Available online at https://journal.rescollacomm.com/index.php/ijqrm/index

International Journal of Quantitative Research and Modeling

e-ISSN 2721-477X
p-ISSN 2722-5046

Vol. 3, No. 3, pp. 113-117, 2022

Investment Portfolio Optimization with a Mean-Variance Model Without Risk-Free Assets

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Abstract

Investment is an allocation of money, stocks, mutual funds, or other valuable resources provided by someone at the present time and held from being used until a specified period to get a profit (return). The higher the return received, the higher the risk. This study studied the Mean-Variance investment portfolio optimization model without risk-free assets to obtain the optimum portfolio. Five shares are used, namely BMRI, AMRT, SSMS, MLPT, and ANTM. The research results obtained optimal portfolio stocks with respective weights BMRI = 0.45741; AMRT=0.17852; SSMS=0.23300; MLPT=0.08475 and ANTM=0.04632. An optimal portfolio composition produces an average return = 0.00207 and variance = 0.00020.

Keywords: Risk, portfolio optimization, mean-variance

1. Introduction

Investment is a delay in current consumption to be included in productive assets for a certain period of time (Ristianawati et al., 2021). Tandelilin (2010) states that investment is a commitment to invest several funds or other resources at this time to obtain several benefits in the future. Investors will choose productive stocks while donating their capital. The productive stocks in question have the greatest returns at certain risks. The portfolio model that emphasizes the return value and portfolio risk is the Markowitz model (Ivanova & Dospatliev, 2017). The Markowitz model is also known as the Mean-Variance model (Deng & Lin, 2010).

In investing, investors can choose to invest their funds at risk, either in non-free assets or in, risk-free assets, or a combination of both assets. Non-risk-free assets are sets for which the actual future rate of return has some bearing. One example of a riskless asset is a stock (Pirvu & Schulze, 2012).

This discussion aims to show the analysis of the Mean-Variance investment portfolio optimization model without risk-free assets. The analysis of portfolio risk and return calculations are then used to determine the weight composition (proportion) of fund allocation in each of the assets that make up the optimum portfolio. It is hoped that it will be useful to add to the literature on investment optimization models and be used as an alternative for investors in optimizing their investment portfolios.

2. Materials and Methods

2.1. Materials

The stocks used in this portfolio are shares of Bank Mandiri (BMRI), Sumber Alfaria Trijaya (AMRT), Sawit Sumbermas Sarana (SSMS), Multipolar Technology (MLPT), and Aneka Tambang (ANTM). Each share was taken from 24 May 2021 – 20 May 2022.

2.2. Methods

In this portfolio, the Mean-Variance method will be used without risk-free assets. Suppose there are N risk-free assets with returns $r_1, r_2, \ldots, r_N$. Assuming that the first and second moments of $r_1, r_2, \ldots, r_N$ exist, the transposed vector of the expected return value is expressed as $\boldsymbol{\mu}^T = (\mu_1, \mu_2, \ldots, \mu_N)$ with $\mu_i = E[r_i], i = 1,2,\ldots,N$ and the covariance matrix is expressed as $\Sigma = (\sigma_{ij})$ where $\sigma_{ij} = Cov(r_i, r_j), i,j = 1,2,\ldots,N$. If the portfolio return is r_p with the transpose weight vector $\mathbf{w}^T = (w_1, w_2, \ldots, w_N)$, and the condition is $\sum_{i=1}^{N} w_i = 1$, then the expected portfolio return using vector notation can be expressed as:

$$\mu_p = E[r_p] = \boldsymbol{\mu}^T\mathbf{w} = \mathbf{w}^T\boldsymbol{\mu} \tag{1}$$

and the portfolio variance can be expressed as:

$$\sigma_p^2 = Var(r_p) = \mathbf{w}^T\Sigma\mathbf{w} \tag{2}$$

In Mean-Variance optimization, the efficient portfolio is defined as follows.
Definition: A portfolio p^* is (mean-variance) efficient if there is no portfolio p with $\mu_p \geq \mu_{p^*}$ and $\sigma_p^2 < \sigma_{p^*}^2$ (Panjer et al., 1998; Rupert, 2004).

