



Determining Pure Premium of Motor Vehicle Insurance with Generalized Linear Models (GLM)

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Abstract

Motor vehicle insurance guarantees protection, coverage, and compensation for the risks of accidents, damages, and loss of motor vehicles. It is crucial for companies to determine appropriate insurance premium rates as a preventive measure to avoid difficulties in meeting claims filed by policyholders. This research aims to determine the pure premium of motor vehicle insurance using the Generalized Linear Models (GLM) method, which utilizes the concept of a general linear relationship between independent variables and the dependent/response variable, as well as identifying motor vehicle characteristics that influence the determination of pure premiums. The data used in this study is from Swedish motor vehicle insurance. The research aims to determine the pure premium in the data by modeling claim frequency using the Poisson distribution and claim severity using the Gamma distribution, depending on the significantly influential characteristics. The Maximum Likelihood Estimation method is employed for parameter estimation. After conducting the research, the estimated parameters β , γ , and the pure premium of motor vehicle insurance are found to be 35,572,223.27 kr, with the characteristics influencing the pure premium being the distance traveled by the vehicle, the insured's geographic zone, and the no-claim bonus.

Keywords: Motor vehicle insurance, pure premium, Poisson distribution, Gamma distribution, claim frequency, claim amount, Generalized Linear Models, Maximum Likelihood

1. Introduction

The increasing number of motor vehicle ownership in Indonesia can contribute to traffic congestion, accidents, and even fatalities. Additionally, it allows for a higher incidence of criminal activities such as motor vehicle theft, leading to financial losses for the vehicle owners. To mitigate such risks, motor vehicle owners can take advantage of motor vehicle insurance products. Essentially, insurance serves as a provider of financial protection services (David, 2015). The fundamental role of insurance is to offer financial coverage in the form of benefits obtained from premiums paid by policyholders during the coverage period (Putra et al., 2021). In nearly all lines of general insurance business, coverage is typically limited to a 12-month period (Suwardi & Purwono, 2021).

The increasing risk of losses accompanying the development of the automotive industry in Indonesia has led motor vehicle insurance companies to compete to become the most sought-after insurance products by the public. As a result, many insurance companies offer low premium prices. However, this can backfire on the companies as their revenue becomes lower than the claims paid out, making it difficult to meet the claims submitted by policyholders. Therefore, determining premiums in motor vehicle insurance is crucial for insurance companies.

The method used to calculate insurance premium for losses is by combining the expected claim frequency with the expected claim amount while considering the influence of risk characteristics. One of these methods is the Generalized Linear Models (GLM) method, referring to McCullagh & Nelder (2019), (Annette J. & Barnett, 2008), and (Hardin & Joseph, 2007). GLM extends the framework of linear regression models from the Normal distribution to a class of distributions from the Exponential family. This allows for modeling a wide range of variable types (count, frequency, etc.) and handling the skewed probability distributions of the data (Kafková & Krivánková, 2014).

2. Literature Review

2.1 Motor Vehicle Insurance

One type of insurance is motor vehicle insurance. In this insurance, there is a combined coverage, which includes coverage for the motor vehicle itself and liability coverage for third parties. Motor vehicle insurance is a specific form of insurance that protects the policyholder against the risk of damage or loss arising from various events related to the ownership of a motor vehicle. Premium is the amount of money that policyholders must pay to the insurance company at regular intervals. The premium charged to the insured includes the pure premium, administrative costs, general expenses, acquisition costs, and the company's profit. A claim is an official request made to the insurance company to request payment based on the terms of the agreement (Wuthrich & Merz, 2012).

2.2 Poisson Distribution

Poisson distribution is a discrete distribution that represents the probability of the number of occurrences of events within a certain standard unit. If Y is a Poisson random variable with $\lambda > 0$, then its probability mass function is defined as:

$$p(Y = y) = \frac{e^{-\lambda} \lambda^y}{y!}, \text{ for } y = 0, 1, 2, \dots, \quad (1)$$

where:

$p(Y = y)$: probability mass function of the Poisson distribution

y : frequency of insured claims

λ : parameter of the Poisson distribution

represented as $Y \sim P(\lambda)$. It has mean and variance of $E(Y) = \lambda$ and $Var(Y) = \lambda$.

