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Optimum Fund Allocation Strategy by Considering the Company's Assets and Liabilities

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Abstract

Investment is essentially placing some funds at present with the expectation of future profits. The basic thing that an investor needs to know is that there is a risk that follows the profit/return. In determining the proper allocation of funds, an investor needs to consider the company's assets and liabilities. Company assets can be in the form of shares, property, and others. Meanwhile, the company's liabilities include debts and other obligations. One of the sectors whose company value has stagnated or increased during the Covid-19 Pandemic is the financial sector. Securities companies are a sub-sector of the financial sector which has a fairly strong position during the Pandemic. This research aims to determine the weight of fund allocation in each company forming the optimum portfolio and to see the effect of the company's assets and liabilities on the formation of the optimum portfolio. One of the methods used is the Lagrange Multiplier method for model formulation. The results of this study show that the optimal portfolio weight of PANS companies is 16.31% with an allocation of funds amounting to IDR163.612.976,00, the optimum portfolio weight of RELI companies is 83.003% with an allocation of funds of IDR830,029,681.00, and the optimum portfolio weight of TRIM companies is 0.636% with the allocation of funds amounting to IDR6,358,243.00. In this study, it was also found that the greater the percentage difference between the company's assets and liabilities, the greater the company's optimum portfolio weight.

Keywords: asset; investment; finance; liability; securities company.

1. Introduction

Investment is essentially placing some funds at present with the expectation of future profits. The basic thing that an investor needs to know is that there is a risk that always follows the return or yield, namely the result obtained from an investment. Risk cannot be avoided when investing, therefore, according to financial experts, it is wise to diversify assets into several investment options (Abreu & Mendes, 2010). Diversification aims to maximize returns and minimize risk by investing in different assets that will each react differently to the same events.

In determining the proper allocation of funds, an investor needs to consider the company's assets and liabilities. The discussion in this paper aims to demonstrate an analysis of the mean-variance and asset-liability investment portfolio optimization model to further seek the optimal weight composition and optimal allocation of funds in the financial sector, especially the securities companies sub-sector.

2. Literature Review

2.1 Lagrange Multiplier Method

The lagrange multiplier method can be used to solve optimization problems with equation constraints. The Lagrange function is defined as (Sinha *et al*., 2019):

$$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i g_i(x), \tag{1}$$

suppose there is an optimization problem with one constraint, the form of the Lagrange function can be written as (Sinha *et al*., 2019; Kalaiarasi et al., 2020):

$$L(x,\lambda) = f(x) + \lambda(g(x) - b), \tag{2}$$

the necessary condition for the problem above is to partially derive from L to x and λ as (Sinha *et al*., 2019):

$$\frac{\partial L}{\partial x_i} = 0, \text{ for } i = 1,2,\dots,n \quad (3)$$
$$\frac{\partial L}{\partial x_i} = 0.$$

2.2 Mean-Variance Investment Portfolio Optimization

Suppose there are N normally distributed stocks with returns $r_1,\dots,r_N$. Then the expected return value vector is given by $\boldsymbol{\mu}^T = (\mu_1,\dots,\mu_N)$ with $\mu_i = E[r_i], i = 1,\dots,N$. The variance-covariance matrix is given by $\boldsymbol{\Sigma} = (\sigma_{ij}), i,j = 1,\dots,N$ with $\sigma_{ij} = Cov(r_i,r_j), i,j = 1,\dots,N$. Portfolio returns with weight vectors $\mathbf{w}^T = (w_1,\dots,w_N)$ where required $\sum_{i=1}^{N} w_i = 1$ is:

$$r_p = \mathbf{w}^T\mathbf{r}, \quad (4)$$

portfolio return expectations are formulated as:

$$\mu_p = E[r_p] = \mathbf{w}^T\boldsymbol{\mu} = \boldsymbol{\mu}^T\mathbf{w}, \quad (5)$$

and portfolio variance is formulated as:

$$\sigma_p^2 = Var(r_p) = \mathbf{w}^T.\boldsymbol{\Sigma}.\mathbf{w}, \quad (6)$$

to get an efficient portfolio, the objective function that is usually used (Pandiangan & Hasbullah, 2021):

$$\text{Maximize } \{2\tau\mu_p - \sigma_p^2\},$$
$$\text{with constraints } \sum_{i=1}^{N} w_i = 1$$

Based on equations (5) and (6), the above model can be written as:

$$\text{Maximize } \{2\tau\boldsymbol{\mu}^T\mathbf{w} - \mathbf{w}^T.\boldsymbol{\Sigma}.\mathbf{w}\}, \quad (7)$$
$$\text{with constraints } \mathbf{e}^T\mathbf{w} = \mathbf{w}^T\mathbf{e} = 1$$

with $\mathbf{e}^T = (1,1\dots,1)$ is a transpose matrix of the unit vectors.

