



## Application of Single Index Model to Determine Optimal Stock Portfolio (A Case Study on IDX30 in 2022)

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### Abstract

Stock represent proof of ownership or participation of an individual or entity in a company. Investors gain profits from shares through capital gains and dividends. The difficulty in selecting an optimal composition of a stock portfolio is a major concern for investors. This study aims to determine the optimal composition of a stock portfolio, calculate the expected returns in the future, and assess the potential risks that investors may encounter later on. The data for this research consists of stocks listed on the IDX30 Index throughout the year 2022, which consistently appear in every six-month evaluation. The analysis is conducted using a single-index model. Based on the findings of this study, the following ten stocks are identified as the optimal portfolio constituents: KLBK with a weight of 17.20%, BBRI with a weight of 17.18%, BBCA with a weight of 17.08%, PTBA with a weight of 12.46%, BBNI with a weight of 9.89%, UNVR with a weight of 8.33%, INKP with a weight of 8.66%, ICBP with a weight of 5.56%, BMRI with a weight of 3.25%, and UNTR with a weight of 0.39%. The expected return from the formed portfolio is 0.1% per day, with a corresponding risk of 0.004%.

**Keywords:** Stocks, IDX3 Index, Single Index Model, Optimal Portofolio

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### 1. Introduction

The business world cannot be separated from investment activities. According to Tandelilin (2010), an investment is the purchase of a number of funds or resources to obtain future profits. An investor is a person who makes an investment. Investing in real or financial assets are two different types of investments (Reilly & Brown, 2011; Fahmi, 2015). Stock is an example of investing in financial assets because it is a risky asset and has an uncertain future rate of return.

Investors want to get the highest possible return by investing in various company shares. However, since stock prices have high fluctuations that are heavily influenced by technical factors and fundamental market conditions, investing in stocks also contains risk elements. Two factors that influence each other in investing in stocks are the level of expected profit and the degree of risk to bear as a result of the influence of such uncertainty (Ward, 1871; Bodie et al., 2020). Investors who want high rates of profit must dare to take risks. As a result, finding, measuring, and managing certain stock risks is very important for those investing in capital markets.

Investors should analyze the various existing stocks according to the latest conditions because investing in stocks involves risk. Then, they should choose the stock that is considered to have the least risk and can produce the best profit. Investors can do so by allocating their funds to various types of stocks known as diversification. With these different types of investments, they can choose the most suitable combination of fund allocation diversification to get the greatest profit while taking the lowest risk.

Investors usually choose the portfolio that has the lowest risk and the highest rate of profit. Portfolio efficiency is the term for a portfolio. An efficient portfolio is a good portfolio but not the best because it only considers one factor, namely the level of profit or risk. An optimal portfolio is a portfolio that combines the best rates of profit and risk or the portfolio investors choose from the many options available in an efficient portfolio. Therefore, an efficient portfolio is a good portfolio but not the best because it only considers one factor. A single index model is one of the best ways to create a portfolio. In 1963, William Sharpe created a single index model. The Single Index model is based on Markowitz's theory that the stock price changes in the direction of the market price index (Elton & Gruber,

2014). Using the Single Index Model, the ranking of each stock is based on Excess Return to Beta. (ERB). The ERB is the ratio between the expected excess return and the risk-free return of the asset with the beta, and the cut-off point is the boundary point that distinguishes which stocks are worthy to enter the portfolio.

The study will find, select, and form the best stock for optimal portfolio consisting of the stocks listed on the Bursa Efek Indonesia and included in the IDX30 index. The list of companies included in the IDX30 index will be compiled from January 1, 2022 to December 31, 2022. This research uses the stock price of www.yahoofinance.com. Analysis is done with the Single Index Model. Investors can use these research results as a reference when deciding to invest their money in a portfolio, or a particular type of stock.

