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Optimization Model in Transportation Based on Linear Programming

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Abstract

This study discusses the development of optimization models in transportation costs and routes and resource distribution based on Linear programming using various methods. This study aims to improve logistics efficiency, maximize the utilization of transportation equipment, infrastructure, operations management, and minimize transportation costs. The methods used include data collection, data processing, and the application of mathematical models to determine the optimal route with iteration methods such as the Simplex Method or Simplex Algorithm, Modified Distribution Method (MODI), Vogel's Approximation Method (VAM), North-West Corner Method, Least Cost Method, and Initial Cost Minimum Method (ICMM). This study successfully shows that this method is able to reduce the cost of reducing carbon emissions, significantly reduce shipping costs and increase the efficiency of goods distribution that can be applied to complex distribution systems, support efficiency, and sustainability of transportation management. Using Linear programming and transportation methods to reduce SME costs and produce more efficient costs and fast solutions. In general,optimization supports economic development, efficiency and sustainability of transportation management.

Keywords: Optimization, Linear Programming, Transportation

1. Introduction

Transportation is one of the important elements in supporting logistics and distribution activities. Efficiency in planning transportation routes can reduce operational costs, travel time, and environmental impacts (Nazry et al., 2025). Transportation is the movement of people or goods from one place to another using a vehicle driven by humans or machines to make it easier for humans to carry out daily activities (Yusup, 2022).

Linear Programming (LP) is a mathematical method used to find the optimal solution (maximum or minimum value) of an objective function, which is a linear function of several variables, by considering a number of constraints that are also expressed in the form of linear equations or inequalities. Linear Programming helps find the best way to allocate limited resources to achieve certain goals, provided that the relationship between the variables and the goals is linear. According to George B. Dantzig (1947) who is called the father of linear programming defines linear programming as a technique that uses linear equations to find the best solution to an optimization problem, with the aim of maximizing or minimizing a linear objective function subject to linear limits.

The importance of efficiency in the distribution of logistics systems, transportation costs, and the challenges faced such as high costs, limited vehicle capacity, and other operational constraints. It is emphasized that the use of mathematical models, especially linear programming, can help optimize delivery routes so that costs can be minimized and efficiency increased.High transportation costs can reduce competitiveness and operational efficiency. Therefore, optimization methods are needed to reduce costs, increase efficiency, and support sustainable economic development.This study aims to develop the model and test its effectiveness in the context of goods distribution by applying various mathematical and analytical methods.transportation algorithms to achieve optimal solutions in the distribution of goods and services.

2. Literature review

Various previous studies such as Welly and Elva (2021) which used Linear Programming and optimization techniques in the field of logistics and distribution on transportation costs such as the simplex and MODI methods for initial solutions and increasing efficiency as wellTransportation algorithms such as Vogel's approximation, North-West Corner, Least Cost, and ICMM. The focus includes applications in vehicle route management, delivery scheduling, waste collection, and urban distribution. With the use of Linear programming for resource allocation. Through sensitivity analysis and loop iteration to ensure optimal solutions. The references show that this approach is effective in improving the efficiency and sustainability of distribution systems.Related literature shows that this combination of methods is effective in reducing costs and improving distribution efficiency, as well as supporting sustainability and reducing carbon emissions.

According to Nazry et al. (2025) developed a linear programming-based transportation route optimization model that aims to minimize total shipping costs while meeting various constraints such as vehicle capacity, customer demand, and delivery time constraints. This model is formulated in the form of objective functions and constraint systems, then solved using computational methods such as Python and Excel Solver.

Sugianto and Susanti (2021) also raised the problem of transportation cost optimization, but with a focus on small and medium enterprises (SMEs) in Batam City. In this study, transportation methods such as Vogel's Approximation Method and Russell's Approximation Method were used to find initial solutions in minimizing the distribution costs of bread products from several production branches to various markets. Sensitivity analysis was conducted to measure the resilience of the solution to changes in cost parameters.