The Lagrangian multiplier function of the portfolio optimization problem according to equation (2) is given by:

$$L(w,\lambda) = 2\tau\boldsymbol{\mu}^T\mathbf{w} - \mathbf{w}^T.\boldsymbol{\Sigma}.\mathbf{w} + \lambda(\mathbf{w}^T\mathbf{e} - 1), \quad (8)$$

or

$$L(w,\lambda) = 2\tau\mathbf{w}^T\boldsymbol{\mu} - \mathbf{w}^T.\boldsymbol{\Sigma}.\mathbf{w} + \lambda(\mathbf{w}^T\mathbf{e} - 1). \quad (9)$$

The optimality conditions of the above equation are:

$$\frac{\partial L}{\partial \mathrm{w}} = 2\tau\boldsymbol{\mu} - 2.\boldsymbol{\Sigma}.\mathbf{w} + \lambda\mathbf{e} = 0 \quad (10)$$
$$\frac{\partial L}{\partial \lambda} = \mathbf{w}^T\mathbf{e} - 1 = 0, \quad (11)$$

equation (10) multiplied by $\boldsymbol{\Sigma}^{-1}$ becomes:

$$2\tau.\boldsymbol{\Sigma}^{-1}.\boldsymbol{\mu} - 2.\boldsymbol{\Sigma}^{-1}.\boldsymbol{\Sigma}.\mathbf{w} + \lambda.\boldsymbol{\Sigma}^{-1}.\mathbf{e} = 0$$
$$2\tau.\boldsymbol{\Sigma}^{-1}.\boldsymbol{\mu} - 2\mathbf{w} + \lambda.\boldsymbol{\Sigma}^{-1}.\mathbf{e} = 0, \quad (12)$$

equation (12) divided by 2 get:

$$\tau.\boldsymbol{\Sigma}^{-1}.\boldsymbol{\mu} - \mathbf{w} + \frac{1}{2}.\lambda.\boldsymbol{\Sigma}^{-1}.\mathbf{e} = 0$$
$$\mathbf{w} = \frac{1}{2}\lambda.\boldsymbol{\Sigma}^{-1}.\mathbf{e} + \tau.\boldsymbol{\Sigma}^{-1}.\boldsymbol{\mu}, \quad (13)$$

equation (13) multiplied by $\mathbf{e}^T$ becomes:

$$\mathbf{e}^T\mathbf{w} = \frac{1}{2}\lambda.\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\mathbf{e} + \tau.\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\boldsymbol{\mu}, \quad (14)$$

because $\mathbf{w}^T\mathbf{e} = \mathbf{e}^T\mathbf{w} = 1$, so:

$$1 = \frac{1}{2}\lambda.\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\mathbf{e} + \tau\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\boldsymbol{\mu}, \quad (15)$$

equation (15) divided by $\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\mathbf{e}$ get:

$$\frac{1}{\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\mathbf{e}} = \frac{1}{2}\lambda + \frac{\tau\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\boldsymbol{\mu}}{\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\mathbf{e}}$$
$$\frac{1}{2}\lambda = \frac{1}{\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\mathbf{e}} - \frac{\tau.\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\boldsymbol{\mu}}{\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\mathbf{e}}, \quad (16)$$

substituting equation (16) into equation (13) gets:

$$\mathbf{w} = \left\{\frac{1}{\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\mathbf{e}} - \frac{\tau.\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\boldsymbol{\mu}}{\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\mathbf{e}}\right\}\boldsymbol{\Sigma}^{-1}.\mathbf{e} + \tau.\boldsymbol{\Sigma}^{-1}.\boldsymbol{\mu}$$
$$\mathbf{w} = \frac{1}{\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\mathbf{e}}\boldsymbol{\Sigma}^{-1}.\mathbf{e} - \frac{\tau.\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\boldsymbol{\mu}}{\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\mathbf{e}}\boldsymbol{\Sigma}^{-1}.\mathbf{e} + \tau.\boldsymbol{\Sigma}^{-1}.\boldsymbol{\mu}$$
$$\mathbf{w} = \frac{1}{\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\mathbf{e}}\boldsymbol{\Sigma}^{-1}.\mathbf{e} + \tau\left\{\boldsymbol{\Sigma}^{-1}.\boldsymbol{\mu} - \frac{\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\boldsymbol{\mu}}{\mathbf{e}^T.\boldsymbol{\Sigma}^{-1}.\mathbf{e}}\boldsymbol{\Sigma}^{-1}.\mathbf{e}\right\}; \tau \geq 0, \quad (17)$$