## 2. Literature Review

### 2.1. Single Index Model Portofolio

William Sharpe created a portfolio analysis model known as the Single Index Model in 1963. This model is based on the idea that the stock price changes in the direction of the market price index (Elton & Gruber, 2014). One of the basics of the Single Index Model is to compare Excess Return to Beta (ERB) with the limiting level or the closing point ( $C^*$ ) of a particular stock. The single index model can be formulated as follows:

$$R_i = \alpha_i + \beta_i R_m + e_i, \tag{1}$$

The single index model can also be expressed in the form of the expected return value as follows (Goetzmann et al., 2014):

$$E(R_i) = \alpha_i + \beta_i E(R_m), \tag{2}$$

With

- $R_i$  : return rate of shares
- $\alpha_i$  : the expected value of the return on the stock is independent of the market return
- $\beta_i$  : the sensitivity of the return of a stock to the return from the market
- $R_m$  : return rate of market
- $e_i$  : residual error is a random variable with a expected value equal to zero
- $E(R_i)$  : the expected value of the stock return
- $E(R_m)$  : the expected value of the market return

### 2.2. Normality Test of data return

The Kolmogorov-Smirnov test is a goodness test for the distribution of data in general. The test was first introduced by the Soviet mathematician Andrei Nikolaevich Kolmogorov in 1933. The Kolmogorov-Smirnov test is as follows:

$$D = \max_{1 \leq i \leq n} (|F(z_i) - F_{n-1}(x_i)|, |F(z_i) - F_n(x_i)|) \tag{3}$$

With

- $F(z)$  : the cumulative distribution function
- $F_n(z)$  : the cumulative distribution function of observationd data

The principle of the normality test using Kolmogorov-Smirnov is to find the greatest resonance (D) of the cumulative distribution function of observational data against the distribution functions. Test criteria if  $D \leq D_k$  then the tested data is distributed normally. In contrast, if  $D > D_k$  then the data is not distributed normally.

### 2.3 Correlation test and t-test

Correlation analysis is used to calculate the strength of the relationship between variables. Referring to this definition, correlation analysis is able to describe the relationship between free variables and bound variables in data. The correlation formula based on a set of data  $(x_i, y_i)$  of size  $n$  can be calculated using the equation as follows:

$$r = \frac{n \sum_{i=1}^n x_i y_i - (\sum_{i=1}^n x_i)(\sum_{i=1}^n y_i)}{\sqrt{(n \sum_{i=1}^n x_i^2 - (\sum_{i=1}^n x_i)^2)(n \sum_{i=1}^n y_i^2 - (\sum_{i=1}^n y_i)^2)}} \tag{4}$$

With

- $n$  : number of observation
- $x_i$  : the value of the data  $i$  on the variable  $x$
- $y_i$  : the value of the data  $i$  on the variable  $y$

r is between  $-1 < r < 1$ , if r is 1 then a positive correlation can be expressed which means that there is a positive relationship between independent variable and dependent variable, whereas if r is -1 then a negative correlations can be stated that means there is an negative relationship between independent variable and dependent variable.

There are two common methods used to test the significance of correlation coefficients. The first method uses the test-t and the second method using the table r. This study used the first method to test the correlation between a free variable and a bound variable. The formula for calculating t can be calculated using the equation as follows:

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \tag{5}$$

With

r : coefficient of correlation

n : number of observation

The hypothesis:

$H_0 : \beta = 0$  (there is no significant correlation between dependent variable and independent variable)

$H_1 : \beta \neq 0$  (there is significant correlation between dependent variable and independent variable)

Significance of  $\alpha = 0.05$

Criteria of Testing:

$H_0$  is rejected when at the level of significance  $\alpha$  the value  $t > t_{table}$ .

### 3. Materials and Methods

#### 3.1. Materials

In this study, the objects are 24 stocks from IDX30 index and the data used in this study are the daily closing price of IDX30 and JCI as many as 246 observations on each stock, and also Bank Indonesia Certificate (SBI) interest rate data from January 1, 2022 to December 31, 2022. The daily closing stock price data obtained at [www.finance.yahoo.com](http://www.finance.yahoo.com) and BI Rate data obtained from <https://www.bi.go.id/>.

The IDX30 stock index is a stock index that falls into the index category used to measure the price performance of stocks that have high liquidity and large market capitalization and are supported by good company fundamentals and evaluated every six months.