To get an efficient portfolio, it means that you have to solve the portfolio optimization problem as follows:

$$\text{maximize } \{2\tau\boldsymbol{\mu}^T\mathbf{w} - \mathbf{w}^T\Sigma\mathbf{w}\} \tag{3}$$

$$\text{condition } \mathbf{e}^T\mathbf{w} = 1$$

with $\mathbf{e}^T = (1,2,\ldots,N)$, $\boldsymbol{\mu}^T\mathbf{w} = \mathbf{w}^T\boldsymbol{\mu}$, and $\mathbf{w}^T\mathbf{e} = \mathbf{e}^T\mathbf{w}$. The Lagrange function of the equation above, where λ is the multiplier, can be expressed as follows:

$$L(\mathbf{w}, \lambda) = (2\tau\mathbf{w}^T\boldsymbol{\mu} - \mathbf{w}^T\Sigma\mathbf{w}) + \lambda(\mathbf{w}^T\mathbf{e} - 1) \tag{4}$$

The above equation uses the necessary conditions of the Kuhn-Tucker theorem $\frac{\partial L}{\partial \mathbf{w}} = 0$ and $\frac{\partial L}{\partial \lambda} = 0$, got:

$$\frac{\partial L}{\partial \mathbf{w}} = 2\tau\boldsymbol{\mu} - 2\Sigma\mathbf{w} + \lambda\mathbf{e} = 0 \tag{5}$$

$$\frac{\partial L}{\partial \lambda} = \mathbf{w}^T\mathbf{e} - 1 = 0 \tag{6}$$

Equation $\frac{\partial L}{\partial \mathbf{w}}$ multiple Σ^{-1} and state in $\mathbf{w}$, then multiply the result $\mathbf{e}^T$. After the solution is done, it is obtained:

$$\mathbf{w} = \frac{1}{\mathbf{e}^T\Sigma^{-1}\mathbf{e}}\Sigma^{-1}\mathbf{e} + \tau\left\{\Sigma^{-1}\boldsymbol{\mu} - \frac{\mathbf{e}^T\Sigma^{-1}\boldsymbol{\mu}}{\mathbf{e}^T\Sigma^{-1}\mathbf{e}}\right\}; \tau \geq 0 \tag{7}$$

When $\tau = 0$ it produces a minimum variance portfolio with weights:

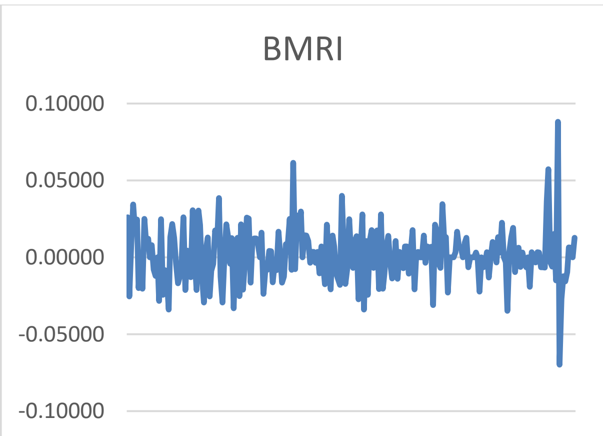
$$\mathbf{w}^{Min} = \frac{1}{\mathbf{e}^T\Sigma^{-1}\mathbf{e}}\Sigma^{-1}\mathbf{e} \tag{8}$$

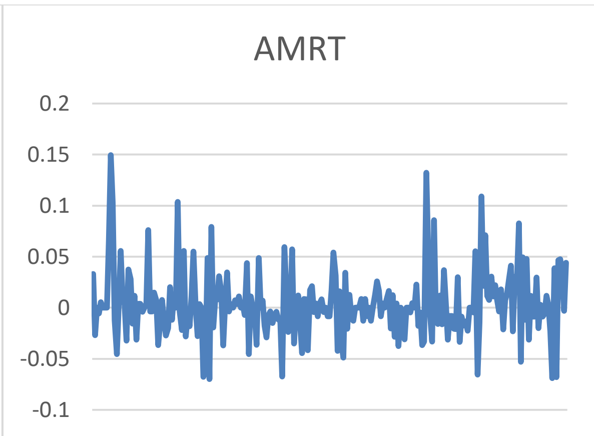
Furthermore, the researchers expanded the Mean-Variance investment portfolio optimization model by including risk-free assets.

