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Investment Portfolio Optimization Model with Mean-Std Deviation

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Abstract

Stock investment is an investment in securities with the hope of getting profits in the future. Investors are expected to make a series of portfolios to get optimal results from investments. This discussion aims to find the weight of the funds invested along with the returns and risks. The method used is the mean + std deviation. The results of this portfolio optimization show that the risk aversion coefficient is 0.1. The optimum weight for investment in each company is KLBF (22.67%), PGAS (8.796%), BBCA (41.77%), ASII (8.24%), and SMAR (18.52%) with a maximum ratio of 8.8% of a return of 0.0881% and a risk of 1.0009%. The results of this portfolio optimization are expected to help investors by dividing the number of funds to be invested by the return and risk.

Keywords: Investment, optimization model, standard deviation

1. Introduction

In recent years, in Indonesia, there has been increasing demand for investment in the capital market because the capital market identifies two interests, namely those who have excess funds and those who need funds. (Fadilah & Witiastuti, 2018). Brigham and Houston (2015) Alternative funding from within the company's profits comes from companies that have been arrested, while funds from outside companies can become debt creditors as well as the nature of funding in the form of shares equity.

Tandelilin (2010) divides stock risk into two types: systematic risk and unsystematic risk. Systematic risk will affect all stocks, while unsystematic risk only affects one class of stock or a certain sector. Investors cannot eliminate the systematic risk that affects all stores in the stock market. Still, investors can reduce unsystematic risk through diversification by forming a portfolio so that the risks borne by investors can be minimized. (Fadilah & Witiastuti, 2018).

This diversification concept has given rise to the Modern Portfolio Theory (MPT), which carries the idea of choosing between two conflicting objectives: risk and return. In this scenario, the importance of the decision-making method emerges so that an investor can choose a financial portfolio within a non-dominated border of portfolios with different risk and return values (Mendonça et al., 2020). Banihashemi and Navidi, (2017) propose a multiobjective model that identifies efficient assets and evaluates inefficient holdings so that the best investments will be selected to construct a financial portfolio.

Stock investment is investing a certain amount of money in securities that show a sign of ownership of a company so that it can provide benefits in the future. To get optimal investment results, investors are expected to make a series of portfolios (Moehring, 2013). Having a portfolio will help investors allocate several funds to achieve optimal profits. Many methods can be used to optimize portfolios, including mean + std deviation. This method maximizes portfolio returns plus the risk load in the form of a standard deviation and risk aversion coefficient, with an investment weight in all stocks totaling 1. The results of this portfolio optimization are expected to help investors by dividing the number of funds to be invested by the returns and risks.

2. Materials and Methods

2.1. Materials

The stock data analyzed is the stock data traded on the capital market in Indonesia through the Indonesia Stock Exchange (IDX). Selected stock data include PT Kalbe Farma Tbk (KLBF), PT Perusahaan Gas Negara Tbk (PGAS), PT Bank Central Asia Tbk (BBCA), PT Astra International Tbk (ASII), and PT Sinar Mas Agro Resources and Technology Tbk(SMAR). Daily stock historical data is accessed via finance.yahoo.com. The data period used is March 31, 2021 – March 31, 2022. software used to make data management easier is Microsoft Excel

2.2. Methods

For efficient portfolio selection, use the mean + std deviation method for the example given a portfolio p with a weight vector w. This is done by finding the maximum value $\mu_p + \frac{\rho}{2}\sigma_p$, with the provision of $\sum_{i=1}^{N} w_i = 1$ and $p > 0$, p parameters called risk tolerance (Basuki et al., 2017). Suppose there are N assets without being risk-free with returns $r_1, \dots, r_N$ exists, the transpose vector of the expected return value is expressed as: $\mu^T = (\mu_1, \dots, \mu_N)$ with $\mu_i = E[r_i], i = 1, \dots, N$ and the covariance matrix is expressed as $\mathbf{\Sigma} = (\boldsymbol{\sigma_{ij}})$ with $\boldsymbol{\sigma_{ij}} = Cov(r_i, r_j), i, j = 1, \dots, N$. If the return portfolio is r_p with the transpose weight vector $\mathbf{w}^T = (\mathbf{w}_1, \dots, \mathbf{w}_N)$, and terms $\sum_{i=1}^{N} w_i = 1$, then the expected portfolio return (average portfolio) using vector notation is as follows:

$$\boldsymbol{\mu}_p = E[r_p] = \boldsymbol{\mu}^T\mathbf{w} = \mathbf{w}^T\boldsymbol{\mu} \tag{1}$$

and the standard deviation is expressed as:

$$\sigma_p = \sqrt{\sigma_p^2} = \sqrt{\mathrm{Var}(r_p)} = \sqrt{\mathbf{w}^T\mathbf{\Sigma}\mathbf{w}} \tag{2}$$

