Available online at https://journal.rescollacomm.com/index.php/ijqrm/index

International Journal of Quantitative Research and Modeling

e-ISSN 2721-477X
p-ISSN 2722-5046

Vol. 3, No. 4, pp. 181-188, 2022

De Moivre Law Application for the Construction of Mortality Tables Based on Indonesian Mortality Tables 2019

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Abstract

The mortality table or often referred to as the life table is the main instrument used by actuaries in building premium and reserve structures for life insurance products, annuities, and pension programs. The mortality table provides a complete description of the mortality rate and life expectancy and shows the pattern of death of a group of people born at the same time based on the age they have reached and plays an important role as a basis for calculating the level of life expectancy in the future. This article aims to find out how to construct a mortality table with reference to the 2019 TMI for men with de Moivre's Law. In the results of the construction with de Moivre's law, the lowest q_x value occurred at the age of 0 years, namely q_x= 0.00900901, while the highest q_x value occurred at the age of 110 years, namely q_x = 1. Based on the construction of the q_x value in the 2019 TMI for men using de Moivre's law, which is compared with the q_x value in the 2019 TMI for men, the results tend to be the same.

Keywords: de Moivre's Law, mortality table, TMI 2019.

1. Introduction

The mortality table or often referred to as the life table is the main instrument used by actuaries in building premium and reserve structures for life insurance products, annuities and pension programs (Pitacco, 2014). The mortality table represents a schema that presents mortality data in terms of probabilities and encapsulates the simple idea that the effect of gradual mortality on shrinking populations can be presented in tabular form.

The role of mortality tables is not only important in actuarial science. Mortality tables are a traditional work tool in the field of demography and have an important role in the field of epidemiology (Utomo, 2021). In demography, mortality tables are often used for descriptive purposes in order to compare mortality rates at different ages, genders, races, times, and places (Thomas et al., 2018), while epidemiologists use mortality tables to determine associated risk factors with morbidity and mortality rates (Handayani et al., 2016).

Within 3 decades, Indonesia has had 4 mortality tables known as the Indonesian Mortality Tables (TMI), with TMI IV as the latest mortality table which was released in October 2019. In accordance with the continuous changes in the demographic structure (Utomo, 2021), the mortality table really needs to be updated, however, it needs to be tested whether the latest mortality table has shown a significant change in the death rate.

The mortality table is a description of the life history of a person or individual with a hypothesis that the number gradually decreases due to death. The mortality table itself consists of two types, namely the complete mortality table and the summary mortality table. The complete mortality table is a complete and detailed mortality table according to one year's age. Meanwhile, the summary mortality table is a mortality table covering all ages, but the details are according to a certain time level, for example five years or ten years. The mortality table that will be used in this article refers to the 2019 TMI for men (Signs, 2015).

In constructing mortality tables, there are several laws that can be used, such as Gompertz, Makeham, de Moivre, and Weibull's laws. However, in this article the author will use de Moivre's Law. De Moivre's law is a mortality law that determines the acceleration of mortality obtained from a uniform distribution. De Moivre's law can be used as a determination of the acceleration of mortality, its probability density function can also be used to determine a person's chance of death and chance of living (Prionggo et al., 2022; Carey, 2001; Rabinovich et al., 2010).

The formulation of the problem that will be studied in this article is how to construct a mortality table with reference to the 2019 TMI for men with de Moivre's Law. The construction is preceded by parameter estimation contained in de Moivre's Law.

Parameter estimation in De Morvie's law was carried out by Mitus (2016) using the maximum likelihood method. In his research, Mitus (2016) also conducted a comparative analysis of the survival function by applying de Moivre's law and Gompertz's law. As for the most recent study, Aulia et al., (2022) conducted parameter estimates using the New Jersey method and applied de Moivre's law in constructing the mortality table. The purpose of writing this article is to find out how to construct a mortality table with reference to the 2019 TMI for men with de Moivre's Law.

2. Literature Review

2.1. Force of Mortality

Mortality or death is one of the three demographic components that can affect the number and age composition of the population, apart from birth/fertility and migration. The high or low level of population mortality will not only affect population growth, but also is a barometer of the level of population health.