3. Results and Discussion

3.1. Stock data analysis

Data of 5 (five) shares as follows:





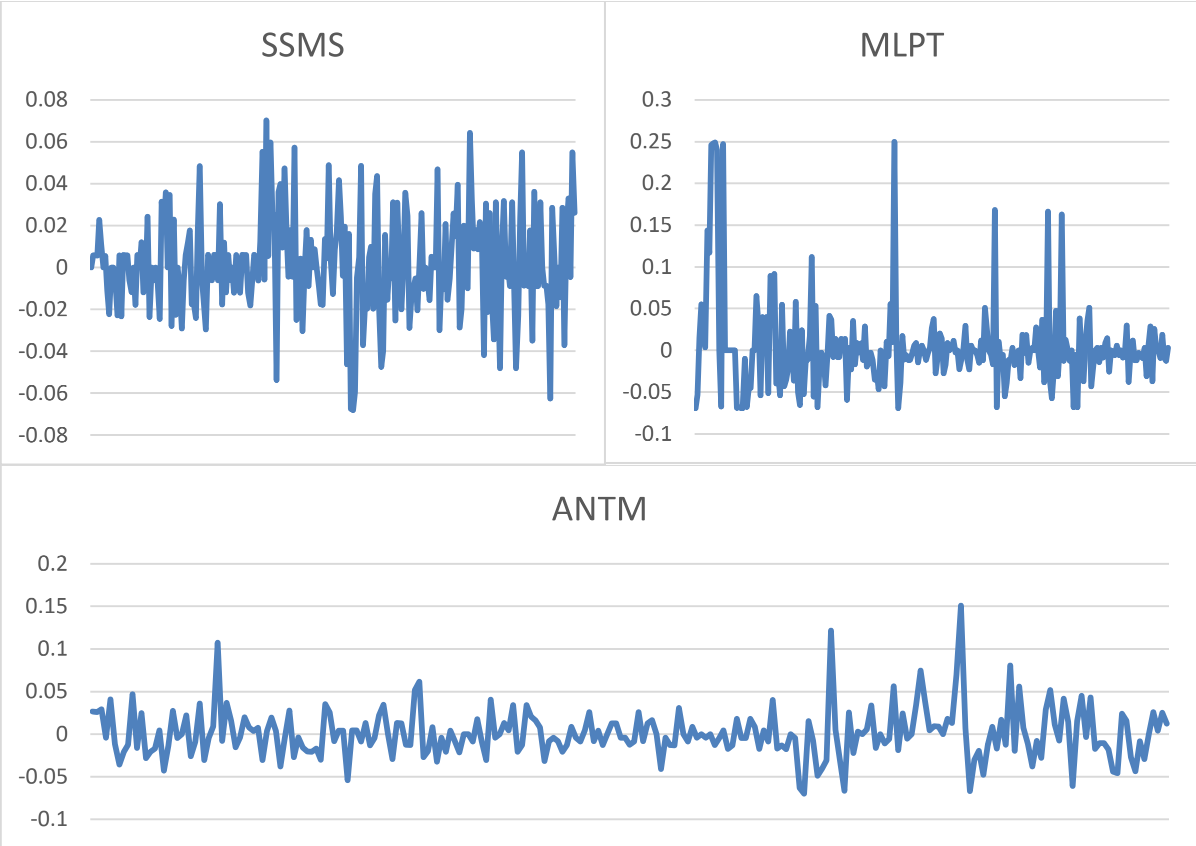


Figure 1: Data of 5 (five) shares

Table 1: 5 (five) shares used

Code	Standard Deviation (σ)	Expectations/Average ($\boldsymbol{\mu}$)	Variance ($\boldsymbol{\sigma}^2$)	Ratio $\left(\frac{\boldsymbol{\mu}}{\boldsymbol{\sigma}^2}\right)$
BMRI	0.01762	0.00152	0.00031	4.91283
AMRT	0.03272	0.00332	0.00107	3.10130
SSMS	0.02404	0.00160	0.00058	2.77038
MLPT	0.05422	0.00441	0.00294	1.50075
ANTM	0.02871	0.00079	0.00082	0.95379

Of the five stocks in Table 1, then the estimated value of the covariance between stocks is determined.