Next, solve the portfolio optimization problem as follows:

$$\text{Maximum } \{\boldsymbol{\mu}^T\mathbf{w} + \tfrac{\rho}{2}\sqrt{\mathbf{w}^T\mathbf{\Sigma}\mathbf{w}}\} \tag{3}$$

$$\text{Constraint } \mathbf{e}^T\mathbf{w} = 1$$

With $\mathbf{e}^T = (1, \dots, 1)$ as many as N unit vectors, $\boldsymbol{\mu}^T\mathbf{w} = \mathbf{w}^T\boldsymbol{\mu}$, and $\mathbf{w}^T\mathbf{e} = \mathbf{e}^T\mathbf{w}$. The Lagrange function of equation (3), where λ is the multiplier, can be expressed as follows:

$$L(\mathbf{w}, \lambda) = \mathbf{w}^T\boldsymbol{\mu} + \tfrac{\rho}{2}\sqrt{\mathbf{w}^T\mathbf{\Sigma}\mathbf{w}} + \lambda(\mathbf{w}^T\mathbf{e} - 1) \tag{4}$$

Equation (4) using necessary conditions $\frac{\partial L}{\partial \mathbf{w}} = 0$ and $\frac{\partial L}{\partial \lambda} = 0$ got:

$$\frac{\partial L}{\partial \mathbf{w}} = \boldsymbol{\mu} + \frac{\rho}{2}\frac{\Sigma\mathbf{w}}{\sqrt{\mathbf{w}^T\Sigma\mathbf{w}}} + \lambda\boldsymbol{e} = \mathbf{0} \tag{5}$$

$$\frac{\partial L}{\partial \lambda} = \mathbf{w}^T\mathbf{e} - 1 = 0 \tag{6}$$

Equation (5) multiply by $\mathbf{\Sigma}^{-1}$ and expressed in w, so we get the equation:

$$\mathbf{w} = \frac{\sqrt{\mathbf{w}^T\mathbf{\Sigma}\mathbf{w}}\,(-2\mathbf{\Sigma}^{-1}\boldsymbol{\mu} - 2\boldsymbol{\lambda}\mathbf{\Sigma}^{-1}\mathbf{e})}{\rho} \tag{7}$$

Equation (7) multiplied by $\mathbf{e}^T$, got the equation:

$$\mathbf{e}^T\mathbf{w} = \frac{\sqrt{\mathbf{w}^T\mathbf{\Sigma}\mathbf{w}}\,(-2\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu} - 2\lambda\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e})}{\rho} \tag{8}$$

Equation (8) with $\mathbf{e}^T\mathbf{w} = \mathbf{w}^T\mathbf{e}$, if substituted in equation (6), obtained:

$$\sqrt{\mathbf{w}^T\mathbf{\Sigma}\mathbf{w}} = \frac{\rho}{(-2\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu}-2\mathbf{e}^T\mathbf{\Sigma}^{-1}\lambda\mathbf{e})} \quad (9)$$

Substitute equation (9) with equation (7) to get:

$$\boldsymbol{w} = \frac{(-2\mathbf{\Sigma}^{-1}\boldsymbol{\mu}-2\lambda\mathbf{\Sigma}^{-1}\mathbf{e})}{(-2\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu}-2\lambda\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e})} \quad (10)$$

Note that equation (5) can be rewritten as:

$$\frac{\rho}{2}.\frac{\mathbf{\Sigma}w}{\sqrt{\mathbf{w}^T\mathbf{\Sigma}\mathbf{w}}} = -\boldsymbol{\mu} - \lambda\mathbf{e} \quad (11)$$

Multiply equation (11) by the got equation:

$$\rho\sqrt{\mathbf{w}^T\mathbf{\Sigma}\mathbf{w}} = -2\boldsymbol{\mu}^T\mathbf{w} - 2\lambda \quad (12)$$