In the mortality table, l_x only describes the state for positive integer x. In fact, as time goes by the number always decreases so that in time intervals, eg $[0, \omega]$, it is possible to perform the differentiation function and x does not have to be a positive integer. During the time interval Δt the number of people who died at the age of $x + \Delta t$ years ${}_{\Delta t}d_x = l_x - l_{x+\Delta t}$.. From this number of deaths, the part for one year is $\frac{l_x - l_{x+\Delta t}}{l_x \Delta t}$, then this result is divided by l_x at the beginning of the year, so one year mortality rate is obtained for each time interval Δt and expressed in the form as the mortality rate in equation (1):

$$\text{Mortality rate} = \frac{l_x - l_{x+\Delta t}}{l_x \Delta t} \tag{1}$$

According to Futami (1993), if $\Delta t \to 0$, the mortality rate in Equation (1) is called the acceleration of mortality (force of mortality) and is denoted by μ_x, namely:

$$\mu_x = \lim_{\Delta t \to 0} \frac{l_x - l_{x+\Delta t}}{l_x \Delta t}$$

$$\mu_x = -\lim_{\Delta t \to 0} \frac{1}{l_x} \left(\frac{l_{x+\Delta t} - l_x}{\Delta t} \right) \tag{2}$$

$$\mu_x = -\frac{1}{l_x} \cdot \frac{d}{d_x} (l_x)$$

$$\mu_x = -\frac{d \ln l_x}{dx}. \tag{3}$$

2.2. Mortality Table

According to Futami (1993), if d_x is the number of individuals who died in the age interval $(x, x+1)$ years and l_x is the number of individuals who lived in the age interval $(x, x+1)$ years, then the value of q_x can be obtained which is the probability of an individual dying in the age interval $(x, x+1)$ years, provided that the individual lives at the beginning of the interval, namely:

$$q_x = \frac{d_x}{l_x}$$

$$= \frac{l_x - l_{x+1}}{l_x}. \tag{4}$$

From the probability of an individual dying in the age interval $(x, x+1)$ the probability of surviving in the interval $(x, x+1)$ or surviving to the age of $x + 1$ can be obtained, namely:

$$p_x = 1 - q_x$$

$$= \frac{l_{x+1}}{l_x}. \quad (5)$$

The expected probability of surviving to t years given the condition of living at age x is

$$ {}_tp_x = \frac{l_{x+t}}{l_x} \quad (6)$$

And the probability that an individual aged x years old will die in t years is

$$ {}_tq_x = 1 - {}_tp_x$$

$$= 1 - \frac{l_{x+t}}{l_x}$$

$$= \frac{l_x - l_{x+t}}{l_x}. \quad (7)$$

With
l_x : the number of people living at the age of x years
d_x : the number of people who died between the ages of x and $x + 1$ year
p_x : the probability that a person currently aged x years will survive to age $x + 1$
q_x : the probability that a person who is currently aged x years old dies before age $x + 1$

In this study, the 2019 TMI adjustment was used with de Moivre's law for male sex. Below (Table 1) the 2019 TMI data issued by the Indonesian Actuary Association (PAI), respectively for male and female gender.