If determined $\tau = 0$, produces a minimum variance portfolio with a weight vector:

$$\mathbf{w}^{Min} = \frac{1}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}}\mathbf{\Sigma}^{-1}.\mathbf{e}, \tag{18}$$

if $\mathbf{z} = \mathbf{\Sigma}^{-1}.\boldsymbol{\mu} - \frac{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\boldsymbol{\mu}}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}}\mathbf{\Sigma}^{-1}\mathbf{e}$ then the weight vector when $\tau > 0$ based on equation (17) can be written as:

$$\mathbf{w} = \mathbf{w}^{Min} + \tau\mathbf{z}, \tag{19}$$

with $\mathbf{w}^{Min}$ is the minimum variance portfolio that depends on the variance-covariance matrix $\mathbf{\Sigma}$ but does not depend on the vector $\boldsymbol{\mu}, \mathbf{\Sigma}^{-1}$ is the inverse of the variance-covariance matrix $\mathbf{\Sigma}$, and $\mathbf{w}$ is the weight vector when $\tau > 0$ that depends on $\mathbf{\Sigma}$ and $\boldsymbol{\mu}$ and has the property $\sum_{i=1}^{N} z_i = 0$.

2.3 Mean-Variance Investment Portfolio Optimization with Assets-Liabilities

a) Asset Liability Model

Surplus is the difference in value between assets and liabilities (Sukono *et al*., 2018), when $t = 0$, the initial surplus is:

$$S_0 = A_0 - L_{0,} \tag{20}$$

with S_0 is a surplus when $t = 0, A_0$ is an asset when $t = 0$, and L_0 is a liability when $t = 0$.

The surplus obtained after one period is (Keel & Muller, 1995) :

$$S_1 = A_1 - L_1 = A_0[1 + r_A] - L_0[1 + r_L], \tag{21}$$

with S_1 is a surplus when $t = 1$, A_1 is an asset when $t = 1, L_1$ is a liability when $t = 1, r_A$ is a return on assets, and r_L is a return on liabilities. Furthermore, if r_s is a surplus return then r_s is formulated by :

$$r_S = \frac{S_1 - S_0}{A_0} \tag{22}$$

b) Mean-Variance Portfolio Optimization Model with Assets-Liabilities

The mean-variance investment portfolio model is developed by considering the liabilities or obligations that must be paid in the future. Let $\mathbf{w}^T = (w_1, \dots, w_N)$ is the weighting vector of the portfolio weighting resulting from the surplus return,$\boldsymbol{\mu}^T = \left(\mu_{S_1}, \dots, \mu_{S_N}\right)$ is the average vector with $\mu_{S_i} = E\left[r_{S_i}\right], i = 1, \dots, N$, $\mathbf{e}^T$ is the unit vector and $\mathbf{\Sigma} = \left(\sigma_{ij}\right), j = 1, \dots, N$ is the variance-covariance matrix of the surplus return with $\sigma_{ij} = Cov\left(r_{S_i}, r_{S_j}\right)$. So that the average surplus return from a portfolio can be determined by:

$$\hat{\mu}_{S_p} = \boldsymbol{\mu}_S^T\mathbf{w}, \tag{23}$$

with $\hat{\mu}_{S_p}$is the average surplus return of a portfolio and $\boldsymbol{\mu}_S^T$ a transpose sector of the average surplus return. The surplus return variance of a portfolio can be determined by:

$$\hat{\sigma}_{S_p}^2 = \mathbf{w}^T.\mathbf{\Sigma}.\mathbf{w}. \tag{24}$$

with $\hat{\sigma}_{S_p}^2$ is the variance of the surplus return of a portfolio. If $\boldsymbol{\gamma}^T = (\gamma_1, \dots, \gamma_N)$ is the covariance vector between return on assets and return on liabilities, then the optimization of surplus return can be expressed by:

$$\text{Maximize } \{2\tau\boldsymbol{\mu}_S^T\mathbf{w} + \mathbf{2}\boldsymbol{\gamma}^T\mathbf{w} - \mathbf{w}^T.\mathbf{\Sigma}.\mathbf{w}\}, \tag{25}$$

$$\text{with constraints } \mathbf{e}^T\mathbf{w} = 1,$$

or

$$\text{Minimize } \{\mathbf{w}^T.\mathbf{\Sigma}.\mathbf{w} - 2\tau\boldsymbol{\mu}_S^T\mathbf{w} - \mathbf{2}\boldsymbol{\gamma}^T\mathbf{w}\}, \tag{26}$$

$$\text{with constraints } \mathbf{e}^T\mathbf{w} = 1$$

Equation (26) can be solved with Lagrange multipliers. The Lagrange function of equation (26) is:

$$L(w, \lambda) = \mathbf{w}^T.\mathbf{\Sigma}.\mathbf{w} - 2\tau\boldsymbol{\mu}_S^T\mathbf{w} - 2\boldsymbol{\gamma}^T\mathbf{w} + \lambda(\mathbf{e}^T\mathbf{w} - 1),$$

or

$$L(w, \lambda) = \mathbf{w}^T.\mathbf{\Sigma}.\mathbf{w} - 2\tau\boldsymbol{w}^T\boldsymbol{\mu}_S - 2\boldsymbol{\gamma}^T\mathbf{w} + \lambda(\mathbf{e}^T\mathbf{w} - 1). \tag{27}$$

