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Investment Portfolio Optimization with Mean-Variance Investment Portfolio Optimization Model Without Risk Free Assets

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Abstract

Forming a portfolio is a strategy that is often carried out by investors in risky investment conditions. Five non-risk free stocks were selected, namely PTBA, IPCM, ANTM, BUMI, and ADMF. The purpose of forming this portfolio is to determine the composition of the weight (proportion) of the allocation of funds in each of these shares in forming the optimum portfolio. The method used is the Mean-Variance investment portfolio optimization model without risk-free assets using the Markowitz approach. Based on the results obtained by the optimum portfolio of the Mean-Variance model without risk-free assets, the average return is 0.00105 and the variance is 0.000067 with a portfolio ratio value of 14.65256. The proportion of fund allocation to PTBA shares = 0.28872; IPCM=0.02484; ANTM=0.00016; EARTH=0.13501; and ADMF=0.55126. It is hoped that the formation of this portfolio optimization model will be useful as an alternative for investors in optimizing the investment portfolio to make it more profitable in the future.

Keywords: PTBA, IPCM, ANTM, BUMI, ADMF

1. Introduction

In investing, investors can choose to invest their funds either only in risk-free assets or only in risk-free assets, or a combination of both assets (Lintner, 1975; Hariharan et al., 2000). A risk-free asset is an asset whose future rate of return can be ascertained at this time, and is indicated by a return variance equal to zero (Köseoğlu & Mercangöz, 2013). Meanwhile, assets without risk are assets whose actual rate of return in the future still contains uncertainty. One example of an asset without risk is stock (Kresta & Zelinková, 2015). Forming a portfolio is a strategy that is often carried out by investors in risky investment conditions. Basically, an investment portfolio in financial assets consists of various investment opportunities available in the capital market. The essence of forming a portfolio is allocating funds to various investment alternatives, so that investment risks can be reduced (minimized) or optimized (Das et al., 2010).

The formation of this investment portfolio aims to demonstrate an analysis of the Mean-Variance investment portfolio optimization model without risk-free assets (Chen et al., 2021). From the results of the analysis of portfolio risk and return calculations, it is then used to determine the weight composition (proportion) of fund-allocation in each of the assets that make up the optimum portfolio (Gökgöz & Atmaca, 2012). After that, it is also numerically analyzed to optimize the Mean-Variance investment portfolio without risk-free assets. It is hoped that the formation of this portfolio optimization model will be useful as an alternative for investors in optimizing their investment portfolio (Brandtner, 2013).

2. Materials and Methods

2.1. Materials

The data used in the optimization of the mean-variance investment portfolio is PTBA.JK-PT Bukit Asam Tbk, IPCM.JK-PT Jasa Armada Indonesia Tbk, ANTM.JK-PT Aneka Tambang Tbk, BUMI.JK-PT Bumi Resources Tbk stock data , and ADMF.JK-PT Adira Dinamika Finance Tbk. Daily historical data can be accessed through the website http://finance.yahoo.com for the period 21 May 2021 - 20 May 2022.

2.2. Methods

Methods include: the stages and formulas that are used in data analysis, arranged sequentially step by step.The method used in this investment portfolio optimization model is mean-variance investment portfolio optimization without risk-free assets. The steps used are as follows. Suppose there are N risk-free assets with returns $r_1, \dots, r_N$. Assuming that the first and second moments of $r_1, \dots, r_N$ exist, the transpose vector of the expected return value is expressed as: $\mu^T = (\mu_1, \dots, \mu_N)$ where $\mu_i = E[r_i], i = 1, \dots, N$ and matrix covariance is expressed as: $\sum = (\sigma_{ij})$ where $\sigma_{ij} = Cov(r_i, r_j), i, j = 1, \dots, N.$ If the portfolio return is $r_p$with the transpose weight vector $w^T = (w_1, \dots, w_N)$, and the condition $\sum_{i=1}^{N} w_i = 1$, then the expected portfolio return using vector notation, can be expressed as:

$$\mu_p = E[r_p] = \mu^T w = w^T \mu \tag{1}$$

And the portfolio variance can be expressed as:

$$\sigma_p^2 = Var(r_p) = w^T \sum w \tag{2}$$

In Mean–Variance optimization, the efficient portfolio is defined as follows.
definition :

$$a\ portfolio\ p^\wedge *\ is\ (Mean - Variance)\ efficient\ if\ it\ does\ not\ exist$$

$portfolio\ p\ with\ \mu_p \geq \mu_p^*\ and\ \sigma_p^2 < \sigma_{p^*}^2$ (Panjer et al., 1998, Rupert,2004).