2.3 Gamma Distribution

The Gamma distribution is often used to represent variables such as claim amounts and annual incomes. The gamma random variable is continuous, non-negative, and right-skewed (Jong & Heller, 2008). Let Y be a random sample from a population following a Gamma distribution with parameters α and β denoted as $G(\alpha, \beta)$. The probability density function for the Gamma distribution is given by:

$$f(Y|\alpha, \beta) = \left(\frac{1}{\beta^\alpha \Gamma(\alpha)} \right) y^{\alpha-1} e^{-\frac{y}{\beta}}, \quad (2)$$

where:

$f(Y)$: PDF of the Gamma distribution

y : amount of insured claims

α, β : parameters of the Gamma distribution

for $y > 0$, $\alpha > 0$, and $\beta > 0$, with a mean and variance of $E(Y) = \alpha\beta$ and $Var(Y) = \alpha\beta^2$.

2.4 Generalized Linear Models (GLM)

Generalized Linear Models (GLM) are an extension of linear regression models, where the distribution of the response variable belongs to the Exponential family, aiming to understand the influence of explanatory variables (X) on the response variable (McCulloch et al., 2011). The goal of the GLM is to estimate the response variable (Y) that depends on the explanations provided by the explanatory variables (X). The observed variable Y , which follows an Exponential family distribution, has a probability function given by (Jong & Heller, 2008):

$$f(y | \theta, \phi) = \exp\left(\frac{y\theta - b(\theta)}{\phi} + c(y, \phi)\right), y \in S, \quad (3)$$

where:

θ : canonical parameter of the response variable

ϕ : dispersion parameter of the response variable.

2.5 Link Function

The link function is a function that connects the random components to the systematic components. Let $x_{1i}, x_{2i}, \dots, x_{ik}$ be the values of k independent variables for the i -th policyholder. The linear predictor η for the i -th policyholder is given by the equation (McCullagh & Nelder, 2019):

$$\eta_i = \beta_0 + \beta_1 x_{i1} + \dots + \beta_k x_{ik} \quad (4)$$

$$\eta_i = \beta_0 + \sum_{j=1}^k \beta_j x_{ij}, \quad (5)$$

with $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, k$, or in matrix form $\boldsymbol{\eta} = \mathbf{X}\boldsymbol{\beta}$, where $\boldsymbol{\eta}$ is a $(n \times 1)$ vector representing the policyholders, $\mathbf{X} = (x_1, x_2, \dots, x_n)^T$ is a $(n \times c)$ matrix of independent variables, $\boldsymbol{\beta}$ is a parameter matrix with $(c \times 1)$

coefficients, and $c = k + 1$. In insurance modeling with categorical independent variables, the function that links $E[Y_i] = \mu_i$ with the linear function $x_i' \beta$ is the logarithmic link function, denoted as:

$$g(\mu_i) = \ln(\mu_i) \quad (6)$$

$$\mu_i = \exp(\beta_0 + \sum_{j=1}^k \beta_k x_{ij}). \quad (7)$$

2.6 Maximum Likelihood Estimation (MLE)

The function $f(y_i; \theta, \phi)$ represents the likelihood function of the Exponential family distribution. The log-likelihood function of $f(y_i; \theta, \phi)$ can be expressed as:

$$L(\theta, \phi) = \sum_{i=1}^n \left(\frac{y_i \theta - b(\theta)}{\phi} + c(y_i, \phi) \right) = \frac{n(\bar{y} \theta - b(\theta))}{\phi} + \sum_{i=1}^n c(y_i, \theta), \quad (8)$$

differentiate the function $L(\theta, \phi)$ to θ :

$$\frac{\partial L(\theta, \phi)}{\partial \theta} = 0$$

$$\frac{\partial}{\partial \theta} \left(\frac{n(\bar{y} \theta - b(\theta))}{\phi} \right) = 0 \Rightarrow b'(\theta) = \bar{y}. \quad (9)$$

The Maximum Likelihood Estimation (MLE) of θ is obtained by solving the equation $b'(\theta) = \mu$, where μ is the mean of y .

2.7 Goodness-of-Fit Tests

In this study, the response variable claim frequency is assumed to follow a Poisson distribution, while the response variables claim amount assumed to follow a Gamma distribution. The distribution of the frequency of claims will be tested using the Kolmogorov-Smirnov test with the following procedure:

a) Hypothesis Testing:

H_0 : The data follows the theoretical distribution.