Table 1: Indonesian Mortality Table 2019

Age	Male	Female	Age	Male	Female	Age	Male	Female
0	0.00524	0.00266	38	0.00139	0.00100	76	0.02369	0.01879
1	0.00053	0.00041	39	0.00155	0.00108	77	0.02738	0.02030
2	0.00042	0.00031	40	0.00175	0.0011 8	78	0.03130	0.02326
3	0.00034	0.00024	41	0.00193	0.00128	79	0.03693	0.02880
4	0.00029	0.00021	42	0.00216	0.00141	80	0.04518	0.03569
5	0.00026	0.00020	43	0.00241	0.00154	81	0.05527	0.04208
6	0.00023	0.00022	44	0.00270	0.00169	82	0.06732	0.04907
7	0.00021	0.00023	45	0.00302	0.00187	83	0.08228	0.05520
8	0.00020	0.00022	46	0.00338	0.00209	84	0.09478	0.06086
9	0.00010	0.00021	47	0.00377	0.00230	85	0.10465	0.06715
10	0.00019	0.00019	48	0.00418	0.00253	86	0.11533	0.07318
11	0.00019	0.00018	49	0.00461	0.00277	87	0.12698	0.08155
12	0.00019	0.00020	50	0.00508	0.00305	88	0.13947	0.09045
13	0.00020	0.00022	51	0.00556	0.00335	89	0.15271	0.10001
14	0.00023	0.00023	52	0.00609	0.00368	90	0.16659	0.10913
15	0.00027	0.00023	53	0.00667	0.00403	91	0.17991	0.11521
16	0.00031	0.00024	54	0.00727	0.00442	92	0.19390	0.12499
17	0.00037	0.00024	55	0.00789	0.00483	93	0.20874	0.13826
18	0.00043	0.00025	56	0.00847	0.00524	94	0.22451	0.15451
19	0.00047	0.00026	57	0.00898	0.00563	95	0.24126	0.17429
20	0.00049	0.00027	58	0.00939	0.00601	96	0.25715	0.19155
21	0.00049	0.00028	59	0.00971	0.00637	97	0.27419	0.20596
22	0.00049	0.00030	60	0.00999	0.00671	98	0.29249	0.22227
23	0.00049	0.00032	61	0.01024	0.00707	99	0.31215	0.23736
24	0.00050	0.00034	62	0.01046	0.00746	100	0.33331	0.25810
25	0.00052	0.00038	63	0.01071	0.00788	101	0.35163	0.28068
26	0.00055	0.00042	64	0.01104	0.00833	102	0.37132	0.30562
27	0.00060	0.00046	65	0.01146	0.00883	103	0.39250	0.33315
28	0.00065	0.00049	66	0.01199	0.00940	104	0.41527	0.36369
29	0.00070	0.00052	67	0.01260	0.01005	105	0.43973	0.39318
30	0.00075	0.00056	68	0.01329	0.01076	106	0.46602	0.42883
31	0.00081	0.00060	69	0.01405	0.01150	107	0.49429	0.46604
32	0.00087	0.00064	70	0.01485	0.01229	108	0.52467	0.50427
33	0.00093	0.00069	71	0.01574	0.01314	109	0.55733	0.54477
34	0.00099	0.00074	72	0.01670	0.01406	110	0.59244	0.58702
35	0.00107	0.00080	73	0.01777	0.01508	111	1.00000	1.00000
36	0.00116	0.00086	74	0.01895	0.01620			
37	0.00127	0.00093	75	0.02026	0.01743			

2.3. de Moivre's law

De Moivre's law was discovered by a scientist named Abraham de Moivre in 1729. According to Futami (1993), de Moivre's law explains that the number of people living at age x can be expressed in the following formula:

$$l_x = l_0 \frac{86-x}{86}. \tag{8}$$

The probability that an individual aged *x* years will live *t* years is as follows:

$$\begin{aligned} \mu_x &= -\frac{1}{l_x} \cdot \frac{d}{dx}(l_x) \\ &= -\frac{1}{l_0 \frac{86-x}{86}} \cdot \frac{d}{dx}\left(l_0 \frac{86-x}{86}\right) \\ &= -\frac{86}{l_0(86-x)} \cdot \frac{d}{dx}\left(\frac{l_0 86}{86} - \frac{l_0 x}{86}\right) \\ &= -\frac{86}{l_0(86-x)}\left(\frac{-l_0}{86}\right) \\ &= \frac{1}{86-x}. \end{aligned} \tag{9}$$

The probability that an individual aged *x* years will live *t* years is as follows:

$$\begin{aligned} {}_tp_x &= \frac{l_{x+t}}{l_x} \\ &= l_0 \frac{\frac{86-(x+t)}{86}}{l_0 \frac{86-x}{86}} \\ &= \frac{86-x-t}{86-x}. \end{aligned} \tag{10}$$

The probability that a person aged *x* years old will die in *t* years is as follows:

$$\begin{aligned} {}_tq_x &= \frac{l_x - l_{x+t}}{l_x} \\ &= \frac{(l_0 \frac{86-x}{86}) - (l_0 \frac{86-(x+t)}{86})}{l_0 \frac{86-x}{86}} \\ &= \frac{\frac{l_0}{86}(86-x) - (86-x-t)}{\frac{l_0}{86}(86-x)} \\ &= \frac{(86-x)-(86-x-t)}{(86-x)} \\ &= \frac{t}{86-x}. \end{aligned} \tag{11}$$

According to Miguel (2009), de Moivre's law assumes that death occurs uniformly during the death interval, that is, the probability density function is $f_X(x) = \frac{1}{\omega}$ for $0 \le x \le \omega$ so we get:

1. Survival function

$$s(x) = \frac{\omega - x}{\omega} \; ; 0 \le x < \omega. \tag{12}$$

2. Accelerated Mortality

$$\mu_x = \frac{1}{\omega - x} \; ; 0 \le x < \omega. \tag{13}$$

3. Life Opportunity

$$ {}_tp_x = \frac{S(x+t)}{S(x)} = \frac{\omega-x-t}{\omega-x} \;; 0 \le t \le \omega - x. \tag{14} $$

