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Portfolio Analysis Using the Markowitz Model with Stock Lot Constraints and Target Returns or Without Target Returns

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Abstract

Stock investment activities are inseparable from returns and risk, so an investor needs expertise to minimize investment risk. One way is by forming an optimal portfolio. The purpose of this research is to determine the number of stock lots in the optimal portfolio. This research analyzes the closing prices of stocks during the research period with the criteria of stocks being listed on the IDX30 index consecutively for 20 periods and belonging to the large cap group (the stock market capitalization exceeds $10 billion). Then the number of stock lots is calculated using the Markowitz model with stock lot constraints and target returns or without target returns. From the selected stocks, an optimal portfolio is formed using Microsoft Excel. Based on the research results, a combination of an optimal portfolio with a target return is ASII: 5, BBCA: 10, BBNI: 23, BBRI: 1, BMRI: 23, TLKM: 93, UNVR: 12, where the risk is 0.000149 and the target expected return is 0.00155. Meanwhile, the optimal portfolio without a target return is ASII: 8, BBCA: 7, BBNI: 32, BBRI: 40, BMRI: 9, TLKM: 62, UNVR: 17, where a risk is 0.000147 and the expected return is 0.00148. This research can be used as a consideration for investors in determining investment portfolios.

Keywords*:* Stocks, returns, optimal portfolio, Markowitz model, IDX30 index

1. Introduction

Investment is investing a number of funds with the aim of getting additional money or positive profits (Adnyana, 2020). The capital market provides investors with a variety of alternative investment options, such as stocks. The capital market is a market where long-term financial instruments are traded (Herlianto, 2013). In investing, there is a return and a risk. Return and risk have a linear relationship, which shows that the higher the expected return, the higher the risk borne by investors while investing (Adnyana, 2020, p. 2). Investors need to be careful in terms of investing because if they make the wrong decision, it can cause a loss. One way for investors to reduce investment risk is to diversify their holdings through the creation of a stock portfolio.

According to Chin et al. (2018), a portfolio is an investment combination that can consist of more than one type of asset. Every investor expects an optimal portfolio, which has the best rate of return at the lowest level of risk and the greatest return at a certain level of risk. Optimal portfolios can be formed with the Markowitz Model. This model emphasizes the relationship between the rate of return and the level of risk. According to Mahayani et al. (2019), the basis of the Markowitz model is to provide input to investors to get maximum profit and avoid risk by diversifying investments or spreading investments.

Investors can buy stock in lots, where one lot consists of 100 stocks. Therefore, the number of stock lots in the model is a constraint. Based on this description, this research discusses the combination of an optimal portfolio using the Markowitz model with stock lot constraints and target returns or without target returns on stocks included in the IDX30 index for 20 periods and included in the large cap group.

2. Literature Review

2.1. Markowitz Portfolio

Portfolio theory was introduced by Harry Markowitz in 1952. Markowitz's portfolio theory states that the rate of return and level of risk must be considered in forming a portfolio. For example, in constructing a portfolio, there are n stocks to be invested in, with the weight of each stock being $w_1, w_2, \ldots, w_n$ which satisfy the following criteria:

$$\sum_{i=1}^{n} w_i = w_1 + w_2 + \cdots + w_n = 1, \quad (1)$$

or it can be written in matrix form as follows:

$$\mathbf{e}^T\mathbf{w} = 1, \quad (2)$$

with

$$\mathbf{e} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \quad (3)$$

Portfolio expected return can be calculated using using the following equation:

$$\mu_p = \sum_{i=1}^{n} (w_i . E(r_i)), \quad (4)$$

with

μ_p: portfolio expected return

n: number of stocks invested

w_i: the weight of stock i in the portfolio

$\bar{r}_i$: expected return of stock i

Meanwhile, portfolio risk in the Markowitz model can be calculated using the variance of the portfolio, using the following formula:

$$V = \mathbf{w}^T\mathbf{Q}\mathbf{w}, \quad (5)$$

where $\mathbf{Q}$ is the matrix of stock return covariance, which can be written as follows:

$$\mathbf{Q} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{pmatrix}, \quad (6)$$

