protection to workers through a social insurance mechanism. The amount of BPJS Employment contributions charged is 5.7% of the wages received per month. The JHT payment scheme uses a joint system: workers pay 2% and the company pays 3.7%.

The issue of National Health Insurance run by BPJS Health is always interesting news to study. Behind the many benefits received by the community from obtaining health services, there is often a budget deficit. Some of the causes of the BPJS deficit are only due to the imbalance between income and expenditure. Until now, the amount of payment claimed for health services is not comparable to the income from BPJS participant contributions. BPJS participant participation has not reached the desired target. As a result, income from contributions is not optimal. Another thing is that the amount of contribution set is lower than it should be, both for class I, II, and III. Although it had increased some time ago, but that is not the actual value.

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