The three studies confirm that the use of mathematical optimization methods in the context of logistics not only increases cost efficiency, but is also able to accommodate the complexity of distribution in various industrial scales, from SMEs to large distribution systems. The linear programming approach and transportation methods remain relevant and effective in providing optimal solutions to modern logistics problems.

3. Materials and Methods

3.1 Material

The main materials studied include data on distance between locations, travel time, vehicle capacity and available resources, the number of goods to be distributed (for example 1,500 loaves of bread) shipping needs, shipping costs and other operational constraints. This data is used as input in a mathematical model to determine the optimal route.

3.2 Method

The main method used in this research is linear programming with simplex and MODI to find the initial solution and optimization involving:

a. Field data collection and processing
b. Preparation of mathematical models in the form of objective functions and constraints
c. Implementation of the model using software such as Python (Pulp/OR-Tools) and MATLAB
d. Model solving to obtain optimal delivery route solutions.

Transportation Methods such as: North-West Corner Rule for initial solution, Least Cost Method and Vogel's Approximation to determine minimum cost allocation, Stepping stone and cuv iterations to improve the solution, Sensitivity analysis to ensure solution stability, and Transportation Algorithms such as ICMM and other algorithms for efficiency and accuracy.

The modeling used in this study is a linear programming model for transportation route optimization. This model includes objective functions and constraints that must be met in order to obtain an optimal solution.

Objective Function:
This function aims to minimize the total shipping cost, which is formulated as:

$$Z = \sum_{i=1}^{N} c_{ij} = \sum_{j=1}^{N} x_{ij}$$

where:

a. c_{ij} is the shipping cost from warehouse (i) to store (j)
b. x_{ij} is the number of items shipped from (i) to (j)

Constraints:
Warehouse capacity:

$$\sum_j x_{ij} \leq i \tag{1}$$

Shop requirements:

$$\sum_i x_{ij} = j \tag{2}$$

Non-negative:
$x_{ij} \geq 0$for all (3) i, j

This model also uses decision variables x_{ij} that must be positive or zero.

ResultsoptimizationThe transportation route is explained in this section which includes the allocation of goods from the starting point to the destination point, the total transportation cost, and the effectiveness analysis of the model used. A simple case study for the modeloptimizationThis linear programming based transportation route shows how linear programming helps solve problems.optimizationsystematically and structured.

4. Results and Discussion

This section explains the results of transportation route optimization including the allocation of goods from the starting point to the destination point, the total transportation cost, and the effectiveness analysis of the applied model. Linear programming helps to solve optimization problems in a systematic and structured way. A company has 3 warehouses, each of which has a certain stock of goods. There are 4 stores that need a certain amount of goods. Each warehouse can ship goods to the stores at a certain cost per unit of goods. The shipping capacity of the warehouse and the needs of the stores are known.

a. Warehouse and Capacity
   Warehouse A: capacity 100 units
   Warehouse B: capacity 150 units
   Warehouse C: capacity 120 units
b. Shops and Supplies
   Shop 1: requires 80 units
   Shop 2: requires 70 units
   Shop 3: requires 100 units
   Shop 4: requires 120 units

Table1: Transportation cost per unit from Warehouse to Store

	Shop 1	Shop 2	Shop 3	Shop 4
Warehouse A	4	6	8	10
Warehouse B	5	7	6	9
Warehouse C	9	4	7	6

To minimize transportation costs by arranging goods from warehouse to store according to warehouse capacity and store needs. Decision variables:
Suppose x_{ij} is the number of items shipped from warehouse i to store j.
x_{11} : number of items sent from Warehouse A to Store 1
x_{12} : number of goods sent from Warehouse A to Store 2
And so on for all combinations of warehouses and stores.

The calculation results using the northwest rule method do not provide optimum results. The method The northwest rule is a method of allocating supply to demand which is done randomly so that does not provide optimum results. The optimization results show that there is an allocation of costs transportation costs of M. M is a very large number so that the allocation of costs transportation of M is impossible to do. Northwest rule method does not provide optimum results because the method does not take into account the costs involved. the smallest.