Then, substituting equations (9) and (10) into equation (12), we get:

$$(-4\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e})\boldsymbol{\lambda}^{\mathbf{2}} + (4\boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\mathbf{e} - 4\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu})\lambda \ + (4\boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu} + \rho^2) = 0 \quad (13)$$

Equation (13) is a quadratic equation in λ, so the root value can be calculated using the ABC formula as follows:

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \text{ with terms } \lambda > 0$$

So obtained:

$$a = (-4\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e}) \qquad b = 4\boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\mathbf{e} - 4\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu} \qquad c = 4\boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu} + \rho^2$$

With $\mathbf{\Sigma}^{-1}$ as the inverse of the covariance matrix Σ, Then get the root value λ:

$$\lambda_{1,2} = \frac{(\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu}-\boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\mathbf{e})\pm\sqrt{(\boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\mathbf{e}-\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu})^2+(\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e})(4\boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu}+\rho^2)}}{(-\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e})} \quad (14)$$

Next, substituting the value of λ into equation (10) is obtained:

$$\mathbf{w}^* = \frac{(-2\mathbf{\Sigma}^{-1}\boldsymbol{\mu}-2\lambda\mathbf{\Sigma}^{-1}\mathbf{e})}{(-2\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu}-2\lambda\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e})}$$

$$\mathbf{w}^* = \frac{\left(-2\mathbf{\Sigma}^{-1}\boldsymbol{\mu}-2\frac{\left(\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu}-\boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\mathbf{e}\right)\pm\sqrt{(\boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\mathbf{e}-\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu})^2+(\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e})(4\boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu}+\rho^2)}}{(-\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e})}\mathbf{\Sigma}^{-1}\mathbf{e}\right)}{\left(-2\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu}-2\frac{(\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu}-\boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\mathbf{e})\pm\sqrt{(\boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\mathbf{e}-\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu})^2+(\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e})(4\boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu}+\rho^2)}}{(-\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e})}\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e}\right)} \quad (15)$$

Furthermore, for sufficient conditions, look for the hessian matrix by second-order derivatives from equations (5) and (6):

$$\frac{\partial^2 L}{\partial^2\mathbf{w}} = \frac{\rho}{2}\,\frac{\mathbf{w}^T\mathbf{\Sigma}\mathbf{w}-(\sum\mathbf{w})^2}{(\mathbf{w}^T\sum\mathbf{w})^{\frac{3}{2}}}; \quad \frac{\partial^2 L}{\partial\mathbf{w}\,\partial\lambda} = \mathbf{e}$$
$$\frac{\partial^2 L}{\partial\lambda\,\partial\mathbf{w}} = \mathbf{e}^T; \quad \frac{\partial^2 L}{\partial^2\lambda} = 0$$

Then the hessian matrix form is obtained:

$$H = \begin{bmatrix} \frac{\rho}{2}\,\frac{\mathbf{w}^T\mathbf{\Sigma}\mathbf{w}-(\sum\mathbf{w})^2}{(\mathbf{w}^T\sum\mathbf{w})^{\frac{3}{2}}} & \mathbf{e} \\ \mathbf{e}^T & 0 \end{bmatrix} \quad (16)$$

Theorem: H is called a negative definite matrix only if it satisfies $(-1)^i\det(H_i) < 0$ with i = 1, 2, ...

Provable:

$i = 1;$

$$(-1)^1 \, det(H_1) = (-1) \left| \frac{\rho}{2} \frac{w^T \Sigma w - (\sum w)^2}{(w^T \sum w)^{\frac{3}{2}}} \right| = - \left| \frac{\rho}{2} \frac{w^T \Sigma w - (\sum w)^2}{(w^T \sum w)^{\frac{3}{2}}} \right| < 0$$