Optimization of the portfolio Markowitz model can be carried out with the criteria of minimizing risk with a target return or without a target return. Optimization of the portfolio Markowitz model by minimizing risk without a target return can be written as **Model 1**:

$$\min V = \mathbf{w}^T\mathbf{Q}\mathbf{w}, \quad (7)$$

with constraints $\sum_{i=1}^{n} w_i = 1$ and $w_i \geq 0$. (8)

Optimization of the portfolio Markowitz model by minimizing risk with a target return can be written as **Model 2:**

$$\min V = \mathbf{w}^T\mathbf{Q}\mathbf{w}, \quad (9)$$

with constraints $\sum_{i=1}^{n} w_i = 1, \sum_{i=1}^{n} (w_i . E(r_i)) = \mu_p,$ and $w_i \geq 0$ (10)

2.2. Portfolio Optimization with Stock Lot Constraints

A lot is an official unit of stock trading that has been determined by the Bursa Efek Indonesia. One lot of stock is equal to 100 stocks. For example, in a portfolio investment with n stocks, investors have funds of M that they want to invest, and the price of the stock i invested is P_i. The number of the stock i that must be purchased in a portfolio is defined as follows:

$$\frac{w_i M}{P_i} \quad (11)$$

So, the number of the stock i to be purchased in lots is defined as the following equation:

$$z_i = \frac{w_i M}{100 P_i} \quad (12)$$

or

$$w_i = 100 \frac{z_i P_i}{M}, \quad (13)$$

with

z_i: number lots of stock i
because the number of stock lots is a non-negative integer, **Model 1** is modified so that the Markowitz model with lot stock constraints and without target return is obtained as **Model 3**:

$$\min V = \mathbf{w}^T\mathbf{Q}\mathbf{w}, \tag{14}$$

with constraints $100\sum_{i=1}^{n} z_i P_i = M$ dan $z_i \in Z^+$ (15)

The Markowitz model with stock lot constraints and target return can be determined by modifying Model 2 to obtain Model 4 as follows:

$$\min V = \mathbf{w}^T\mathbf{Q}\mathbf{w}, \tag{16}$$

with constraints $100\sum_{i=1}^{n} z_i P_i = M$, $\frac{100}{M}\sum_{i=1}^{n} z_i P_i E(r_i) = \mu_p$ dan $z_i \in Z^+$ (17)

2.3. Generalized Reduced Gradient

Lasdon *et al*. (1978) stated that the Generalized Reduced Gradient (GRG) is an algorithm for solving nonlinear programming problems that was developed from the Reduced Gradient (RG) algorithm. The general form of the GRG is as follows:

$$\min f(x), \tag{18}$$

with constraints $h_k(x) = 0, k = 1, \dots, K$ dan $x \geq 0$, (19)
where $\boldsymbol{K}$ is the number of constraints.

3. Materials and Methods

3.1. Materials

This research uses closing price stock data from stocks included IDX30 for 20 consecutive periods spanning February 2012 - January 2022 and their market capitalization value is more than \$10 billion in rupiah worth IDR 148,8755 trillion. The data analyzed is the closing price of stock from 1 September 2021 to 31 August 2022, taken from the website www.finance.yahoo.com. The formation of an optimal portfolio using the Markowitz model with stock lot constraints and target returns or without target returns is calculated using the GRG method with the help of the Solver program in Microsoft Excel.