Table 2: The 5-stock covariance estimator used

No	Code	BMRI	AMRT	SSMS	MLPT	ANTM
1	BMRI	0.00031	0.00006	0.00006	0.00003	0.00007
2	AMRT	0.00006	0.00107	0.00004	0.00037	0.00016
3	SSMS	0.00006	0.00004	0.00058	-0.00001	0.00015
4	MLPT	0.00003	0.00037	-0.00001	0.00293	0.00001
5	ANTM	0.00007	0.00016	0.00015	0.00001	0.00082

3.2. Formation of Investment Portfolio Optimization with a Mean-Variance Model Without Risk-Free Assets

From the 5 (five) stocks used in Table 1., the estimator of the average value $\boldsymbol{\mu}_i, (i = 1,2,\dots,5)$ is formed by the average transpose vector

$$\boldsymbol{\mu}^T = (0.00152 \quad 0.00332 \quad 0.00160 \quad 0.00441 \quad 0.00079),$$

then the unit transpose vector is formed $\mathbf{e}^T = (1 \quad 1 \quad 1 \quad 1 \quad 1)$. Furthermore, from Table 2, the estimator of the variance value is $\sigma_i^2, (i = 1,2,\dots,5)$ with the covariance estimator between stock returns. Used to form the covariance matrix $\sum$.

$$\Sigma = \begin{bmatrix} 0.00031 & 0.00006 & 0.00006 & 0.00003 & 0.00007 \\ 0.00006 & 0.00107 & 0.00004 & 0.00037 & 0.00016 \\ 0.00006 & 0.00004 & 0.00058 & -0.00001 & 0.00015 \\ 0.00003 & 0.00037 & -0.00001 & 0.00293 & 0.00001 \\ 0.00007 & 0.00016 & 0.00015 & 0.00001 & 0.00082 \end{bmatrix}$$

$$\sum^{-1} = \begin{bmatrix} 3380.75 & -146.32 & -316.51 & -12.55 & -211.06 \\ -146.32 & 1020.02 & -13.19 & -128.08 & -185.51 \\ -316.51 & -13.19 & 1849.60 & 11.53 & -296.81 \\ -12.55 & -128.08 & 11.53 & 357.95 & 21.62 \\ -211.06 & -185.51 & -296.81 & 21.62 & 1325.71 \end{bmatrix}$$

The inverse covariance matrix $\sum^{-1}$ is used to calculate the composition of the efficient portfolio weights.

3.3. Investment Portfolio Optimization Process

The problem of Mean-Variance optimization without risk-free assets is compiled using the vectors $\boldsymbol{\mu}^T$, $\mathbf{e}^T$, and the matrix $\sum^{-1}$, the weight vector $\mathbf{w}$ is calculated using the previous equation. The risk tolerance τ with the condition that $\tau \geq 0$ in the investment portfolio optimization is simulated by taking several values that meet the requirements $\mathbf{e}^T\mathbf{w} = 1$. Taking risk tolerance is stopped if, for a risk tolerance value after being substituted into the previous equation, it produces a weight that is not a positive real number and fulfills $\mathbf{e}^T\mathbf{w} = 1$.

Table 3: The process of optimizing a Mean-Variance investment portfolio without risk-free assets