H_1 : The data does not follow the theoretical distribution.

b) Determining the significance level: $\alpha = 0.05$.

c) Determining the test statistic:

$$D = \max |F_n(x) - F_0(x)|. \quad (10)$$

d) Determining the test criteria: Reject H_0 if the calculated D exceeds the critical D or if the P-Value is less than α .

The response variable claim amount will be tested using the Chi-Square test with the following procedure:

a) Test Statistic:

$$\chi_h^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}. \quad (11)$$

b) Determining the hypotheses:

H_0 : The data follows the specified distribution.

H_1 : The data does not follow the specified distribution.

c) Determining the significance level: $\alpha = 0.05$.

d) Determining the test criteria: $\chi_h^2 < \chi_{tabel}^2$ or P-Value $> \alpha$, where χ_{tabel}^2 is the critical chi-square value.

The testing of the distribution of the response variable will be conducted with the assistance of EasyFit software.

2.8 Wald Test

The parameter of the policyholder characteristics will be tested using the Wald test to determine if the independent variables have a partial effect on the response variable. The procedure is as follows:

a) Hypothesis Testing:

$H_0: \beta_j = 0$

$H_1: \beta_j \neq 0$

b) Determining the significance level: $\alpha = 0.05$.

c) Calculating the test statistic:

$$W = \frac{\hat{\beta}_j}{\sqrt{\text{Var}(\beta)}} \quad (12)$$

d) H_0 is rejected if $W < X_{(1-\frac{\alpha}{2})(1)}^2$ or $W > X_{(\frac{\alpha}{2})(1)}^2$, or H_0 is rejected if P-Value $< \alpha$.

3. Materials and Methods

3.1. Materials

The research objective of this study is to determine the pure premium of motor vehicle insurance using the Generalized Linear Models (GLM) method. The supporting data used to obtain the model for the pure premium of motor vehicle insurance is the Swedish Motor Insurance dataset, which includes characteristics such as claim amounts and claim frequencies.

The data is presented in a Comma Separated Value (CSV) file format and is processed using Microsoft Excel software. The determination of the pure premium is assisted by the EasyFit software for distribution fitting tests and the SPSS software for significance testing and parameter estimation calculations.

3.2. Methods

- Selection of the response variable distribution and link function.
- Goodness-of-Fit Tests for the distribution models. The Kolmogorov-Smirnov test is used to assess the distribution fit for the claim frequency model. For the claim amount model, the Chi-Square test is employed to evaluate the goodness-of-fit.
- Significance testing of policyholder characteristics on the response variable. The Wald test is used to examine the significance of the policyholder characteristics in determining the pure premium of motor vehicle insurance, as described in equation.
- Estimation of response variable parameters using Maximum Likelihood Estimation. The parameters β for the Poisson distribution and γ for the Gamma distribution are estimated using Maximum Likelihood Estimation.
- Determination of pure premium by multiplying two components: the expected claim frequency and the expected claim amount.
- Interpretation of research results.
- Drawing conclusions and recommendations from the research.

4. Results and Discussion

4.1 Research Data

The data used in this study consists of 100 motor vehicle insurance policies from Sweden, presented in ordinal data format. The data includes five policyholder characteristics or independent variables (X) and two response variables (Y), namely claim frequency and claim amount (in Swedish Krona or SEK). The statistical description of the independent variables along with their means can be seen in the following Table 1.

Table 1: Descriptive Statistics of Independent Variables

Independent Variables	Category	Mean
Kilometres (X_1)	(1, 2, ..., 5)	2.22
Zone (X_2)	(1, 2, ..., 6)	2.6
Bonus (X_3)	(1, 2, ..., 7)	3.6
Make (X_4)	(1, 2, ..., 8)	4.7
Insured (X_5)	(1, 2, ..., 10)	2.86

- Kilometres (X_1) the number of kilometres travelled per year; a numeric vector with levels 1 (less than 10000), 2 (from 10000 to 15000), 3 (15000 to 20000), 4 (20000 to 25000) or 5 (more than 25000).
- Zone (X_2) geographical zone (only in motorins); a numeric vector with levels 1 to 6.
- Bonus (X_3) no claim bonus; a numeric vector equal to the number of years plus one since the last claim.
- Make (X_4) the make of vehicle; a numeric vector with levels from 1 to 8 representing eight common car models.
- Insured (X_5) the number of insured in policy-years; a numeric vector.