3.2. Methods

This research uses a quantitative approach based on concrete data in the form of numbers studied to produce a conclusion. The concrete data in this study is stock price data included in IDX30, which is then calculated by the lot composition of stocks that make the optimal portfolio using the Markowitz model. In this research, sample selection was carried out by means of purposive sampling. Data processing uses a solver program that already exists in the Microsoft Excel software. The following are the steps to determine the optimal portfolio:

1. Choose stocks to be included in a portfolio according to predetermined criteria.
2. Collect the closing price of stocks data from www.finance.yahoo.com during the research period, from 1 September 2021 to 31 August 2022.
3. Calculate the return from each stock using the following formula:
$$r_{it} = \frac{P_{it} - P_{i(t-1)}}{P_{i(t-1)}} \tag{20}$$
4. Calculate the expected return from each stock using the following formula:
$$E(r_i) = \frac{\left(\sum_{t=1}^{m} r_{it}\right)}{m}, \tag{21}$$
5. Calculate the risk from each stock using the following formula:
$$\sigma_i^2 = \frac{\sum_{t=1}^{m}(r_{it} - E(r_i))^2}{m}, \tag{22}$$
6. Calculate the covariance between two stocks in the portfolio with the following formula:
$$\sigma_{ij} = \frac{\sum_{t=1}^{m}(r_{it} - E(r_i))\left(r_{jt} - E(r_j)\right)}{m} \tag{23}$$
7. Create a **Q** matrix, namely the stock covariance matrix, with matrix entries as in equation (6).
8. Determine the optimal portfolio combination equation and calculate portfolio risk using (14), (15), (16), and (17) with the GRG algorithm, calculate expected portfolio return using equation (4) and calculate the ratio of target return or portfolio return to risk using Microsoft Excel.

4. Results and Discussion

In this research, seven stocks were chosen based on the criteria, these stocks are listed in Table 1.

Table 1: List of selected stocks

No.	Code	Stock Name
1	ASII	Astra International Tbk.
2	BBCA	Bank Central Asia Tbk.
3	BBNI	Bank Negara Indonesia (Persero) Tbk.
4	BBRI	Bank Rakyat Indonesia (Persero) Tbk.
5	BMRI	Bank Mandiri (Persero) Tbk.
6	TLKM	Telkom Indonesia (Persero) Tbk.
7	UNVR	Unilever Indonesia Tbk.

Then collect the daily closing price of the seven stocks selected during the research period. Then calculate stock returns using equation (20). Stock returns are used to calculate expected return using equation (21) and stock risk using equation (22). The results of these calculations can be seen in Table 2.

Tabel 2: Expected return and stock risk

Stock code	Expected return $E(r_i)$	Risk (σ^2)
ASII	0.00144	0.00043
BBCA	0.00102	0.00022
BBNI	0.00212	0.00038
BBRI	0.00097	0.00027
BMRI	0.00175	0.00031
TLKM	0.00141	0.00028
UNVR	0.00076	0.00062

Then calculate the covariance between stocks using equation (23). After that, forming a stock return covariance matrix referring to equation (6), and the results are as follows:

$$\mathbf{Q} = \begin{bmatrix} 0.00043 & 0.00011 & 0.00015 & 0.00009 & 0.00015 & 0.00009 & 0.00006 \\ 0.00011 & 0.00022 & 0.00015 & 0.00011 & 0.00011 & 0.00007 & 0.00008 \\ 0.00015 & 0.00015 & 0.00038 & 0.00017 & 0.00017 & 0.00009 & 0.00011 \\ 0.00009 & 0.00011 & 0.00017 & 0.00027 & 0.00014 & 0.00005 & 0.00007 \\ 0.00015 & 0.00011 & 0.00017 & 0.00014 & 0.00030 & 0.00006 & 0.00007 \\ 0.00009 & 0.00007 & 0.00009 & 0.00005 & 0.00006 & 0.00028 & 0.00002 \\ 0.00006 & 0.00008 & 0.00011 & 0.00007 & 0.00007 & 0.00002 & 0.00062 \end{bmatrix}$$

In this research, the investment fund is set at Rp100.000.000,00, and the stock price considered is the stock price at the end of the research period, which is August 31, 2022. The values for calculating the Markowitz portfolio model are in Table 4.

