



# Calculation of Term Life Insurance Premium Reserves with Fackler Method and Canadian Method

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## Abstract

Every individual around the world goes through the life cycle of birth and continues their journey with unique experiences. The uncertainty of the future, which includes both happiness and calamity, is a universal aspect of human life. Life risks, such as illness and death, are an unavoidable reality for every individual in this world. Life insurance is one of the solutions to manage these risks, with term life insurance being one of the options. The focus of this research lies on term life insurance, with the aim of calculating premium reserves using the Fackler and Canadian methods. This research is concerned with the process of calculating premium reserves, and the results show that the Fackler method produces a larger premium reserve value compared to the Canadian method. Recommendations are given to companies to use the Fackler Method in calculating term life insurance premium reserves to avoid potential losses that could occur if using the Canadian method. The choice of premium calculation method is a strategic key in effective risk management for the company.

*Keywords:* life insurance, term life insurance, premium reserve, Fackler method, Canadian method.

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## 1. Introduction

All living things in the world already have their own time, which means that we will never know what will happen in the future. Like happiness to disaster, we cannot predict when and where it will happen. Every human being in the world will definitely feel this. If a disaster occurs, there will be costs that must be incurred for treatment outside the budget that has been prepared. Death is an event that will definitely happen to all humans in the world and do not know when it will happen. All of these things can be experienced by every human being, where and when, so efforts are needed to minimize these unexpected risks.

As in 2019, there was a disaster that befell Indonesia, namely the Corona Virus or commonly known as COVID-19. This virus has changed human life from all aspects, starting from health, education, finance and many others. Some humans affected by the Corona Virus must get proper and fast treatment where the person must pay for the medical expenses themselves even though the government provides some assistance for the treatment. One of the efforts to meet the needs and reduce uncertain risks in the future, insurance is one of the things needed for humans. Insurance is a form of financial protection for humans so that funds are protected from possible losses.

Life insurance companies will usually cover payments in the event of a disaster or death. The insurance company will bear a greater loss at the end of the policy year, because the higher the age of the insured, the higher the death rate. The life insurance that will be is term life insurance.

Research on insurance premium reserves has previously been conducted by Faturachman, *et al.* (2018) who calculated life insurance premium reserves using the Canadian method and Ibrahim, *et al.* who calculated joint life insurance premium reserves using the Canadian method. Based on the explanation above, the purpose of this study is to determine the results of the calculation of term life insurance premium reserves using the Fackler and Canadian methods.

## 2. Literature Review

### 1) Life Insurance

Life insurance can be defined as an agreement (insurance policy) between the insurance company (insurer) and the policy owner (insured) that requires the insured to pay an obligation (premium) to the insurer requires the insured to pay an obligation (premium) to the insurer with a certain amount. The insurer allocates the premium paid by the insured as compensation or benefits to be returned to the insured, company operating costs, and premium reserves. Apart from functioning as compensation, premium reserves can also be used in the event of a claim (Himmah, 2015).

### 2) Term Life Insurance

Term life insurance is an insurance where the policyholder, from the approval of the insurance contract until a certain period of time, dies, and the sum insured will be paid (Futami, 1993).

### 3) Term Life Annuity

A term life annuity is a life annuity where payments are made at a certain period of time. The initial term life annuity calculation formula is (Futami, 1993):

$$\ddot{a}_{x:\overline{n}|} = \frac{N_x - N_{x+n}}{D_x} \quad (1)$$

where  $\ddot{a}_{x:\overline{n}|}$ : initial term life annuity with a term of  $n$  years,  $N_x$ : the accumulated cash value of payments ( $v$ ) at age  $x$  years with the number of humans surviving at age  $x$  years, and  $D_x$ : the product of the cash value of payment ( $v$ ) at age  $x$  years with the number of people who survive at age  $x$  years.

### 4) Acommutation symbol

Commutation symbols are values to simplify the calculation of annuities, premiums, premium reserves, and other insurance value calculations. There are several commutation symbols, namely (Futami, 1993):

$$D_x = v^x l_x \quad (2)$$

$$C_x = v^{x+1} d_x \quad (3)$$

$$N_x = D_x + D_{x+1} + D_{x+2} + \dots + D_\omega \quad (4)$$

$$M_x = C_x + C_{x+1} + C_{x+2} + \dots + C_\omega \quad (5)$$

where  $D_x$ : the product of the cash value of payment ( $v$ ) at age  $x$  years with the number of people who survive at age  $x$  years,  $v^x$ : the discount factor for  $x$  years,  $l_x$ : the number of people who survive at age  $x$  years,  $C_x$ : the product of the cash value of the payment ( $v$ ) to the age of  $(x + 1)$  years with the number of people who die at age  $x$  years before reaching  $(x + 1)$  years,  $v^{x+1}$ : the discount factor for  $(x + 1)$  years,  $d_x$ : the number of people who die at the age of  $x$  years before reaching  $(x + 1)$  years,  $N_x$ : the accumulated cash value of payments ( $v$ ) at age  $x$  years with the number of humans surviving at age  $x$  years, and  $M_x$ : the sum of the cash value of the payment ( $v$ ) to the age of  $(x + 1)$  years with the number of people who die at age  $x$  years before reaching  $(x + 1)$  years.