#### 3.2. Methods

This research uses a quantitative and applicative approach through a case study conducted, namely the closing price of IDX30 stocks, which is then calculated by single index model to make the optimal portfolio. In this research, sample selection was carried out by means of purposive sampling. Calculations are assisted by Microsoft Excel, SPSS, and EasyFit software. The following are the steps to determine the optimal portfolio:

1. Choose stocks to be included in a portofolio according to predetermined criteria.
2. Collect the closing price of stocks data from [www.finance.yahoo.com](http://www.finance.yahoo.com) during the research period, from 1 January 2022 to 31 December 2022.
3. Calculate the return and expected return from each stock using the following formula:

$$r_{i,t} = \frac{P_t - P_{t-1}}{P_{t-1}}; i = 1,2, \dots, 24; t = 1,2, \dots, 245 \tag{6}$$

$$E(R_i) = \frac{\sum_{t=1}^n R_{i,t}}{n} \tag{7}$$

4. Calculate the return and expected return market using the following formula:

$$r_{m,t} = \frac{P_t - P_{t-1}}{P_{t-1}}; t = 1,2, \dots, 245 \tag{8}$$

$$E(R_m) = \frac{\sum_{t=1}^n R_{m,t}}{n} \tag{9}$$

5. Calculate the risk market and the risk from each stock using the following formula:

$$\sigma_i^2 = \frac{\sum_{t=1}^n (R_{i,t} - E(R_i))^2}{n-1} \tag{10}$$

$$\sigma_m^2 = \frac{\sum_{t=1}^n (R_{m,t} - E(R_m))^2}{n-1} \tag{11}$$

6. Perform a normality test of the return data on each stock using the following formula:

$$D = \max_{1 \leq i \leq n} (|F(z_i) - F_{n_i-1}(x_i)|, |F(z_i) - F_{n_i}(x_i)|) \tag{12}$$

7. Perform a correlation test between independent variable and dependent variable using the following formula:

$$r = \frac{n\sum_{i=1}^n x_i y_i - (\sum_{i=1}^n x_i)(\sum_{i=1}^n y_i)}{\sqrt{(n\sum_{i=1}^n x_i^2 - (\sum_{i=1}^n x_i)^2)(n\sum_{i=1}^n y_i^2 - (\sum_{i=1}^n y_i)^2)}} \tag{13}$$

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \tag{14}$$

8. Calculate the beta and alpha from each stock using the following formula:

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} \tag{15}$$

$$\alpha_i = E(R_i) - \beta_i E(R_m) \tag{16}$$

9. Calculate the variance of the residual error using the following formula:

$$\sigma_{ei}^2 = \frac{\sum_{i=1}^n (R_{i,t} - \alpha_i - \beta_i R_{m,t})^2}{n-1} \tag{17}$$

10. Calculate the excess return to beta using the following formula:

$$ERB_i = \frac{E(R_i) - R_{BR}}{\beta_i} \tag{18}$$

11. Calculate the beta and alpha portofolio using the following formula:

$$\alpha_p = \sum_{i=1}^n w_i \alpha_i \tag{19}$$

$$\beta_p = \sum_{i=1}^n w_i \beta_i \tag{20}$$

12. Calculate the return and risk portofolio using the following formula:

$$E(R_p) = \alpha_p + \beta_p E(R_m) \tag{21}$$

$$\sigma_p^2 = \beta_p^2 \sigma_m^2 + \sum_{i=1}^n w_i^2 \sigma_{ei}^2 \tag{22}$$

#### 4. Results and Discussion

In this research, twenty-four stocks were chosen based on the criteria, these stocks are listed in Table 1.