Table 4: The values for the calculation of the portfolio Markowitz model

Stock Code	Number Lots of Stock (z_i)	Stock Price (P_i)	Stock Weight (w_i)	Expected Return $E(r_i)$	z_iP_i	$z_iP_i\bar{r}_i$
ASII	z_1	6975	$0{,}00698z_1$	0,00144	$6975z_1$	$10.01326z_1$
BBCA	z_2	8200	$0{,}00820z_2$	0,00102	$8200z_2$	$8.36536z_2$
BBNI	z_3	8525	$0{,}00853z_3$	0,00212	$8525z_3$	$18.05114z_3$
BBRI	z_4	4340	$0{,}00434z_4$	0,00097	$4340z_4$	$4.22973z_4$
BMRI	z_5	8850	$0{,}00885z_5$	0,00175	$8850z_5$	$15.44748z_5$
TLKM	z_6	4560	$0{,}00456z_6$	0,00141	$4560z_6$	$6.45212z_6$
UNVR	z_7	4590	$0{,}00459z_7$	0,00076	$4590z_7$	$3.50014z_7$

The values that have been obtained are substituted into the Markowitz model with stock lot constraints and target returns or without target returns. The Markowitz model with stock lot constraints and no target return refers to equations (14) and (15) as follows:

$$\text{Min } V = \mathbf{w}^T\mathbf{Q}\mathbf{w}$$

with constraints

$100(6975z_1+8200z_2+8525z_3+4340z_4+8850z_5+4560z_6+4590z_7) = 100{,}000{,}000$ and $z_i \in Z^+$.

The Markowitz model with stock lot constraints and target returns refers to equations (16) and (17) as follows:

$$\text{Min } V = \mathbf{w}^T\mathbf{Q}\mathbf{w}$$

with constraints

$100(6975z_1+8200z_2+8525z_3+4340z_4+8850z_5+4560z_6+4590z_7) = 100{,}000{,}000$;

$\frac{1}{1.000.000}(7.063339z_1 + 6.582468z_2 + 14.843543z_3 + 3.058698z_4 + 12.756506z_5 + 5.187877z_6 + 0.780973z_7) = \mu_p$;

and $z_i \in Z^+$.

Furthermore, in determining the number of stock lots in the optimal portfolio using the GRG algorithm with the Solver program contained in Microsoft Excel. The results of the calculation of the combination stock lots of the portfolio model without a target return for ASII, BBCA, BBNI, BMRI, TLKM, and UNVR are respectively: 8, 7, 32, 40, 9, 62, 17 with a risk of 0,000147. Then the expected return portfolio is calculated using equation (4) to obtain a result of 0,00148 and the ratio value of the target portfolio return to risk is 10,09757.

In the portfolio model with a target return, it is calculated for several different target returns. The target return starts with a value of 0,00100 and is carried out repeatedly with the addition of a target return value of 0,00005, the determination of the portfolio combination with the target return is stopped if the specified return value does not provide a feasible solution. Calculating the combination of the number of stock lots and risk using equations (16) and (17). From the calculations that have been done, an efficient portfolio is selected. The determination of the target returns and the results of calculating the number lots of stock in the efficient portfolio are given in Table 5.

Table 5: Efficient portfolio of the Markowitz model with lot stock constraints and target returns

Expected Return Target	ASII	BBCA	BBNI	BBRI	BMRI	TLKM	UNVR	Risk	Ratio
0,00120	7	24	7	31	8	59	48	0.000131	9.15838
0,00140	0	12	10	16	23	95	24	0.000136	10.26957
0,00155	5	10	23	1	23	93	12	0.000149	10.38580
0,00165	6	3	26	0	32	92	2	0.000163	10.12280
0,00170	5	0	29	0	44	66	6	0.000170	9.98423
0,00180	0	0	40	4	56	28	4	0.000206	8.73814
0,00190	4	0	80	3	4	53	0	0.000243	7.81714
0,00200	0	0	82	1	33	1	0	0.000283	7.06976
0,00205	3	0	105	5	6	1	1	0.000334	6.14004

The results of Table 5 show that the target return value of the efficient portfolio is in the interval $0{,}00120 \leq \mu_p \leq 0{,}00205$ because for a target return of 0,00210, the Solver program in Microsoft Excel cannot find a feasible solution. The relationship between return and risk can be seen in Figure 1 in the form of an efficient frontier graph made using Microsoft Excel.