The calculation results using the least cost and vogel methods give the same results. This is because both methods are carried out by finding the smallest transportation costs. Both methods have similarities, namely focusing on the smallest costs so that they aim to achieve optimum values. However, the vogel method is a better method because it provides an overview of basic and non-basic variables.

In linear programming, basic variables are variables that have values while non-basic variables are variables whose values are equal to zero. If we look at the results of advanced optimization using MODI, we get several variables that have allocations and do not have allocations. Variables that have allocations are basic variables and variables that do not have allocations are non-basic variables.

If the mathematical model of transportation is presented in the form of a linear program, it will have the form of the equation is as follows:

$$Min\ Z = c_{11}x_{11} + c_{22}x_{22} + \cdots + c_{ij}x_{ij} + Ms_1 + \cdots + Ms_{m+j} + Ms_n,$$

were :
c : cost
x : number of products
M : coefficient
And the constraint function is as follows:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} = s_i$$
$$\sum_{j=1}^{n}\sum_{i=1}^{m} x_{ij} = d_j$$

Based on this equation, the transportation method can be solved using simplex method. If the simplex equation is placed in a simplex table, it will the following results were obtained:

Table 2. Simplex method for solving transportation problems

Basic Variables	Equality	Coefficient								Right Side
		Z	…	x_{ij}	…	Z_i	…	Z_{m+j}	…	
Z	(0)	-1		C_{i+j}		M		M		0
	(1)									
Z_i	(i)	0		1		1				
Z_{m+j}	$(m+j)$	0		1				1		
	$(m+n)$									

After simplex iteration, the objective function equation will change to:

Table 3: Simplex iteration for transportation problems

Basic Variables	Equality	Coefficient								Right Side
		Z	…	x_{ij}	…	Z_i	…	Z_{m+j}	…	
Z	(0)	-1		C_{i+j} $-u_i$ $-v_j$		$M-u_i$		$M-v_j$		$-\sum_{i=1}^{m} s_i u_i$ $-\sum_{j=1}^{n} d_j v_j$

The northwest rule, least cost and vogel methods provide solutions to simplex iteration. If the table is compared to the simplex table, then the columns that do not get allocation are non-basic variables and the columns that get allocation are basic variables. If the variables are non-basic, then the value $c_{ij} - u_i - v_j$ will be zero and vice versa. Optimization tests are performed to ensure whether there are still non-basic variable coefficients that are still negative. If there are still coefficients non-basic variables that still have negative values, then iteration must still be carried out with using loops. The principle of loops is basically to change nonbasic variables that are still negative coefficient becomes positive or changes to a basic variable.

Through the research conducted, it was successfully shown that the linear programming-based route optimization model is able to contribute to significantly reducing shipping costs, increasing distribution efficiency, and can be applied to complex multi-warehouse and store distribution systems. Through the implementation of software such as Python and MATLAB which provide practical solutions, the use of optimization methods has succeeded in reducing transportation costs from the initial figure to around IDR 33,630–IDR 33,980. The resulting distribution solution is efficient, reducing total costs and increasing demand fulfillment. With transportation methods such as Vogel's approximation and iterative algorithms, it is able to provide fast and accurate solutions. The impact on reducing carbon emissions and increasing operational efficiency is also achieved. In the SME study, transportation cost optimization helps manage distribution more effectively and efficiently.

5. Conclusion

Through mmodeloptimizationLinear programming-based routes are effective in reducing costs and improving logistics efficiency. This model is flexible and can be applied to various distribution systems, supporting sustainability and more optimal transportation management.In using algorithms such as Vogel's approximation and the stepping stone method, it is possible to find minimum cost solutions quickly.optimizationThis supports economic development, sustainability, and environmental impact reduction. The application of mathematical models and transportation algorithms has been proven to be able to optimally manage supply and demand, as well as provide practical benefits for SMEs and logistics systems.

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