$i = 2;$

$$(-1)^2 \, det(H_2) = (1) \begin{vmatrix} \frac{\rho}{2} \frac{w^T \Sigma w - (\sum w)^2}{(w^T \sum w)^{\frac{3}{2}}} & \boldsymbol{e} \\ \boldsymbol{e}^T & 0 \end{vmatrix} = \frac{\rho}{2} \frac{w^T \Sigma w - (\sum w)^2}{(w^T \sum w)^{\frac{3}{2}}} . 0 - \boldsymbol{e}^T . \boldsymbol{e}$$

$$= 0 - (\boldsymbol{e}^T . \boldsymbol{e})$$

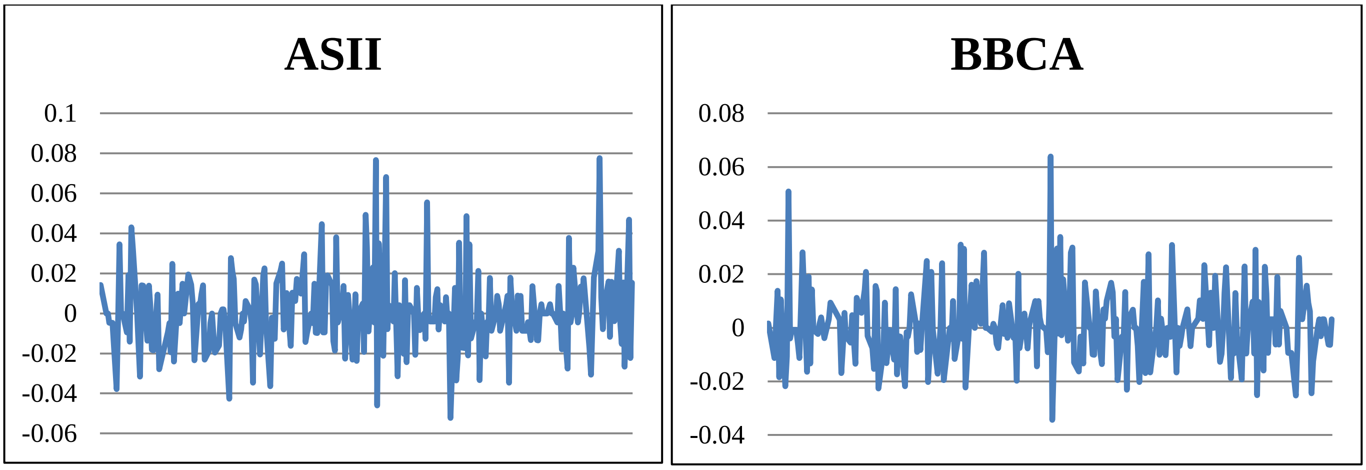
$$= -(\boldsymbol{e}^T . \boldsymbol{e}) < 0$$

So it is proven that H is a negative definite matrix because it satisfies $(-1)^i \text{det}(H_i) < 0$ with i = 1, 2. Based on this, it can be concluded that the necessary and sufficient conditions for optimality are met: $\nabla L(\mathbf{w}^*) = 0$ and $H(\mathbf{w}^*)$ negative definite, so $\mathbf{w}^*$ It is the maximum value. With the help of Microsoft Excel, the data obtained is then analyzed using average returns, return plots, and descriptive statistics (Ruppert, 2004). Then formed the optimization of the investment portfolio using the mean + std deviation method and obtained an investment portfolio table with an efficient surface graph and a graph of the portfolio ratio

3. Results

3.1. Stock Data Analysis

The formation of return charts for five stocks, namely, PT Kalbe Farma Tbk (KLBF), PT Perusahaan Gas Negara Tbk (PGAS), PT Bank Central Asia Tbk (BBCA), PT Astra International Tbk (ASII), and PT Sinar Mas Agro Resources and Technology Tbk (SMAR, which will be presented in Figure 1