The conditions for optimization in equation (27) are obtained by the first derivative, namely:

$$\frac{\partial L}{\partial w} = 2\mathbf{\Sigma}\mathbf{w} - 2\tau\boldsymbol{\mu}_S - 2\boldsymbol{\gamma}^T + \lambda\mathbf{e} = 0 \tag{28}$$

$$\frac{\partial L}{\partial \lambda} = \mathbf{w}^T\mathbf{e} - 1 = 0, \tag{29}$$

equation (28) multiplied by $\mathbf{\Sigma}^{-1}$ becomes:

$$2.\mathbf{\Sigma}^{-1}.\mathbf{\Sigma}\mathbf{w} - 2\tau.\mathbf{\Sigma}^{-1}\boldsymbol{\mu}_S - 2.\mathbf{\Sigma}^{-1}.\boldsymbol{\gamma}^T + \lambda.\mathbf{\Sigma}^{-1}.\mathbf{e} = 0$$

$$2\mathbf{w} - 2\tau.\mathbf{\Sigma}^{-1}.\boldsymbol{\mu}_S - 2.\mathbf{\Sigma}^{-1}.\boldsymbol{\gamma}^T + \lambda.\mathbf{\Sigma}^{-1}.\mathbf{e} = 0, \tag{30}$$

equation (30) divided by 2 becomes:

$$\mathbf{w} - \tau.\mathbf{\Sigma}^{-1}.\boldsymbol{\mu}_S - \mathbf{\Sigma}^{-1}.\boldsymbol{\gamma}^T + \frac{1}{2}.\lambda.\mathbf{\Sigma}^{-1}.\mathbf{e} = 0$$

$$\mathbf{w} = \tau.\mathbf{\Sigma}^{-1}.\boldsymbol{\mu}_S + \mathbf{\Sigma}^{-1}.\boldsymbol{\gamma}^T - \frac{1}{2}.\lambda.\mathbf{\Sigma}^{-1}.\mathbf{e}, \tag{31}$$

equation (31) multiplied by $\mathbf{e}^T$becomes:

$$\mathbf{e}^T\mathbf{w} = \tau.\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\boldsymbol{\mu}_S + \mathbf{e}^T.\mathbf{\Sigma}^{-1}.\boldsymbol{\gamma}^T - \frac{1}{2}.\lambda.\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}, \quad (32)$$

because $\mathbf{e}^T\mathbf{w} = 1$, gets:

$$1 = \tau.\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\boldsymbol{\mu}_S + \mathbf{e}^T.\mathbf{\Sigma}^{-1}.\boldsymbol{\gamma}^T - \frac{1}{2}.\lambda.\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}, \quad (33)$$

equation (33) divided by $\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e}$ becomes:

$$\frac{1}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}} = \frac{\tau.\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\boldsymbol{\mu}_S}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}} + \frac{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\boldsymbol{\gamma}^T}{\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e}} - \frac{\frac{1}{2}.\lambda.\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}}$$

$$\frac{1}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}} = \frac{\tau.\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\boldsymbol{\mu}_S}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}} + \frac{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\boldsymbol{\gamma}^T}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}} - \frac{1}{2}\lambda$$

$$-\frac{1}{2}\lambda = \frac{1}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}} - \frac{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\boldsymbol{\gamma}^T}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}} - \frac{\tau.\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\boldsymbol{\mu}_S}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}}, \quad (34)$$

substituting equation (34) into equation (31) gets:

$$\mathbf{w} = \tau.\mathbf{\Sigma}^{-1}.\boldsymbol{\mu}_S + \mathbf{\Sigma}^{-1}.\boldsymbol{\gamma}^T + \left\{\frac{1}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}} - \frac{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\boldsymbol{\gamma}^T}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}} - \frac{\tau.\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\boldsymbol{\mu}_S}{\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e}}\right\}\mathbf{\Sigma}^{-1}\mathbf{e}$$

$$\mathbf{w} = \tau.\mathbf{\Sigma}^{-1}.\boldsymbol{\mu}_S + \mathbf{\Sigma}^{-1}.\boldsymbol{\gamma}^T + \frac{1}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}}\mathbf{\Sigma}^{-1}.\mathbf{e} - \frac{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\boldsymbol{\gamma}^T}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}}\mathbf{\Sigma}^{-1}.\mathbf{e} - \frac{\tau.\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\boldsymbol{\mu}_S}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}}\mathbf{\Sigma}^{-1}\mathbf{e}$$