4.2 Data Distribution Model Hypothesis

4.2.1 The Kolmogorov-Smirnov Test for Claim Frequency Data

The Kolmogorov-Smirnov test was conducted to test the goodness-of-fit of the claim frequency distribution model in this study, with the assistance of EasyFit software. The results of the goodness-of-fit test for the claim frequency distribution model can be observed in the diagram below.

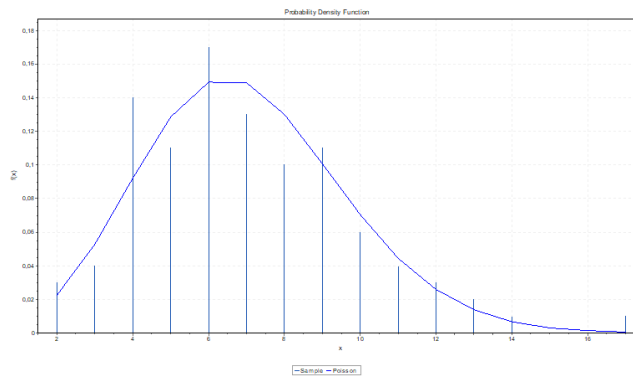


Figure 1: The Kolmogorov-Smirnov test diagram for claim frequency data.

Based on Figure 1, it can be observed that the data follows a Poisson distribution. The horizontal line represents the observed claim frequency data (y_{1i}), while the vertical line represents the Probability Density Function (PDF) of the data according to the Poisson distribution. The obtained parameter λ is 6.98 with $\alpha = 0.05$, and the P-Value is greater than α , specifically 0.05375. Therefore, we can conclude that the data follows a Poisson distribution.

4.2.1 The Chi-Square Test for Claim Frequency Data

The Chi-Square test was conducted to test the goodness-of-fit of the claim amount distribution model in this study, with the assistance of EasyFit software. The results of the goodness-of-fit test for the claim amount distribution model can be observed in the histogram below.

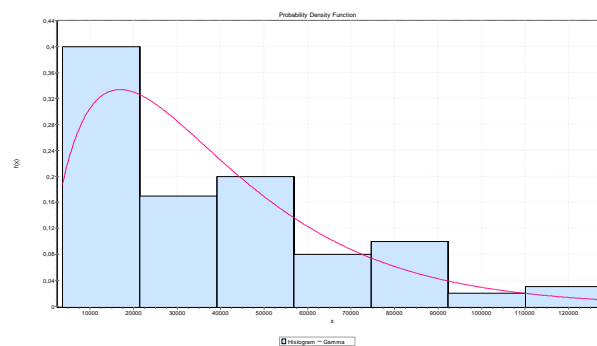


Figure 2: The Chi-Square test histogram for claim amount data.

Based on Figure 2, it can be observed that the data follows a Gamma distribution. The horizontal line represents the observed claim amount data (y_{2i}), while the vertical line represents the Probability Density Function (PDF) of the data according to the Gamma distribution with $\alpha = 1.7826$ and $\beta = 21.650$. The obtained P-Value is greater than α , specifically 0.4103, which means that the null hypothesis (H_0) is not rejected. Therefore, we can conclude that the data follows a Gamma distribution.

4.3 The Significance Test and Parameter Estimation of the Policyholder Characteristics

4.3.1 The Significance Test and Parameter Estimation of the Policyholder Characteristics

Parameters on Claim Frequency

Furthermore, a significance test of the parameters of the policyholder characteristics on claim frequency was conducted to determine which characteristics have a significant impact on claim frequency. The results of the significance test can be seen in the following Table 2.

Table 2: The significance test of the parameters of the policyholder characteristics on claim frequency was conducted with a significance level $\alpha = 0.05$.

Parameter	Wald Chi-Square	df	Sig.
Intercept (β_0)	2837.567	1	0.000
Kilometres	54.528	4	0.000
Zone	37.660	5	0.000
Bonus	0.565	6	0.997
Make	1.410	7	0.985
Insured	2.540	8	0.960

Based on the results of the significance test in Table 2, it can be observed that the policyholder characteristics of Kilometres and Zone have a significant impact on claim frequency. However, the characteristics of Bonus, Make, and Insured do not have a significant impact on claim frequency and are therefore excluded from the model.

Then, the parameter estimation of the significant policyholder characteristics, namely Kilometres and Zone, on claim frequency was conducted using Maximum Likelihood Estimation (MLE) and the Newton-Raphson method. The parameter estimation was performed with the assistance of SPSS software. In the parameter estimation of the significant independent variables, the data were transformed into binary form. The results of the parameter estimation for claim frequency, assisted by SPSS software, are presented in Table 3 below.