4. Chance of Death

$$ {}_tq_x = \frac{t}{\omega-x} \;; 0 \le t \le \omega - x. \tag{15} $$

3. Research Methods

The data used in this study is the q_x value at TMI 2019 for men and data processing is done using Excel software. The steps taken in this study, namely:

1) Estimate the parameters of de Moivre's law which will be used to construct the de Moivre Mortality Table;
2) Find the value of q_x "to " $x = \omega$ based on de Moivre's law;
3) Constructing the de Moivre Mortality Table;
4) Creating illustrative charts for de Moivre's legal mortality tables; and
5) Concluded the construction results of the Indonesian Mortality Table (TMI) using de Moivre's law.

4. Results and Discussion

4.1. 2019 TMI Construction According to de Moivre's Law

Based on de Moivre's law, to determine the number of living at age x can be expressed in the formula

$$ l_x = l_0 \frac{\omega-x}{\omega} \;, 0 \le x \le \omega. \tag{16} $$

Based on equation (16) it can be obtained the acceleration of mortality, namely

$$ \mu_x = \frac{1}{\omega-x}. \tag{17} $$

Based on equation (16), the probability that a person aged *x* years will die at age $x + t$ years is obtained

$$ {}_tq_x = \frac{t}{\omega-x}. \tag{18} $$

Based on equation (18), the probability that a person aged $x + t$ years will die for the next 1 year is obtained

$$ q_{x+t} = \frac{1}{\omega-x-t}. \tag{19} $$

Construction of TMI 2019 Based on de Moivre's Law for Men Constructing TMI 2019 based on de Moivre's Law for men is by defining an initial value (radix/cohort) of $l_0 = 100{,}000$, using equation (1), then $l_1 = l_0 \frac{111-1}{111} = 99{,}099.10$.

Then the number of people who die between the ages of 0 and 1 year will be determined by using equation (3) de Moivre's law, then:

$$ {}_1q_0 = \frac{1}{111-0} = 0.00900901 $$

The calculation is carried out until $x = \omega$ to obtain the value of q_x.

4.2. Construction Results Based on de Moivre's Law for Men

The construction results of the 2019 TMI using de Moivre's law for men can be seen in Table 2.

Table 2: De Moivre Mortality Table for males based on the 2019 TMI

x	q_x **(Moivre)**	*X*	q_x **(Moivre)**	x	q_x **(Moivre)**
0	0.00900901	38	0.01369863	76	0.02857143
1	0.00909091	39	0.01388889	77	0.02941176

2	0.00917431	40	0.01408451	78	0.03030303
3	0.00925926	41	0.01428571	79	0.03125000
4	0.00934579	42	0.01449275	80	0.03225806
5	0.00943396	43	0.01470588	81	0.03333333
6	0.00952381	44	0.01492537	82	0.03448276
7	0.00961538	45	0.01515152	83	0.03571429
8	0.00970874	46	0.01538462	84	0.03703704
9	0.00980392	47	0.01562500	85	0.03846154
10	0.00990099	48	0.01587302	86	0.04000000
11	0.01000000	49	0.01612903	87	0.04166667
12	0.01010101	50	0.01639344	88	0.04347826
13	0.01020408	51	0.01666667	89	0.04545455
14	0.01030928	52	0.01694915	90	0.04761905
15	0.01041667	53	0.01724138	91	0.05000000
16	0.01052632	54	0.01754386	92	0.05263158
17	0.01063830	55	0.01785714	93	0.05555556
18	0.01075269	56	0.01818182	94	0.05882353
19	0.01086957	57	0.01851852	95	0.06250000
20	0.01098901	58	0.01886792	96	0.06666667
21	0.01111111	59	0.01923077	97	0.07142857
22	0.01123596	60	0.01960784	98	0.07692308
23	0.01136364	61	0.02000000	99	0.08333333
24	0.01149425	62	0.02040816	100	0.09090909
25	0.01162791	63	0.02083333	101	0.10000000
26	0.01176471	64	0.02127660	102	0.11111111
27	0.01190476	65	0.02173913	103	0.12500000
28	0.01204819	66	0.02222222	104	0.14285714
29	0.01219512	67	0.02272727	105	0.16666667
30	0.01234568	68	0.02325581	106	0.20000000
31	0.01250000	69	0.02380952	107	0.25000000
32	0.01265823	70	0.02439024	108	0.33333333
33	0.01282051	71	0.02500000	109	0.50000000
34	0.01298701	72	0.02564103	110	1.00000000
35	0.01315789	73	0.02631579	111	Tidak terdefinisi
36	0.01333333	74	0.02702703		
37	0.01351351	75	0.02777778		