$$\mathbf{w} = \frac{1}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}}\mathbf{\Sigma}^{-1}.\mathbf{e} + \tau.\mathbf{\Sigma}^{-1}.\boldsymbol{\mu}_S - \frac{\tau.\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\boldsymbol{\mu}_S}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}}\mathbf{\Sigma}^{-1}.\mathbf{e} + \mathbf{\Sigma}^{-1}.\boldsymbol{\gamma}^T - \mathbf{\Sigma}^{-1}.\boldsymbol{\gamma}^T\frac{\mathbf{e}^T}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}}\mathbf{\Sigma}^{-1}.\mathbf{e}$$

$$\mathbf{w} = \frac{1}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}}\mathbf{\Sigma}^{-1}.\mathbf{e} + \tau\left\{\mathbf{\Sigma}^{-1}.\boldsymbol{\mu}_S - \frac{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\boldsymbol{\mu}_S}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}}\mathbf{\Sigma}^{-1}.\mathbf{e}\right\} + \mathbf{\Sigma}^{-1}.\boldsymbol{\gamma}^T\left\{1 - \frac{\mathbf{e}^T}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}}\mathbf{\Sigma}^{-1}.\mathbf{e}\right\}; \tau \geq 0, \quad (35)$$

when $\tau = 0$ produces a minimum variance portfolio with weights:

$$\mathbf{w}^{Min,L} = \frac{1}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}}\mathbf{\Sigma}^{-1}.\mathbf{e} + \mathbf{\Sigma}^{-1}.\boldsymbol{\gamma}^T\left\{1 - \frac{\mathbf{e}^T}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}}\mathbf{\Sigma}^{-1}.\mathbf{e}\right\}. \quad (36)$$

The first part of the equation (36) is:

$$\mathbf{w}^{Min} = \frac{1}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}}\mathbf{\Sigma}^{-1}.\mathbf{e}, \quad (37)$$

the second part of the equation (36) is:

$$\mathbf{z}^L = \mathbf{\Sigma}^{-1}.\boldsymbol{\gamma}^T\left\{1 - \frac{\mathbf{e}^T}{\mathbf{e}^T.\mathbf{\Sigma}^{-1}.\mathbf{e}}\mathbf{\Sigma}^{-1}.\mathbf{e}\right\}, \text{ with } \sum_{i=1}^{N} z_i^L = 0, \quad (38)$$

so that the equation for the minimum variance portfolio with liabilities can be expressed by:

$$\mathbf{w}^{Min,L} = \mathbf{w}^{Min} + \mathbf{z}^L, \quad (39)$$

for $\tau > 0$ based on equation (35) obtained:

$$\mathbf{w}^* = \mathbf{w}^{Min,L} + \tau\mathbf{z}^* \quad (40)$$

$\mathbf{z}^* = \left\{\mathbf{\Sigma}^{-1}\boldsymbol{\mu} - \frac{\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e}}\mathbf{\Sigma}^{-1}\mathbf{e}\right\}$ and $\sum_{i=1}^{N} z_i^* = 0$.

2.4 Allocation of Funds for Each Company Based on Optimum Portfolio Weight

Let the funds to be invested are denoted B, and the optimal portfolio weight for each stock is w_a, and the allocated weight for each stock is w_{B_a} then the formula for weight/proportion of fund allocation is:

$$w_{B_a} = B \times w_a \quad (41)$$

3. Materials and Methods

3.1. Materials

The object of this study is monthly stock closing data in the financial sector, especially securities companies, namely PANS, RELI, and TRIM companies, as well as data on the assets and liabilities of the three companies for the period 1 January 2020 – 1 September 2022. Calculations in this research were assisted by Microsoft Excel 2019 software.

3.2. Methods

The steps in conducting this research are:

a. Collect all the required data obtained from the website https://finance.yahoo.com and the financial reports for each share.
b. Optimizing the mean-variance investment portfolio as in equations (7) to (19) until the equation obtains the optimum portfolio weight.

c. Optimizing the mean-variance investment portfolio and asset liabilities as in equation (26) to equation (40) until the optimum portfolio weight is obtained.
d. Calculating the allocation of funds based on the optimum portfolio weight based on equation (41).
e. Compare the results of the first method with the second method and conclude.