To get an efficient portfolio, it means that you have to solve the portfolio optimization problem as follows:

$$\text{Maximize } \{2\tau\mu^T w - w^T \sum w\}, \}, \text{condition } e^T w = 1 \tag{3}$$

With $e^T = (1, \dots, N), \mu^T w = w^T \mu$ and $w^T e = e^T w$.The lagrange function of equation (3) where λ is the multiplier, can be expressed as follows:

$$L(w, \lambda) = (2\tau w^T \mu - w^T \sum w) + \lambda(w^T e - 1) \tag{4}$$

Equation (4) using the necessary conditions of the Khun-Tucker theorem $\frac{\partial L}{\partial \lambda} = 0$ and $\frac{\partial L}{\partial w} = 0$, we get:

$$\frac{\partial L}{\partial w} = 2\tau\mu - 2\sum w + \lambda e = 0 \tag{5}$$

$$\frac{\partial L}{\partial \lambda} = w^T e - 1 = 0 \tag{6}$$

Equation (5) multiply $\sum^{-1}$ and express it in w, then multiply the result e^T,after solving it, we get:

$$w = \frac{1}{e^T \sum^{-1} e} \sum^{-1} e + \tau \left\{ \sum^{-1} \mu - \frac{e^T \sum^{-1} \mu}{e^T \sum^{-1} e} \sum^{-1} e \right\}; \tau \geq 0 \tag{7}$$

When $\tau = 0$ it produces a minimum variance portfolio with weights:

$$w^{min} = \frac{1}{e^T \sum^{-1} e} \sum^{-1} e$$

3. Results and Discussion

3.1. Analysis of stock data (show Return Plot, Descriptive Statistics, and other necessary analyzes)

With the help of Microsoft Excel, descriptive statistics are obtained as shown in the following Table 1.

Table 1: Data descriptive statistics

Stock name	PTBA	IPCM	ANTM	BUMI	ADMF

Mean μ	0.00288	0.00054	0.00068	0.00109	0.00010
Standard Error	0.00147	0.00159	0.00184	0.00243	0.00063
Median	0.00000	0.00000	-0.00396	0.00000	0.00000
Mode	0.00000	0.00000	0.00000	0.00000	0.00000
Standard Deviation	0.02282	0.02474	0.02870	0.03779	0.00986
Sample Variance σ^2	0.00052	0.00061	0.00082	0.00143	0.00010
Kurtosis	1.53705	4.51716	4.49570	4.46746	18.05592
Skewness	0.61006	1.18441	1.14824	-1.10614	-2.88645
Range	0.14562	0.18869	0.22073	0.26455	0.09360
Minimum	-0.05639	-0.06952	-0.06971	-0.19048	-0.06780
Maximum	0.08923	0.11917	0.15102	0.07407	0.02581
Sum	0.69614	0.12961	0.16372	0.26462	0.02446
Count	242.00000	242.00000	242.00000	242.00000	242.00000
Confidence Level (95.0%)	0.00289	0.00313	0.00363	0.00479	0.00125
Rasio $\frac{\mu}{\sigma^2}$	5.54633	0.87853	0.82467	0.76567	1.04291