**Table 1:** List of selected stocks

No.	Stock Code	Stock Name	Sector
1	ADRO	Adaro Energy Tbk.	Materials
2	ANTM	Aneka Tambang (Persero) Tbk.	Materials
3	ASII	Astra International Tbk.	Misc Industry
4	BBCA	Bank Central Asia Tbk.	Finance
5	BBNI	Bank Negara Indonesia (Persero) Tbk.	Finance
6	BBRI	Bank Rakyat Indonesia (Persero) Tbk.	Finance
7	BMRI	Bank Mandiri (Persero) Tbk.	Finance
8	BUKA	Bukalapak Tbk.	Technology
9	CPIN	Charoen Pokphand Indonesia Tbk.	Consumer Goods
10	EMTK	Elang Mahkota Teknologi Tbk.	Industry
11	ICBP	Indofood CBP Sukses Makmur Tbk.	Consumer Goods
12	INDF	Indofood Sukses Makmur Tbk.	Consumer Goods
13	INKP	Indah Kiat Pulp and Paper Tbk.	Misc Industry
14	KLBF	Kalbe Farma Tbk.	Consumer Goods
15	MDKA	Merdeka Coper Gold Tbk.	Materials
16	PGAS	Perusahaan Gas Negara (Persero) Tbk.	Infrastructure & Transportation
17	PTBA	Bukit Asam Tbk.	Materials
18	SMGR	Semen Indonesia (Persero) Tbk.	Chemical Industry
19	TBIG	Tower Bersama Infrastructure Tbk.	Infrastructure & Transportation
20	TINS	Timah Tbk.	Materials
21	TLKM	Telekomunikasi Indonesia (Persero) Tbk.	Infrastructure & Transportation
22	TOWR	Sarana Menara Nusantara Tbk.	Infrastructure & Transportation
23	UNTR	United Tractors Tbk.	Trade, Service & Investment
24	UNVR	Unilever Indonesia Tbk.	Consumer Goods

Then collect the daily closing price of the twenty-four stocks selected during the research periode. Then calculated stock returns using equation (1). Stock returns are used to calculate expected return stock using equation (2), stock risk using equation (5), and expected return market using equation (4). The results of these calculations can be seen in Table 2.

**Table 2:** Expected return, stock risk, beta, alpha, and excess return to beta

Stock Code	$E(R_i)$	$\sigma_i^2$	$E(R_m)$
ADRO	0.00239	0.00082	-0.00006
ANTM	-0.00020	0.00097	-0.00006
ASII	0.00014	0.00032	-0.00006
BBCA	0.00074	0.00021	-0.00006
BBNI	0.00145	0.00031	-0.00006
BBRI	0.00081	0.00026	-0.00006
BMRI	0.00155	0.00030	-0.00006
BUKA	-0.00122	0.00151	-0.00006
CPIN	-0.00003	0.00037	-0.00006
EMTK	-0.00294	0.00118	-0.00006
ICBP	0.00074	0.00027	-0.00006
INDF	0.00038	0.00016	-0.00006
INKP	0.00052	0.00043	-0.00006
KLBF	0.00120	0.00032	-0.00006
MDKA	0.00046	0.00097	-0.00006
PGAS	0.00119	0.00052	-0.00006
PTBA	0.00158	0.00053	-0.00006
SMGR	-0.00018	0.00046	-0.00006
TBIG	-0.00098	0.00034	-0.00006
TINS	-0.00060	0.00067	-0.00006
TLKM	-0.00031	0.00026	-0.00006
TOWR	0.00002	0.00029	-0.00006
UNTR	0.00096	0.00046	-0.00006
UNVR	0.00068	0.00050	-0.00006

Based on the calculation of the expected value of stock returns, it turns out that eight stocks have a negative return expectation value. Henceforth, the eight stocks are not included in the optimal portfolio formation. In addition, we also obtained the expected market return value during the observation period of  $-0.00006$  with a risk of  $0.00005$ .

Then, a stock return data normality test will be carried out. The normality test was carried out to determine whether the IDX30 daily stock closing price return data is normally distributed. A summary of the results of the Normality Test on IDX30 stock return data are in Table 3.

**Table 3:** Normality test for stocks data returnss

Stock Code	$D$	$D_k$	Decision	Note
ADRO	0.10922	0.08689	$H_0$ rejected	data returns are not normally distributed
ASII	0.07470	0.08689	$H_0$ accepted	data returns are normally distributed
BBCA	0.05926	0.08689	$H_0$ accepted	data returns are normally distributed
BBNI	0.09747	0.08689	$H_0$ rejected	data returns are not normally distributed
BBRI	0.05614	0.08689	$H_0$ accepted	data returns are normally distributed
BMRI	0.08986	0.08689	$H_0$ rejected	data returns are not normally distributed
ICBP	0.06089	0.08689	$H_0$ accepted	data returns are normally distributed
INDF	0.08334	0.08689	$H_0$ accepted	data returns are normally distributed