4. Results and Discussion

4.1. Mean-Variance Investment Portfolio Optimization

a) Mean Vector and Unit Vector

Before determining the average vector, the average return of 3 stocks is calculated first using the Excel 2019 software in Table 1:

Table 1: Data on average individual stock returns

Num	Code	Average
1	PANS	0.0118
2	RELI	0.0410
3	TRIM	0.0312

Based on the average value in Table 1, the average vector is formed as:

$$\boldsymbol{\mu}^T = [0{,}0118 \quad 0{,}0410 \quad 0{,}0312]$$

Because there are 3 stocks analyzed, the unit vector consists of 3 elements with a value of 1 is:

$$\mathbf{e}^T = [1 \quad 1 \quad 1]$$

b) Variance-Covariance Matrix

Before forming the variance-covariance matrix, it is necessary to determine the covariance of individual stock returns between 3 stocks. The results of these calculations are shown in Table 2.

Table 2: Covariance of individual stock returns between 3 stocks

	PANS	RELI	TRIM
PANS	0.0117	0.0028	-0.0024
RELI	0.0028	0.0164	0.0022
TRIM	-0.0024	0.0022	0.0227

After obtaining the stock variance and covariance values in Table 2, the variance-covariance matrix is formed as:

$$\boldsymbol{\Sigma} = \begin{bmatrix} 0.0117 & 0.0028 & -0.0024 \\ 0.0028 & 0.0164 & 0.0022 \\ -0.0024 & 0.0022 & 0.0227 \end{bmatrix}$$

The variance-covariance matrix above is determined by the inverse using Microsoft Excel 2019 software to produce:

$$\boldsymbol{\Sigma}^{-1} = \begin{bmatrix} 91.488 & -17.031 & 11.200 \\ -17.031 & 65.023 & -8.167 \\ 11.200 & -8.167 & 46.115 \end{bmatrix}$$

c) Calculation Process

In this process, the tau value (τ) or risk tolerance value is determined using a simulation with values starting from $0 \leq \tau \leq 0.37$ with an increase of 0.01. The process of determining the optimum weight is carried out using equation (17) is given in Table 3.

Table 3: Mean-variance investment portfolio optimization data

Num	τ	PANS	RELI	TRIM	$\sum w$	Portfolio Average (μ_p)	Portfolio Variance (σ_p^2)	Risk (σ_p)	Ratio (μ_p/σ_p^2)
1	0	0.49051	0.22805	0.28144	1	0.0239	0.0057	0.0757	4.1774
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
12	0.11	0.34539	0.36672	0.28789	1	0.0281	0.0062	0.0787	4.5426
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
38	0.37	0.00238	0.69448	0.30313	1	0.0380	0.0109	0.1045	3.4754
39	0.38	-0.01081	0.70709	0.30372	1	0.0384	0.0112	0.1059	3.4210
					MIN	0.0239	0.0057	0.0757	3.4754
					MAX	0.0380	0.0109	0.1045	4.5426

Based on Table 3, the minimum portfolio occurs when the value of $\tau = 0$ with a portfolio average of 0.0239 and a portfolio variance of 0.0057, the maximum portfolio occurs when the value of $\tau = 0.37$ with a portfolio average of 0.0380 and a portfolio variance of 0. 0109, and the optimum portfolio is the portfolio that has the largest ratio when the value of $\tau = 0.11$ with a portfolio average of 0.0281 and a portfolio variance of 0.0062.

From the average portfolio and risk values in Table 3, an efficient frontier graph can be formed. The efficient frontier is an efficient surface where portfolios are located whose average return is commensurate with the risk (Gusliana & Salih, 2022). The efficient frontier graph is given in Figure 1.

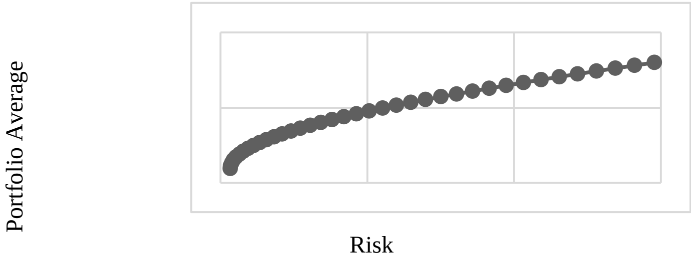

Figure 1: Efficient portfolio surface graph

From Figure 1 it can be seen that the higher the portfolio risk, the higher the average portfolio return.

4.2. Mean-Variance Investment Portfolio Optimization with Assets-Liabilities

a) Calculation of Return Surplus

In this section, the calculation of return surplus involves data on return on assets and return on liabilities. The asset and liability data analyzed is in the form of quarterly data for the period January 2020 to September 2022. The percentage difference between each company's assets and liabilities can be seen in Table 4.

Table 4: The percentage of the difference between the assets and liabilities of each company

Num	PANS	RELI	TRIM
1	62.55%	79.78%	27.08%
2	59.50%	80.36%	35.43%
3	63.65%	76.54%	28.81%
4	46.17%	78.23%	30.73%
5	57.39%	79.42%	24.13%
6	60.86%	80.41%	33.11%
7	51.22%	78.92%	27.59%
8	70.87%	78.37%	48.20%
9	68.31%	75.49%	35.89%
10	71.81%	78.38%	32.47%
11	69.25%	74.95%	32.41%

The calculation of return surplus refers to equation (22). The results of calculating the return surplus can be seen in Table 5.