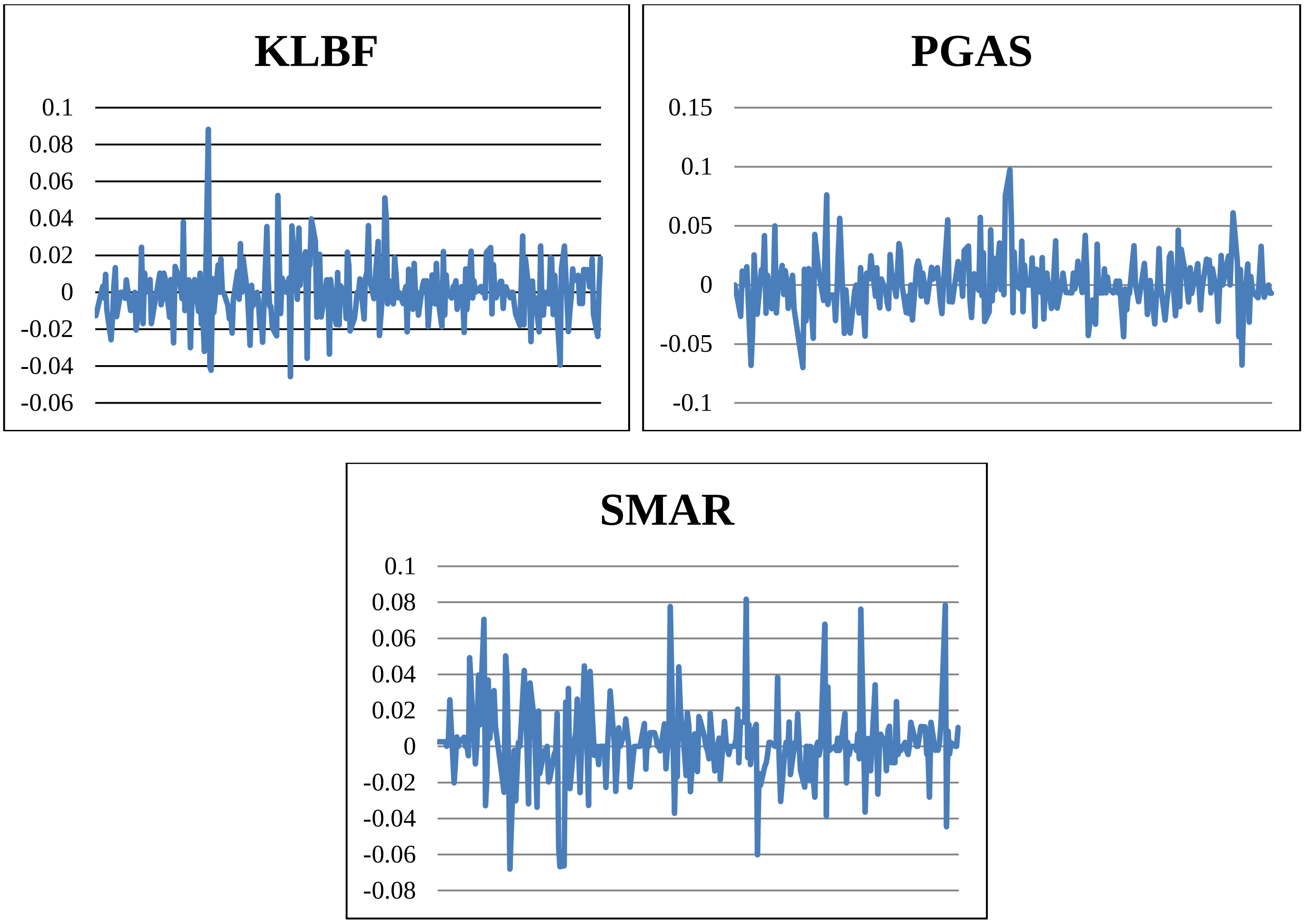


Figure 1: Stock Return Chart Period April 2021 to March 2022

The results of the estimated distribution, expectations, and variance of returns from the five stocks, along with the ratio between expectations and variances of returns, can be seen in Table 1

Table 1: Distribution Estimation, Expectation, and Stock Return Variance

Share Name	Expectation μ	Standard Deviation σ	Variance σ^2	Ratio μ/σ
KLBF	0.00037	0.01681	0.00028	1.29634
PGAS	0.00053	0.02413	0.00058	0.90709
BBCA	0.00105	0.01366	0.00019	5.65525
ASII	0.00113	0.01968	0.00039	2.92623
SMAR	0.00117	0.02178	0.00047	2.47529

3.2. Formation of Investment Portfolio Optimization with the Mean+StdDev Model

Average Vector $\boldsymbol{\mu}$

$$\boldsymbol{\mu} = \begin{bmatrix} 0.00037 \\ 0.00053 \\ 0.00105 \\ 0.00113 \\ 0.00117 \end{bmatrix}$$

$$\boldsymbol{\mu}^T = [0.00037 \quad 0.00053 \quad 0.00105 \quad 0.00113 \quad 0.00117]$$