INKP	0.08949	0.08689	$H_0$ rejected	data returns are not normally distribute
KLBF	0.06545	0.08689	$H_0$ accepted	data returns are normally distributed
MDKA	0.10387	0.08689	$H_0$ rejected	data returns are not normally distributed
PGAS	0.09413	0.08689	$H_0$ rejected	data returns are not normally distributed
PTBA	0.08744	0.08689	$H_0$ rejected	data returns are not normally distributed
TOWR	0.10929	0.08689	$H_0$ rejected	data returns are not normally distributed
UNTR	0.07260	0.08689	$H_0$ accepted	data returns are normally distributed
UNVR	0.12765	0.08689	$H_0$ rejected	data returns are not normally distributed

It was found that seven stocks had normally distributed return data, namely ASII, BBCA, BBRI, ICBP, INDF, KLBF, UNTR. While the remaining nine stocks have data returns that are not normally distributed, namely ADRO, BBNI, BMRI, INKP, MDKA, PGAS, PTBA, TOWR, and UNVR. For nine stocks that are not normally distributed, data returns will be transformed by transforming the data returns into natural logarithmic forms. The results of the Kolmogorov-Smirnov test, after being transformed by natural logarithms on stock return data that are not normally distributed are presented in Table 4.

**Table 4:** Normality test results after transformed by the natural logarithm

Stock Code	Sig-value	Criteria	Note
ADRO	0.20	$0.20 > 0.05$	data returns are normally distributed
BBNI	0.20	$0.20 > 0.05$	data returns are normally distributed
BMRI	0.20	$0.20 > 0.05$	data returns are normally distributed
INKP	0.20	$0.20 > 0.05$	data returns are normally distributed
MDKA	0.00	$0.00 < 0.05$	data returns are not normally distributed
PGAS	0.20	$0.20 > 0.05$	data returns are normally distributed
PTBA	0.20	$0.20 > 0.05$	data returns are normally distributed
TOWR	0.07	$0.07 > 0.05$	data returns are normally distributed
UNVR	0.20	$0.20 > 0.05$	data returns are normally distributed

It was found that of the nine data returns that had been transformed into natural logarithmic form and then retested with the Kolmogorov-Smirnov test, it showed that eight of the results were normally distributed; namely, ADRO, BBNI, BMRI, INKP, PGAS, PTBA, TOWR, and UNVR and 1 was not normally distributed, namely MDKA shares. Therefore, MDKA shares were excluded from the calculation. After recalculating the expected return on the transformed data return stocks, it is found that stocks with the TOWR code have a value of  $E(R_i) < 0$ , so TOWR is eliminated from the candidate portfolio. Thus, there will be 14 stocks that will become candidates for forming an optimal portfolio, namely ASII, BBCA, BBRI, ICBP, INDF, KLBF, UNTR, ADRO, BBNI, BMRI, INKP, PGAS, PTBA, and UNVR. To check whether the IHSG price movement affects the price of securities or stocks, a correlation test will be carried out. The results are in Table 5.

**Table 5:** Calculation of correlation coefficient and t-test

Stock Name	$r$	$t$	$t_{table}$	Decision
ADRO	-0.00145	-0.02266	-1.65112	$H_0$ accepted
ASII	0.00894	0.13933	-1.65112	$H_0$ accepted
BBCA	0.04148	0.64711	-1.65112	$H_0$ accepted
BBNI	0.01835	0.28609	-1.65112	$H_0$ accepted
BBRI	0.05224	0.81545	-1.65112	$H_0$ accepted
BMRI	0.00636	0.09917	-1.65112	$H_0$ accepted
ICBP	0.01988	0.30997	-1.65112	$H_0$ accepted
INDF	-0.04309	-0.67235	-1.65112	$H_0$ accepted
INKP	0.10051	1.57470	-1.65112	$H_0$ accepted
KLBF	0.04402	0.68694	-1.65112	$H_0$ accepted

PGAS	-0.02615	-0.40771	-1.65112	$H_0$ accepted
PTBA	0.04785	0.74680	-1.65112	$H_0$ accepted
UNTR	0.00227	0.03531	-1.65112	$H_0$ accepted
UNVR	0.08076	1.26306	-1.65112	$H_0$ accepted