4.3. Graph Illustration of Mortality Table

The de Moivre mortality table for men based on the 2019 TMI is illustrated in graphical form and compared to the 2019 TMI graph for men in Figure 1. Figure 1 shows a graph of the probability of dying at age x at the 2019 TMI for men and the construction results with de Moivre's law. It can be seen that TMI 2019 has the lowest chance of dying at the age of 10 to 12 years, namely 0.00019. Meanwhile, the de Moivre Mortality Table has the lowest value at the age of 0 years, namely 0.00900901.

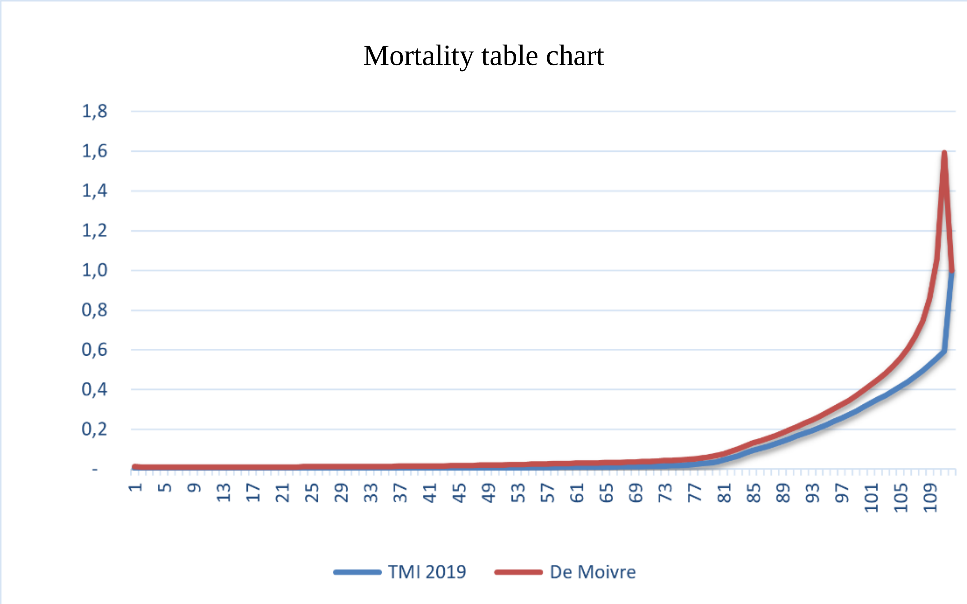

Figure 1: Graph of Probability of Death for TMI 2019 and De Moivre Mortality Table for Males

In the de Moivre Mortality Table, the value or probability of a person dying at age x continues to increase with age, from 0 to 110 years, and when a person is 111 years old the probability of death is undefined. Whereas in TMI 2019 there is an increase and decrease in a person's chance of dying at age x, but at the age of 8 years the value of the chance of dying continues to increase with age up to 111 years.

5. Conclusions and Recommendations

The construction of values in the 2019 TMI for men uses de Moivre's law by defining the initial value $l_0 = 100{,}000$ and $\omega = 111$ to produce ${}_1q_0 = 0.00900901$. Based on the construction of the q_x value in the 2019 TMI for men using de Moivre's law which is compared visually with the q_x value in the 2019 TMI table for men, results tend to be the same.

In TMI 2019, the lowest chance of dying occurs at the age of 10 to 12 years, namely 0.00019. Meanwhile, the de Moivre Mortality Table has the lowest value at the age of 0 years, namely 0.00900901.

Furthermore, in the de Moivre table, the value or probability of dying for someone at age x continues to increase with age, from 0 to 110 years, but there is a decrease when a person is 111 years old where the chance of dying is undefined or has a very high value. Whereas in TMI 2019 there is an increase and decrease in a person's chance of dying at age x, but at the age of 8 years the value of the chance of dying continues to increase with age up to 111 years.

Suggestions for further research are to find parameter values in de Moivre's law which are carried out by other methods, such as maximum likelihood and Bayesian.

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