Average Vector **e**

$$\mathbf{e} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\mathbf{e}^T = [1 \quad 1 \quad 1 \quad 1 \quad 1]$$

Variance and covariance data of the five stocks will be presented in Table 2

Table 1: Variances and Covariances of selected stocks

Share Name	KLBF	PGAS	BBCA	ASII	SMAR
KLBF	0.00028	0.00001	0.00004	0.00006	0.00001
PGAS	0.00001	0.00058	0.00004	0.00009	0.00007
BBCA	0.00004	0.00004	0.00019	0.00011	0.00002
ASII	0.00006	0.00009	0.00011	0.00039	0.00004
SMAR	0.00001	0.00007	0.00002	0.00004	0.00047

Covariance Matrix **Σ**

$$\mathbf{\Sigma} = \begin{bmatrix} 0.00028 & 0.00001 & 0.00004 & 0.00006 & 0.00001 \\ 0.00001 & 0.00058 & 0.00004 & 0.00009 & 0.00007 \\ 0.00004 & 0.00004 & 0.00019 & 0.00011 & 0.00002 \\ 0.00006 & 0.00009 & 0.00011 & 0.00039 & 0.00004 \\ 0.00001 & 0.00007 & 0.00002 & 0.00004 & 0.00047 \end{bmatrix}$$

Inverse Covariance Matrix $\mathbf{\Sigma}^{-1}$

$$\mathbf{\Sigma}^{-1} = \begin{bmatrix} 3733.74702 & 36.21702 & -668.50648 & -380.29943 & -2.16424 \\ 36.21702 & 1825.81465 & -208.73767 & -367.72202 & -225.41000 \\ -668.50648 & -208.73767 & 6566.95896 & -1672.81798 & -81.25369 \\ -380.29943 & -367.72202 & -1672.81798 & 3221.41237 & -148.06559 \\ -2.16424 & -225.41000 & -81.25369 & -148.06559 & 2166.09249 \end{bmatrix}$$

3.3. Investment Portfolio Optimization Process

The result of optimizing an efficient Mean+Std Deviation investment portfolio is shown in Table 3.

Table 3: Mean+Std Deviation Investment Portfolio Optimization Results

ρ	KLBF	PGAS	BBCA	ASII	SMAR	w^Te	μ_p	σ_p^2	μ_p/σ_p^2
0.1	0.226722	0.08796	0.41773	0.08237	0.18521	1.00000	0.000881	0.010009	0.088013
0.2	0.231651	0.089935057	0.414623	0.080359	0.183432	1.00000	0.000876	0.010004	0.087581
0.3	0.236592	0.091911842	0.411512	0.07834	0.181645	1.00000	0.000871	0.009998	0.087146
0.4	0.240764	0.093581184	0.408884	0.076635	0.180135	1.00000	0.000867	0.009994	0.086779
0.5	0.244151	0.094936003	0.406751	0.075251	0.178911	1.00000	0.000864	0.009990	0.086481
0.6	0.246896	0.096034013	0.405023	0.07413	0.177918	1.00000	0.000861	0.009987	0.086239
0.7	0.249142	0.096932867	0.403608	0.073211	0.177106	1.00000	0.000859	0.009985	0.086041
0.8	0.251006	0.097678275	0.402434	0.07245	0.176432	1.00000	0.000857	0.009983	0.085877
0.9	0.252571	0.098304515	0.401449	0.07181	0.175866	1.00000	0.000856	0.009981	0.085739
1	0.253902	0.098837041	0.40061	0.071266	0.175384	1.00000	0.000854	0.009979	0.085621
1.1	0.255046	0.09929486	0.39989	0.070799	0.17497	1.00000	0.000853	0.009978	0.085520
1.2	0.25604	0.099692332	0.399264	0.070393	0.174611	1.00000	0.000852	0.009977	0.085432
1.3	0.25691	0.100040448	0.398716	0.070037	0.174296	1.00000	0.000852	0.009976	0.085356
1.4	0.257678	0.100347734	0.398232	0.069723	0.174019	1.00000	0.000851	0.009975	0.085288
1.5	0.258361	0.10062089	0.397802	0.069444	0.173772	1.00000	0.000850	0.009975	0.085227
1.6	0.258972	0.100865247	0.397418	0.069195	0.173551	1.00000	0.000850	0.009974	0.085174
1.7	0.259521	0.10108509	0.397071	0.06897	0.173352	1.00000	0.000849	0.009973	0.085125
1.8	0.260018	0.101283904	0.396758	0.068767	0.173172	1.00000	0.000848	0.009973	0.085081
1.9	0.26047	0.101464548	0.396474	0.068582	0.173009	1.00000	0.000848	0.009972	0.085041
2	0.260882	0.10162939	0.396215	0.068414	0.17286	1.00000	0.000848	0.009972	0.085005
2.1	0.261259	0.101780405	0.395977	0.06826	0.172724	1.00000	0.000847	0.009971	0.084971
2.2	0.261606	0.101919255	0.395758	0.068118	0.172598	1.00000	0.000847	0.009971	0.084941
2.3	0.261926	0.102047345	0.395557	0.067987	0.172482	1.00000	0.000847	0.009971	0.084912