Table 5: Surplus return data

No	PANS	RELI	TRIM
1	0.0097	0.0063	-0.0108
2	0.0115	0.0084	0.0070
3	0.0075	0.0090	0.0036
4	-0.0100	0.0132	0.0068
5	0.0210	-0.0020	0.0019
6	0.0315	0.0159	0.0121
7	0.0308	0.0098	0.0061
Average	0.0146	0.0087	0.0038

b) Mean Vector and Unit Vector

Based on the average value in Table 5, the average vector is formed as:

$$\boldsymbol{\mu}^{\boldsymbol{T}} = [0{,}0146 \quad 0{,}0087 \quad 0{,}0038]$$

Becuse there are 3 data analyzed, the unit vector consists of 3 elements with a value of 1 is:

$$\mathbf{e}^T = [1 \quad 1 \quad 1]$$

c) Gamma Vector and the Variance-Covariance Matrix

Before determining the gamma vector, the covariance between asset returns and liability returns is determined first because the gamma vector is the covariance vector between asset returns and liability returns. Calculation of the covariance between return on assets and return on liabilities with the help of Microsoft Excel 2019 software produces Table 6.

Table 6: Asset return covariance data and liability returns

Num	Covariance
1	0.1239
2	0.0057
3	0.1063

After obtaining the covariance between return on assets and return on liabilities, a gamma vector can be created as:

$$\gamma^T = [0.1239 \quad 0.0057 \quad 0.1063]$$

Furthermore, to make the variance-covariance matrix, it is necessary to determine the covariance between the three return surpluses first. Calculation of the covariance between the three return surpluses with the help of Microsoft Excel 2019 software produces Table 7.

Table 7: Covariance return surplus data

	PANS	RELI	TRIM
PANS	0.000182	-0.000004	0.000022
RELI	-0.000004	0.000028	0.000019
TRIM	0.000022	0.000019	0.000044

Based on the values in Table 7, the variance-covariance matrix of the return surplus is formed as:

$$\mathbf{\Sigma} = \begin{bmatrix} 0.000182 & -0.000004 & 0.000022 \\ -0.000004 & 0.000028 & 0.000019 \\ 0.000022 & 0.000019 & 0.000044 \end{bmatrix}$$

The variance-covariance matrix above is determined by the inverse using Microsoft Excel 2019 software to produce:

$$\mathbf{\Sigma}^{-1} = \begin{bmatrix} 6179.039 & 4229.490 & -4857.952 \\ 4229.490 & 52839.372 & -24199.028 \\ -4857.952 & -24199.028 & 35095.557 \end{bmatrix}$$

d) Calculation Process

In this process, the tau value (τ) is determined using a simulation with values starting from $0 \leq \tau \leq 0.00065$ with an increase of 0.00005. The process of determining the optimum weight is carried out using equation (35) given in Table 8.

Table 8: Data on optimization of the mean-variance portfolio and assets and liabilities

Num	τ	PANS	RELI	TRIM	$\sum w$	Portfolio Average (μ_p)	Portfolio Variance (σ_p^2)	Risk (σ_p)	Ratio (μ_p/σ_p^2)
1	0	0.1248	0.7393	0.1358	1	0.0087	0.0000225	0.0047	388.6653
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
14	0.00065	0.1636	0.8300	0.0064	1	0.0096	0.0000230	0.0048	416.4305
15	0.0007	0.1666	0.8370	-0.0036	1	0.0097	0.0000231	0.0048	417.6769
					MIN	0.0087	0.0000225	0.0047	388.6653
					MAX	0.0096	0.0000230	0.0048	416.4305

Based on Table 8, the minimum portfolio occurs when the value of $\tau = 0$ with a portfolio average of 0.0087 and a portfolio variance of 0.0000225, the maximum portfolio occurs when the value of $\tau = 0.00065$ with a portfolio average of 0.0096 and a portfolio variance of 0. 0000230, and the optimum portfolio is the portfolio that has the largest ratio when the value of $\tau = 0.00065$ with an average portfolio of 0.0096 and a portfolio variance of 0.0000230.

From the average portfolio and risk values in Table 8, an efficient frontier graph can be formed. The efficient frontier graph is given in Figure 2.