T	BMRI	AMRT	SSMS	MLPT	ANTM	$\mathbf{w}^T\mathbf{e}$	μ_p	σ_p^2	μ_p/σ_p^2
0	0.41405	0.25538	0.23653	0.12295	-0.02890	1	0.00238	0.00025	9.51967
0.1	0.42272	0.24000	0.23582	0.11531	-0.01386	1	0.00232	0.00024	9.75004
0.2	0.43139	0.22463	0.23512	0.10767	0.00119	1	0.00226	0.00023	9.94883
0.3	0.44006	0.20926	0.23441	0.10003	0.01623	1	0.00220	0.00022	10.10779
0.4	0.44874	0.19389	0.23371	0.09239	0.03128	1	0.00213	0.00021	10.21854
0.5	**0.45741**	**0.17852**	**0.23300**	**0.08475**	**0.04632**	**1**	**0.00207**	**0.00020**	**10.27300**
0.6	0.46608	0.16314	0.23229	0.07711	0.06137	1	0.00201	0.00020	10.26408
0.7	0.47476	0.14777	0.23159	0.06947	0.07641	1	0.00195	0.00019	10.18623
0.8	0.48343	0.13240	0.23088	0.06183	0.09146	1	0.00189	0.00019	10.03614
0.9	0.49210	0.11703	0.23018	0.05419	0.10650	1	0.00183	0.00019	9.81317
1	0.50078	0.10165	0.22947	0.04655	0.12155	1	0.00177	0.00019	9.51967
1.1	0.50945	0.08628	0.22877	0.03891	0.13659	1	0.00171	0.00019	9.16096
1.2	0.51812	0.07091	0.22806	0.03127	0.15164	1	0.00165	0.00019	8.74498
1.3	0.52680	0.05554	0.22735	0.02363	0.16668	1	0.00159	0.00019	8.28179
1.4	0.53547	0.04016	0.22665	0.01599	0.18173	1	0.00153	0.00020	7.78273
1.5	0.54414	0.02479	0.22594	0.00835	0.19677	1	000147	0.00020	7.25969
1.6	0.55282	0.00942	0.22524	0.00071	0.21182	1	0.00140	0.00021	6.72422
1.7	**0.56149**	**-0.00595**	**0.22453**	**-0.00693**	**0.22686**	**1**	**0.00134**	**0.00022**	**6.18700**

A series of efficient portfolios are on the efficient frontier. The efficient frontier is an efficient surface on which portfolios are located and whose returns are commensurate with the risks. The efficient frontier curve and the ratio between the average return and the portfolio variance are shown in the following Figure 2.

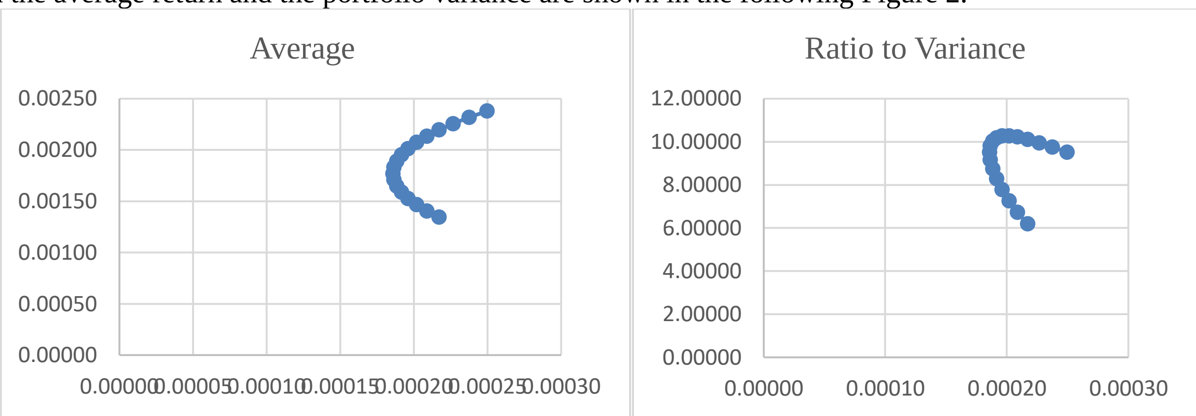


Figure 2: The efficient frontier curve and the ratio between the average return and the portfolio variance

3.4. Discussion

Analysis of the results of the optimization process is carried out by paying attention to the results in Table 3 which are explained as follows:

a. Risk tolerance, taking risk tolerance only for values 0≤τ≤1.6. This is because the value of risk tolerance τ> 1.6 produces a negative weight.

b. The minimum portfolio obtained an average return of 0.00238 with a variance of 0.00025.
c. The maximum portfolio obtained an average return of 0.00140 with a variance of 0.00021.
d. The optimum portfolio obtained an average return of 0.00207 with a variance of 0.00020.
e. The optimum portfolio weight, the proportion of shares BMRI = 0.45741; AMRT=0.17852; SSMS=0.23300; MLPT=0.08475 and ANTM=0.04632.

4. Conclussion

The Mean-Variance model without risk-free assets can be used to determine the optimum portfolio weight. There are five shares used, namely BMRI, AMRT, SSMS, MLPT, and ANTM shares, with optimum weights sequentially 0.45741, 0.17852, 0.23300, 0.08475, and 0.04632. With the average return and optimum variance, respectively 0.00207 and 0.00020.

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