2.4	0.262223	0.102165876	0.39537	0.067866	0.172375	1.00000	0.000846	0.009970	0.084886
2.5	0.262498	0.102275875	0.395197	0.067754	0.172276	1.00000	0.000846	0.009970	0.084862
2.6	0.262754	0.102378227	0.395036	0.067649	0.172183	1.00000	0.000846	0.009970	0.084839
2.7	0.262992	0.102473702	0.394886	0.067552	0.172097	1.00000	0.000846	0.009969	0.084818
2.8	0.263215	0.102562968	0.394745	0.06746	0.172016	1.00000	0.000845	0.009969	0.084798
2.9	0.263424	0.10264661	0.394613	0.067375	0.171941	1.00000	0.000845	0.009969	0.084780
3	0.263621	0.102725142	0.39449	0.067295	0.17187	1.00000	0.000845	0.009969	0.084763
3.1	0.263805	0.102799017	0.394373	0.067219	0.171803	1.00000	0.000845	0.009969	0.084746
3.2	0.263979	0.102868637	0.394264	0.067148	0.17174	1.00000	0.000845	0.009968	0.084731
3.3	0.264144	0.102934358	0.39416	0.067081	0.17168	1.00000	0.000844	0.009968	0.084716
3.4	0.264299	0.102996499	0.394063	0.067018	0.171624	1.00000	0.000844	0.009968	0.084703
3.5	0.264446	0.103055343	0.39397	0.066958	0.171571	1.00000	0.000844	0.009968	0.084690
3.6	0.264586	0.103111146	0.393882	0.066901	0.171521	1.00000	0.000844	0.009968	0.084677
3.7	0.264718	0.103164137	0.393799	0.066846	0.171473	1.00000	0.000844	0.009968	0.084665
3.8	0.264844	0.103214524	0.393719	0.066795	0.171427	1.00000	0.000844	0.009967	0.084654
3.9	0.264964	0.103262493	0.393644	0.066746	0.171384	1.00000	0.000844	0.009967	0.084644
4	0.265078	0.103308214	0.393572	0.066699	0.171343	1.00000	0.000844	0.009967	0.084634
4.1	0.265187	0.103351841	0.393503	0.066655	0.171303	1.00000	0.000843	0.009967	0.084624
4.2	0.265291	0.103393515	0.393438	0.066612	0.171265	1.00000	0.000843	0.009967	0.084615
4.3	0.265391	0.103433364	0.393375	0.066571	0.171229	1.00000	0.000843	0.009967	0.084606
4.4	0.265486	0.103471505	0.393315	0.066532	0.171195	1.00000	0.000843	0.009967	0.084598
4.5	0.265578	0.103508045	0.393257	0.066495	0.171162	1.00000	0.000843	0.009967	0.084589
4.6	0.265665	0.103543084	0.393202	0.066459	0.17113	1.00000	0.000843	0.009967	0.084582
4.7	0.265749	0.103576711	0.393149	0.066425	0.1711	1.00000	0.000843	0.009966	0.084574
4.8	0.26583	0.10360901	0.393098	0.066392	0.171071	1.00000	0.000843	0.009966	0.084567
4.9	0.265908	0.103640059	0.39305	0.06636	0.171043	1.00000	0.000843	0.009966	0.084560
5	0.265982	0.103669928	0.393003	0.06633	0.171016	1.00000	0.000843	0.009966	0.084554

A series of efficient portfolios are at the forefront of an efficient surface where the portfolio's return is commensurate with the risk, which will be presented in Figure 2.