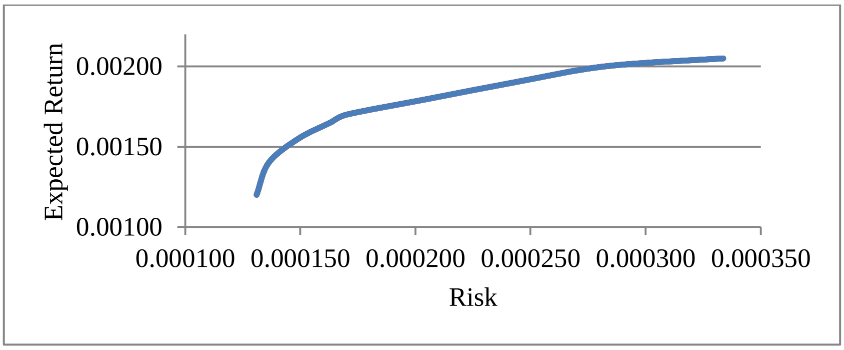

Figure 1: Efficient frontier graph

Selection of the optimal portfolio using the ratio of the portfolio return target to risk. Referring to the Table 5 Ratio column, the optimal portfolio is the one that has the highest ratio value, namely 10,38580. This optimal portfolio provides a risk of 0.000149 with a target return of 0,00155 with a combination of the lots of ASII, BBCA, BBNI, BMRI, TLKM, and UNVR, respectively: 5, 10, 23, 1, 23, 93, 12.

Referring to Table 5, the relationship between the portfolio risk value and the ratio can be seen. In this research, for a risk value of 0.000131 to 0.000149, the greater the risk value, the greater the ratio. Whereas for the risk value of 0.000149 to 0.000334, the greater the risk value, the smaller the ratio value obtained. This relationship is illustrated in Figure 2, which was made using Microsoft Excel.

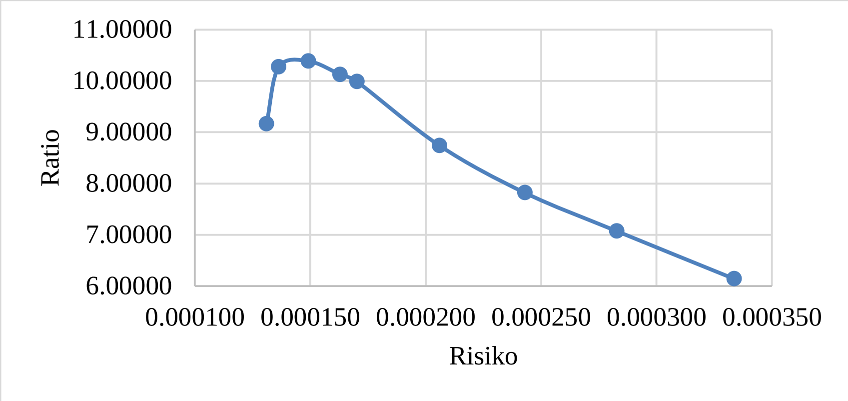

Figure 2: Ratio graph

In this research, a portfolio without target returns has a ratio value of 10.09757. While the optimal portfolio with a target return has a ratio value of 10.38580. Using the ratio of the expected return target or the expected return portfolio to risk, the most optimal portfolio of the two choices is the optimal portfolio with a target return because it gives a large ratio value.

5. Conclussion

Based on the results of the research, the optimal portfolio with a target returns or without a target return gives different results. The combination of lots of stock ASII, BBCA, BBNI, BMRI, TLKM, and UNVR in a portfolio with a target return is 5, 10, 23, 1, 23, 93, and 12, with a risk of 0.000149 and an expected return of 0.00155. Without a target return, the combination of lots of stock ASII, BBCA, BBNI, BBRI, BMRI, TLKM, and UNVR in the portfolio is 8, 7, 32, 40, 9, 62, and 17, with a risk of 0.000147 and an expected return of 0.00148. Portfolio with a target return provide a greater return value than portfolios without a target return.

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