With the return plot of each stock as follows

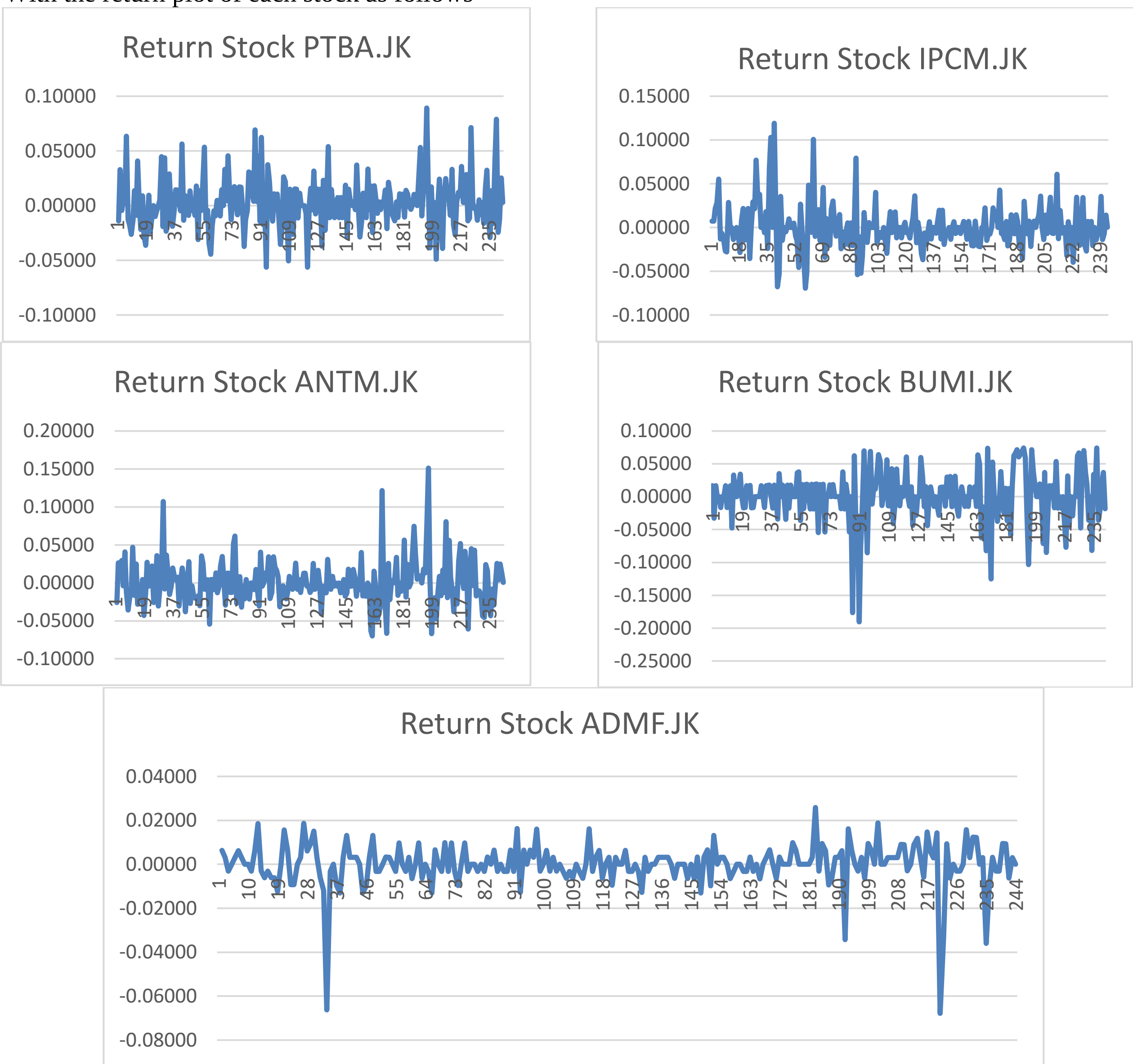

Figure 1: Movement of stock returns

3.2. Formation of Investment Portfolio Optimization with the selected Model (Return expected return vector, unit vector e, variance, covariance matrix Σ, Inverse covariance matrix Σ^{-1}, and other entities needed

Using Microsoft Excel, the covariance values between selected stocks are determined, as shown in the following Table 2.

Table 2: Covariance value between stocks

	PTBA	IPCM	ANTM	BUMI	ADMF
PTBA	0.00052	0.00006	0.00025	-0.00038	0.00001
IPCM	0.00006	0.00061	0.00010	0.00002	0.00005
ANTM	0.00025	0.00010	0.00082	-0.00017	0.00003
BUMI	-0.00038	0.00002	-0.00017	0.00142	-0.00002
ADMF	0.00001	0.00005	0.00003	-0.00002	0.00010

Of the five selected stocks, the estimator of the average value $\mu_i, (i = 1, 2, 3, 4,5)$ is formed by the vector transpose average $\mu^T = (0.00288 \quad 0.00054 \quad 0.00068 \quad 0.00109 \quad 0.00010)$. Then a unit transpose vector is formed $e^T = (1 \quad 1 \quad 1 \quad 1 \quad 1)$. Furthermore, from the estimator table the variance value $\sigma_i^2, (i = 1,2,3,4,5)$ together with the covariance estimator between stock returns, is used to form the covariance matrix $\sum$ and the inverse covariance matrix $\sum^{-1}$ can be determined. The covariance matrix $\sum$ and the inverse covariance matrix $\sum^{-1}$ are expressed as follows:

$$\sum = \begin{pmatrix} 0.00052 & 0.00006 & 0.00025 & -0.00038 & 0.00001 \\ 0.00006 & 0.00061 & 0.00010 & 0.00002 & 0.00005 \\ 0.00025 & 0.00010 & 0.00082 & -0.00017 & 0.00003 \\ -0.00038 & 0.00002 & -0.00017 & 0.00142 & -0.00002 \\ 0.00001 & 0.00005 & 0.00003 & -0.00002 & 0.00010 \end{pmatrix}$$

$$\sum^{-1} = \begin{pmatrix} 2749.11 & -211.69 & -667.50 & 659.77 & 180.42 \\ -211.69 & 1754.57 & -143.54 & -105.01 & -836.13 \\ -667.50 & -143.54 & 1441.28 & -10.96 & -252.48 \\ 659.77 & -105.01 & -10.96 & 881.35 & 174.28 \\ 180.42 & -836.13 & -252.48 & 174.28 & 10819.61 \end{pmatrix}$$

3.3. Investment Portfolio Optimization Process (Generate investment portfolio tables, efficient surface graphs and portfolio ratio graphs)

Using the vector μ^T, e^T and the matrix $\sum^{-1}$, then the weight vector w is calculated using equation (7). Risk tolerance τ provided that $\tau \geq 0$ in investment portfolio optimization, simulated by taking several valuessatisfy the conditions $e^T w = 1$. Taking the risk tolerance value is stopped, if for athe risk tolerance value after being substituted into equation (7) produces a weight $w_i(i = 1,2,3,4,5)$ which is not a positive real number and satisfies $e^T w = 1$. With the help of excel, a table of the Mean-Variance investment portfolio optimization process Table 3 without risk-free assets is obtained as follows:

Table 3: Mean-Variance investment portfolio optimization process

τ	PTBA	IPCM	ANTM	BUMI	ADMF	$w^T e$	μ_p	σ_p^2	$\frac{\mu_p}{\sigma_p^2}$
0.00	0.73138	0.00378	-0.09558	0.25473	0.10570	1.00000	0.00233	0.000183	11.05944
0.10	0.67605	0.00641	-0.08361	0.23976	0.16139	1.00000	0.00217	0.000159	11.84785
0.20	0.62072	0.00904	-0.07165	0.22480	0.21709	1.00000	0.00201	0.000137	12.67917
0.30	0.56539	0.01167	-0.05968	0.20983	0.27278	1.00000	0.00185	0.000118	13.52369
0.40	0.51005	0.01431	-0.04771	0.19487	0.32848	1.00000	0.00169	0.000102	14.32634
0.50	0.45472	0.01694	-0.03574	0.17991	0.38417	1.00000	0.00153	0.000089	14.99534
0.60	0.39939	0.01957	-0.02377	0.16494	0.43987	1.00000	0.00137	0.000079	15.39313
0.70	0.34406	0.02221	-0.01181	0.14998	0.49556	1.00000	0.00121	0.000072	15.34073
0.80	**0.28872**	**0.02484**	**0.00016**	**0.13501**	**0.55126**	**1.00000**	**0.00105**	**0.000067**	**14.65256**
0.90	0.23339	0.02747	0.01213	0.12005	0.60696	1.00000	0.00089	0.000066	13.21168
1.00	0.17806	0.03010	0.02410	0.10509	0.66265	1.00000	0.00073	0.000067	11.05944
1.10	0.12273	0.03274	0.03607	0.09012	0.71835	1.00000	0.00057	0.000072	8.42893
1.20	0.06739	0.03537	0.04804	0.07516	0.77404	1.00000	0.00041	0.000079	5.66976

1.30	0.01206	0.03800	0.06000	0.06019	0.82974	1.00000	0.00024	0.000089	3.10847
1.40	-0.04327	0.04064	0.07197	0.04523	0.88543	1.00000	0.00008	0.000183	0.94749

A series of efficient portfolios are on the efficient frontier. The efficient frontier is an efficient surface on which portfolios are located whose returns are commensurate with the risks. Efficient frontier curve and the ratio between the average return to the portfolio variance looks like in the following Figure 2.