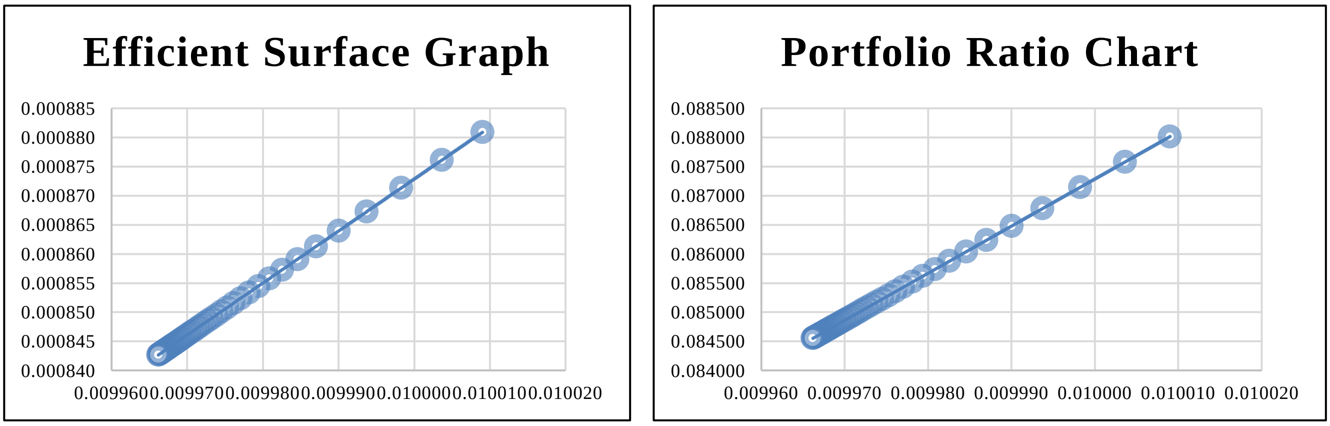


Figure 2: Efficient Frontier and Mean-Variance Portfolio Ratio

4. Discussion

Table 3 it is taking the risk tolerance value for the value of. From the optimization of the portfolio, it can be seen that the greater the risk taken, the greater the weight of the ratio of the average portfolio to the maximum standard deviation obtained. Obtained a maximum ratio of 8.8% of a return of 0.0881% and a risk of 1.0009% with a risk aversion coefficient of 0.1. At a risk aversion coefficient of 0.1, the optimum weight is obtained for investment in each company, namely KLBF (22.67%), PGAS (8.796%), BBCA (41.77%), ASII (8.24%), and SMAR (18.52%). However, suppose investors do not want to take too high a risk. In that case, it can be advised to take a risk aversion coefficient of 1.6, which is the midpoint of the ratio weight with the optimum portfolio for investment in each company, namely KLBF (25.897%), PGAS (10.09%), BBCA (39.74%), ASII (6.92%), and SMAR (17.35%) with a maximum ratio of 8.52% of the return of 0.085% and risk of 0.997%.

From the efficient surface graph, it can be seen that the average value of the portfolio with the risk measure of the investment portfolio produces a graph that continues to rise, indicating that the higher the average portfolio value, the higher the risk measure. Furthermore, it can be seen through the portfolio ratio graph, and it can be seen that the risk size value of the investment portfolio with the ratio weight produces a graph that continues to rise, indicating that the higher the portfolio risk size, the higher the ratio weight

5. Conclusion

From the five selected stock data, namely, PT Kalbe Farma Tbk (KLBF), PT Perusahaan Gas Negara Tbk (PGAS), PT Bank Central Asia Tbk (BBCA), PT Astra International Tbk (ASII), and PT Sinar Mas Agro Resources and Technology Tbk (SMAR), obtained an optimum portfolio with a risk aversion coefficient of 0.1. At a risk aversion coefficient of 0.1, the optimum weight is acquired for investment in each company, namely KLBF (22.67%), PGAS (8.796%), BBCA (41.77%), ASII (8.24%), and SMAR (18.52%) with a maximum ratio of 8.8% of a return of 0.0881% and a risk of 1.0009%.

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