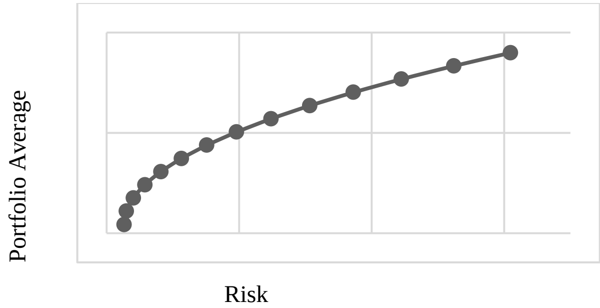

Figure 2: Efficient portfolio surface graph

From Figure 2 it can be seen that the higher the portfolio risk, the higher the average portfolio return.

4.3. Allocation of Funds for Each Company Based on Optimum Portfolio Weight

The allocation of funds for each company is calculated based on the weight obtained by referring to equation (41). If it is assumed that an investor has funds of IDR1.000.000.000,00. then the allocation of funds can be seen in Table 9.

Table 9: Optimum fund allocation data based on optimum portfolio weight with the mean-variance investment portfolio model

τ	Portfolio Minimum		PANS	RELI	TRIM
0	Weight (%)	100%	49.051%	22.805%	28.144%
	Fund allocation	IDR1,000,000,000.00	IDR490,506,406.00	IDR228,054,202.00	IDR281,439,393.00
τ	Portfolio Maksimum		PANS	RELI	TRIM
0.37	Weight (%)	100%	0.238%	69.448%	30.313%
	Fund allocation	IDR1,000,000,000.00	IDR2,384,989.00	IDR694,482,912.00	IDR303,132,099.00
τ	Portfolio Optimum		PANS	RELI	TRIM
0.11	Weight (%)	100%	34.539%	36.672%	28.789%
	Fund allocation	IDR1,000,000,000.00	IDR345,389,228.00	IDR366,722,197.00	IDR287,888,576.00

From Table 9, it can be seen that the optimum portfolio is obtained when $\tau = 0.11$ with the weight and allocation of funds for each share as:

1. PANS 34.539% with the allocation of funds IDR345,389,228.00
2. RELI 36.672% with the allocation of funds IDR366,722,197.00
3. TRIM 28.789% with the allocation of funds IDR287,888,576.00.

From these results, it can be concluded that the largest weight is found in RELI company shares (PT. Reliance Sekuritas Indonesia Tbk) and the smallest weight is in TRIM company shares (PT. Trimegah Sekuritas Indonesia Tbk). Next, the calculation results are displayed using the Mean-variance Investment portfolio model and Asset Liability in Table 10.

Table 10: Optimum fund allocation data is based on the optimum portfolio weight of the mean-variance investment portfolio model and asset liabilities

τ	Portfolio Minimum		PANS	RELI	TRIM
0	Weight (%)	100%	12.485%	73.933%	13.582%
	Fund allocation	IDR1,000,000,000.00	IDR124,847,122.00	IDR739,329,350.00	IDR135,823,527.00
τ	Portfolio Maksimum		PANS	RELI	TRIM
0.00065	Weight (%)	100%	16.361%	83.003%	0.636%
	Fund allocation	IDR1,000,000,000.00	IDR163,612,076.00	IDR830,029,681.00	IDR6,358,243.00
τ	Portfolio Optimum		PANS	RELI	TRIM
0.00065	Weight (%)	100%	16.361%	83.003%	0.636%
	Fund allocation	IDR1,000,000,000.00	IDR163,612,076.00	IDR830,029,681.00	IDR6,358,243.00

From Table 10, it can be seen that the optimum portfolio is obtained when $\tau = 0.00065$ with the weight and allocation of funds for each share as:

1. PANS 16.361% with the allocation of funds IDR163,612,076.00
2. RELI 83.003% with the allocation of funds IDR830,029,681.00
3. TRIM 0.636% with the allocation of funds IDR6,358,243.00.

In this method assets and liabilities affect the weight of a company, the greater the percentage difference between the assets and liabilities of a company, the greater the weight, and vice versa. This is following the data in Table 4 which shows that the percentage difference between the value of the assets and liabilities of RELI companies has the largest percentage difference and TRIM companies have the smallest difference.

5. Conclussion

Based on the analysis results, the optimum portfolio weight of each company in the nominated securities company sub-sector, namely for PANS, RELI, and TRIM stocks respectively is 16.361%, 83.003%, and 0.636%. Based on the optimum portfolio weight that has been obtained, an allocation of funds for each company in the securities company sub-sector that has been nominated, namely the PANS, RELI, and TRIM companies respectively IDR163,612,076.00, IDR830,029,681.00, and IDR6,358,243.00, respectively. The effect of the company's assets and liabilities on portfolio optimization is that the greater the percentage difference between the company's assets and liabilities, the greater the resulting weight, and vice versa. In this study, the largest percentage difference is RELI company and the smallest percentage difference is TRIM company which results in RELI company having the biggest weight and TRIM company having the smallest weight.

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