Table 3: Parameter estimation of the policyholder characteristics that significantly affect claim frequency.

No	Parameter	β	Std. Error	Wald Chi-Square	df	Sig.
1	(Intercept)	1.865	0.7302	6.525	1	0.011
2	[Kilometres=1]	0.764	0.4394	3.022	1	0.082
3	[Kilometres=2]	0.893	0.4368	4.177	1	0.041
4	[Kilometres=3]	0.809	0.4179	3.745	1	0.053
5	[Kilometres=4]	0.572	0.2937	3.799	1	0.051
6	[Kilometres=5]	0	0		0	
7	[Zone=1]	-0.635	0.3635	3.047	1	0.081
8	[Zone=2]	-0.753	0.2315	10.592	1	0.001
9	[Zone=3]	-0.489	0.1714	8.158	1	0.004
10	[Zone=4]	-0.387	0.2016	3.69	1	0.055
11	[Zone=5]	-0.162	0.1826	0.791	1	0.374
12	[Zone=6]	0	0		0	

Based on Table 3, the GLM model for motor vehicle claim frequency is as follows:

$$\hat{\mu}_{f_i} = \exp(1.865 + 0.764x_{11} + \dots - 0.572x_{14} - 0.635x_{21} + \dots - 0.162x_{25}). \quad (13)$$

The Kilometres variable, which represents the distance traveled by the vehicle per year, leads to an increase in the estimated claim frequency. For example, for vehicles in the category 1 distance range, it will result in an increase in the estimated claim frequency by $\exp(0.764)$ times. On the other hand, the geographic zone of the vehicle leads to a decrease in the estimated claim frequency. For instance, vehicles in zone category 2 will lower the estimated claim frequency by $\exp(-0.753)$ times.

4.3.2 The Significance Test and Parameter Estimation of the Policyholder Characteristics Parameters on Claim Amount

The significance test of the parameters of the policyholder characteristics on claim amount was conducted to determine which characteristics have a significant impact on claim amount. The results of the significance test can be seen in the following Table 4.

Table 4: The significance test of the parameters of the policyholder characteristics on claim amount was conducted with a significance level $\alpha = 0.05$

Parameter	Wald Chi-Square	df	Sig.
Intercept (β_0)	37068.631	1	0.000
Kilometres	45.203	4	0.000
Zone	31.957	5	0.000
Bonus	15.973	6	0.014
Make	13.287	7	0.065
Insured	7.276	8	0.507

Based on the results of the significance test conducted with the help of SPSS, as shown in Table 4, it can be observed that the characteristics of the policyholder that significantly influence the claim amount are Kilometres, Zone, and Bonus. However, the characteristics of Make and Insured do not have a significant impact, and therefore, they are excluded from the model.

Then, the parameter estimation of the significant policyholder characteristics, namely Kilometres, Zone, and Bonus on claim amount was conducted using Maximum Likelihood Estimation (MLE) and the Newton-Raphson method. The parameter estimation was performed with the assistance of SPSS software. In the parameter estimation of the significant independent variables, the data were transformed into binary form. The results of the parameter estimation for claim amount, assisted by SPSS software, are presented in Table 5 below.

Table 5: Parameter estimation of the policyholder characteristics that significantly affect claim amount

No	Parameter	γ	Std. Error	Wald Chi-Square	df	Sig.
1	(Intercept)	10.990	10.675	105.99	1	0.000
2	[Kilometres=1]	0.374	0.497	0.565	1	0.452
3	[Kilometres=2]	0.578	0.490	1.391	1	0.238
4	[Kilometres=3]	0.763	0.454	2.822	1	0.093
5	[Kilometres=4]	-0.034	0.296	0.013	1	0.909
6	[Kilometres=5]	0	0	0	0	
7	[Zone=1]	-0.707	0.465	2.306	1	0.129
8	[Zone=2]	-0.846	0.369	5.228	1	0.022
9	[Zone=3]	-0.663	0.304	4.743	1	0.029
10	[Zone=4]	-0.107	0.346	0,095	1	0.758
11	[Zone=5]	-0.750	0.336	4,974	1	0.026
12	[Zone=6]	0	0	0	0	
13	[Bonus=1]	-0.304	0.739	0.169	1	0.681
14	[Bonus=2]	-0.606	0.703	0.742	1	0.389
15	[Bonus=3]	-0.695	0.728	0.911	1	0.340
16	[Bonus=4]	-0.470	0.729	0.416	1	0.519
17	[Bonus=5]	-0.070	0.724	0.009	1	0.923
18	[Bonus=6]	-0.549	0.738	0.554	1	0.457
19	[Bonus=7]	0	0	0	0	