It is obtained that fourteen stocks have a  $t \geq t_{table}$  value so that  $H_0$  is accepted, which means that the correlation between the independent variables and the dependent variable is correlated significant. In order to estimate whether a stock has a rate of return that exceeds the Indonesian bank interest rate, it is necessary to make a comparison between the average return obtained from stocks compared to interest rates, meaning that these stocks can provide a rate of return above that given when only investing through bank deposit. Excess Return to Beta (ERB) is the difference from the expected return value with a risk-free return on assets. This ERB value is the value used as the basis for determining which stocks are included in the optimal portfolio. Share that is included in this calculation are stocks with a positive expected return value, and the expected return value is greater than the return on risk-free assets. ERB with a negative value will cause investment losses when compared to depositing money in a bank that is more profitable. Therefore, only 17 shares will enter the next stage. The ERB values will be sorted from the highest value to the lowest value to determine the cut-off point value. A comparison between the expected returns of each stock and the risk-free rate of return, and also calculations of excess return to beta of each stocks is presented in Table 6.

**Table 6:** Comparison of Stock's Expected Return with Risk-Free Return

Stock Name	$E(R_i)$	$R_f$	$\beta_i$	$ERB_i$
ADRO	0.00239	0.00017	-0.01225	-0.18091
ASII	0.00014	0.00017	0.02198	-0.00123
BBCA	0.00074	0.00017	0.08137	0.00695
BBNI	0.00145	0.00017	0.04721	0.02705
BBRI	0.00081	0.00017	0.11542	0.00557
BMRI	0.00155	0.00017	0.01317	0.10454
ICBP	0.00074	0.00017	0.04435	0.01280
INDF	0.00038	0.00017	-0.07459	-0.00281
INKP	0.00052	0.00017	0.27502	0.00126
KLBF	0.00120	0.00017	0.10751	0.00958
PGAS	0.00119	0.00017	-0.07312	-0.01396
PTBA	0.00158	0.00017	0.14858	0.00950
UNTR	0.00096	0.00017	0.00663	0.11877
UNVR	0.00068	0.00017	0.24351	0.00208

Based on table 6, ASII shares have an average return below the average return on risk-free assets, and this means that investing in ASII shares in 2022 is still less profitable than depositing money in a bank. Stocks with negative ERB values cause investment losses compared to depositing money in more profitable banks. Therefore, only ten shares will enter the next stage, namely BBCA, BBNI, BBRI, BMRI, ICBP, INKP, KLBF, PGAS, PTBA, UNTR, and UNVR. The ERB values will be sorted from the highest to the lowest to determine the cut-off point value. After sorting the stock erb values from largest to smallest, then calculate the values of  $A_i$ ,  $B_i$ , and  $C_i$  to determine the cut-off point. Calculations of  $A_i$ ,  $B_i$ , and  $C_i$  are given in Table 7.

**Table 7:** Calculation of  $A_i$ ,  $B_i$ ,  $C_i$ , and cut-off point

Stock Name	$ERB_i$	$A_i$	$B_i$	$C_i$
UNTR	0.11877	0.01127	0.09485	0.00000
BMRI	0.10454	0.06061	0.57979	0.00000
BBNI	0.02705	0.19299	7.13450	0.00001
ICBP	0.01280	0.09353	7.30839	0.00002
KLBF	0.00958	0.34496	35.99054	0.00004
PTBA	0.00950	0.40393	42.51675	0.00006

BBCA	0.00695	0.22153	31.88732	0.00007
BBRI	0.00557	0.28219	50.64844	0.00009
UNVR	0.00208	0.25327	121.81997	0.00010
INKP	0.00126	0.22677	180.52365	0.00011

From the calculations in Table 7, it can be seen that the cut-off point value is 0.00011 which is the limit for determining which stocks will be included in the portfolio optimal. Stocks included in the optimal portfolio are stocks that have an ERB value greater than or equal to the cut-off point. Ten stocks were selected with an ERB value above the cut-off point. The stocks selected for the optimal portfolio are UNTR, BMRI, BBNI, ICBP, KLBF, PTBA, BBCA, BBRI, UNVR, and INKP, to calculate the return on the proportion of funds ( $W_i$ ) from each share, first calculate the weighted scale ( $Z_i$ ). The results of calculating the proportion of funds and the weighted scale can be seen in Table 8.