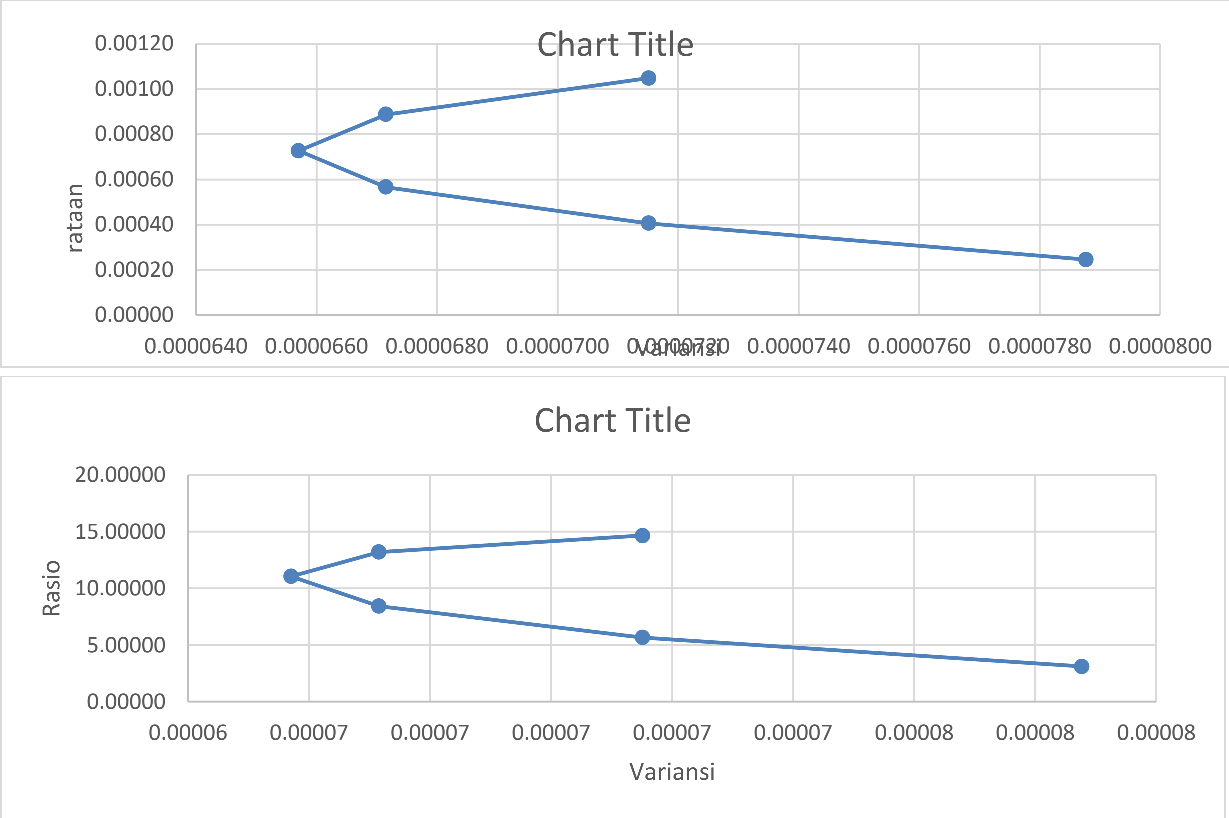

Figure 2: The efficient frontier curve and the ratio between the average return and the portfolio variance

3.4. Discussion (Discussion) -> analysis of the results of forming an optimal investment portfolio

Analysis of the results of forming a mean-variance investment portfolio without risk-free assets is carried out by taking into account the numerically important characteristics in the table

1. The risk tolerance of the mean variance model without risk-free assets ranges from $0.80 \leq \tau \leq 1.3$
2. The minimum portfolio of the Mean-Variance model without risk-free assets obtains an average return of 0.00024 with a variance of 0.000089;
3. For the maximum portfolio of the Mean-Variance model without risk-free assets, the average return is 0.00105 with a variance of 0.000067;
4. The optimum portfolio of the Mean-Variance model without risk-free assets obtained an average return of 0.00105 and a variance of 0.000067 with a portfolio ratio value of 14.65256;
5. The optimum portfolio weight for the Mean-Variance model without risk-free assets is the proportion of PTBA shares = 0.28872, IPCM = 0.02484; ANTM=0.00016; EARTH=0.13501; ADMF=0.55126

4. Conclusion

The formation of an investment portfolio can be done using the mean-variance investment portfolio optimization model without risk-free assets to determine the proportion of selected shares, the optimal proportion of the five selected stocks is 28.872% for PTBA, 2.484% for IPCM, 0.016% for ANTM, 13.501 % for BUMI, and 55.126% for ADMF.

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