The obtained GLM model for the claim amount of motor vehicle insurance is as follows:

$$\hat{\mu}_{jk_i} = \exp(10.990 + 0.374x_{11} + \dots - 0.034x_{14} - 0.707x_{21} + \dots + 0.750x_{25} - 0.304x_{31} + \dots - 0.549x_{34}). \quad (14)$$

Based on the model and Table 5 above, it can be observed that the Kilometres variable, representing the distance traveled by the vehicle per year, leads to a decrease in the estimated claim amount for Category 4 or a distance between 20,000 – 25,000 km by a factor of $\exp(-0.034)$. Similarly, the geographic zone and bonus variables also result in a decrease in the estimated claim amount.

4.4 Calculating the Pure Premium of Motor Vehicle Insurance

To calculate the pure premium of motor vehicle insurance, the multiplication of two expectations is performed, namely the expectation of claim frequency and the expectation of claim amount. The estimation of parameters for motor is presented in Table 6 below.

Table 6: Parameter estimation of the policyholder characteristics that significantly affect pure premium.

No	Parameter	Claim Frequency (β_{ij})	Claim Amount (γ_{ij})	Pure Premium ($\beta_{ij} + \gamma_{ij}$)
1	Intercept	1.865	10.99	12.855
2	Kilometres=1	0.764	0.374	1.138
3	Kilometres=2	0.893	0.578	1.471
4	Kilometres=3	0.809	0.763	1.572
5	Kilometres=4	0.572	-0.034	0.538
6	Kilometres=5		0	0
7	Zone=1	-0.635	-0.707	-1.342
8	Zone=2	-0.753	-0.846	-1.599
9	Zone=3	-0.489	-0.663	-1.152
10	Zone=4	-0.387	-0.107	-0.494
11	Zone=5	-0.162	-0.75	-0.912
12	Zone=6		0	0
13	Bonus=1	-	-0.304	-0.304
14	Bonus=2	-	-0.606	-0.606
15	Bonus=3	-	-0.695	-0.695
16	Bonus=4	-	-0.470	-0.470
17	Bonus=5	-	-0.070	-0.070
18	Bonus=6	-	-0.549	-0.549
19	Bonus=7	-	0	0

Based on Table 6, the Generalized Linear Models (GLM) model for pure premium of motor vehicle insurance is obtained as follows:

$$\hat{\mu}_{p_i} = \exp(12.855 + 1.138x_{11} + \dots + 0.538x_{14} - 1.342x_{21} + \dots - 0.912x_{25} - 0.304x_{31} + \dots - 0.549x_{36}). \quad (15)$$

According to the model equation (15) and Table 6, the pure premium is determined by considering the policyholder characteristics, namely Kilometres, Zone, and Bonus. The Kilometres variable, which represents the distance traveled by the vehicle per year, leads to an increase in the premium. On the other hand, the geographic zone of the vehicle and the bonus have a decreasing effect on the premium. The total pure premium ($\sum \hat{\mu}_{p_i}$) to be paid by the policyholder of motor vehicle insurance is calculated to be 35,572,223.27 kr with an average of 355,722.23 kr.

5. Conclusion

- In this study, the frequency of claims data follows a Poisson distribution, and the estimated parameters for the frequency of claims are $\beta_0 = 1.865$, β_{1i} increases the estimated frequency of claims, and β_{2i} decreases the estimated claims, except for β_{21} . On the other hand, the number of claims data follows a Gamma distribution, and the obtained parameters are $\gamma_0 = 10.990$. γ_{1i} increases the estimated number of claims, except for γ_{14} , γ_{2i} decreases the estimated number of claims, and γ_{3i} also decreases the estimated number of claims.
- The policyholder's characteristics, namely Kilometres (X_1), Zone (X_2), and Bonus (X_3), significantly influence the determination of the pure premium for motor vehicle insurance.
- The total amount of the pure premium for motor vehicle insurance is obtained by multiplying the expected frequency of claims and the expected value, resulting in 35,572,223.27 kr, with an average of 355,722.23 kr.

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