**Table 8:** Calculations of weighted scale and proportions of funds.

Stock Name	$Z_i$	$w_i$
UNTR	24.29104	0.00389
BMRI	202.72672	0.03246
BBNI	617.87465	0.09894
ICBP	347.49135	0.05564
KLBF	1074.01463	0.17198
PTBA	777.94258	0.12457
BBCA	1066.89077	0.17084
BBRI	1072.75702	0.17178
UNVR	520.16524	0.08329
INKP	540.97093	0.08662

Referring to Table 8, ten optimal stocks are obtained to form a stock portfolio in 2022, namely the UNTR stock code of 0.39%, BMRI of 3.25%, BBNI of 9.89%, ICBP of 5.56%, KLBF of 17.20%, PTBA 12.46%, BBCA 17.08%, BBRI 17.18%, UNVR 8.33%, and INKP 8.66%. The expected return portfolio is the average expected return for each stock in the portfolio formation. An expected return portfolio is helpful in knowing the rate of return obtained by investors in the future. Following are the results of portfolio alpha and beta calculations in Table 9.

**Table 9:** Calculations of alpha and beta portfolio

Stock Name	$w_i$	$\alpha_i$	$\beta_i$	$\alpha_p$	$\beta_p$
UNTR	0.00389	0.00096	0.00663	0.00000	0.00003
BMRI	0.03246	0.00155	0.01317	0.00005	0.00043
BBNI	0.09894	0.00145	0.04721	0.00014	0.00467
ICBP	0.05564	0.00074	0.04435	0.00004	0.00247
KLBF	0.17198	0.00121	0.10751	0.00021	0.01849
PTBA	0.12457	0.00159	0.14858	0.00020	0.01851
BBCA	0.17084	0.00074	0.08137	0.00013	0.01390
BBRI	0.17178	0.00082	0.11542	0.00014	0.01983
UNVR	0.08329	0.00069	0.24351	0.00006	0.02028
INKP	0.08662	0.00053	0.27502	0.00005	0.02382

Based on Table 9, PT Indah Kiat Pulp and Paper Tbk (INKP) has the highest beta value of 0.27502, while PT Indofood Sukses Makmur Tbk (INDF) has a lowest beta of -0,07459. PT Adaro Energy Indonesia Tbk (ADRO) has the highest beta value of 0.00198, while PT Astra International Tbk (ASII) has a lowest beta rating of 0.00014.

The return portfolio is an indication of the amount of profit obtained. The portfolio was formed. Calculate the expected value using the following formula.

$$E(R_p) = \alpha_p + \beta_p E(R_m)$$



The expected return value of the portfolio is as follows.

$$E(R_p) = 0.00102 + 0.12242(-0.00005) = 0.00101 = 0.101\%$$

The next step is to calculate the portfolio risk. Portfolio risk is the risk influenced by the market or related to beta, market return variants, and the weighted average of the residual error variants of each stock. Mathematically, the portfolio risk can be written as follows.

$$\sigma_p^2 = \beta_p^2 \sigma_m^2 + \sum_{i=1}^n w_i^2 \sigma_{ei}^2$$

Based on the formula above, portfolio risk calculation has four components, namely the portfolio beta square value, the market return variation, the quarter value of the proportion or weight of each stock, and the residual error variation of each share. Here is the portfolio risk calculation:

$$\sigma_p^2 = ((0.122421)^2 \cdot 0.000054) + 0.000042 = 0.000043 = 0.004\%$$

Based on the above calculation, the portfolio formed yielded a profit rate of 0.101% per day with a risk of 0.004% per day. The expected return value is higher when compared to the market's expected return, which is -0.006% per day. In addition, the portfolio's hail return of 0.101% per day turned out to be greater than the risk-free asset return of only 0.017% per day. Investing in such optimal portfolio stocks provides a greater rate of profit than just investing through bank deposits.

## 5. Conclusion

Based on the research, there are 10 optimal stocks and meet the criteria for optimal portfolio formation with the following proportions of funds KLBF by 17.20%, BBRI by 17.18%, BBCA by 17.08%, PTBA by 12.46%, BBNI by 9.89%, INKP by 8.66%, UNVR by 8.33%, ICBP by 5.56%, BMRI by 3.25%, UNTR by 0.39%. The expected return that investors will get is 0.101% per day and the risk that investors will face is 0.004%.

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