### 5) Mortality Table

The table obtained from observations of mortality rates based on certain age groups is called a mortality table (Futami, 1993). Mortality tables are used to calculate the right premium. There are several notations used in the mortality table, namely  $x, n, l_x, {}_n d_x, {}_n q_x$ , and  ${}_n p_x$  (Coale and Demeny, 1983).

$${}_n d_x = l_x - l_{x+n} \quad (6)$$

$${}_n q_x = \frac{d_{x+n}}{l_{x+n}} \quad (7)$$

$${}_n p_x = 1 - {}_n q_x \quad (8)$$

where  $x$ : age,  $n$ : the difference between age  $x$  and the next age  $x$ ,  $l_x$ : the number of individuals who survive at exactly  $x$  years of age,  ${}_n d_x$ : the number of deaths of  $l_x$  that occur between age  $x$  and  $(x + n)$  years,  ${}_n q_x$ : the probability that a person exactly  $x$  years of age dies before reaching  $(x + n)$  years, and  ${}_n p_x$ : the probability that a person exactly  $x$  years of age lives to reach his/her  $(x + n)$  year birthday.

## 6) Single Net Premium

The single net premium in term life insurance is denoted by  $A_{x:\overline{n}|}^1$  where  $x$  is a person's age and  $n$  is the period and the sum insured to be paid is 1. If he dies in the first year, the cash value paid is  $\frac{v d_x}{l_x}$  and if he dies in the second year, the cash value paid is  $\frac{v^2 d_{x+1}}{l_x}$  and so on. The formula for single net premium in term life insurance is (Futami, 1993):

$$A_{x:\overline{n}|}^1 = S \cdot \frac{M_x - M_{x+n}}{D_x} \quad (9)$$

where  $S$ : compensation,  $M_x$ : the sum of the cash value of the payment ( $v$ ) to the age of  $(x + 1)$  years with the number of people who die at age  $x$  years before reaching  $(x + 1)$  years, and  $D_x$ : the product of the cash value of payment ( $v$ ) at age  $x$  years with the number of people who survive at age  $x$  years.

## 7) Annual Net Premium

The annual net premium in term life insurance is the premium paid by the insured at the beginning of each year, the amount of which can be the same or change every year. The annual net premium of term life insurance is denoted by  $P_{x:\overline{n}|}^1$  where  $x$  is the age of a person and  $n$  is the time period and the sum insured to be paid is 1. The formula can be expressed by the following equation (Futami, 1993):

$$P_{x:\overline{n}|}^1 = \frac{A_{x:\overline{n}|}^1}{\ddot{a}_{x:\overline{n}|}} \quad (10)$$

where  $A_{x:\overline{n}|}^1$ : the single net premium in term life insurance and  $\ddot{a}_{x:\overline{n}|}$ : initial term life annuity with a term of  $n$  years.

## 8) Premium Reserve

Premium reserve is the amount of money collected by insurance companies obtained from the difference between the value of compensation and the cash value of payments at a time of coverage in preparation for claim payments (Futami, 1993).

## 9) Interest

Interest is a payment made by the borrower of money to the lender in return for the use of borrowed money. The amount of interest is determined based on the principal amount of the loan, the term, and the interest rate. There are two ways of calculating interest, namely single interest and compound interest (Futami, 1993).

## 10) Fackler Method

David Parks Fackler is an American actuary who first introduced the formula to the Fackler method (Mashitah, et al. 2013). The Fackler method is very useful for calculating premium reserves for several consecutive years. The reserve value sought is year  $(t + 1)$ . The premium reserve at the end of year  $(t + 1)$  is (Januarti, et al. 2019):

$${}_tV = P \cdot {}_t u_x - {}_t k_x \quad (11)$$

$${}_{t+1}V = u_{x+t} ({}_tV + P) - k_{x+t} \quad (12)$$

where  ${}_tV$ : premium reserve at the end of year  $t$ ,  $P$ : annual net premium,  ${}_t u_x$ : the amount of premium and interest from  $x$  years to  $t$  years,  ${}_t k_x$ : the amount of insurance cost and interest from  $x$  years to  $t$  years,  ${}_{t+1}V$ : premium reserve at the end of year  $(t + 1)$ ,  $u_{x+t}$ : the value of premium and interest from  $x$  years to  $t$  years,  $k_{x+t}$ : the cost of insurance and interest from  $x$  years to  $t$  years.

## 11) Canadian Method

The Canadian method is a method of calculating reserves by equalizing the net premium for the first year of the Canadian method with the difference between the net premium for whole life insurance policies and the natural premium. Calculation of premium reserves using the Canadian method, so it is expressed as follows (Menge and Fischer, 1985):

$$\beta^{can} = P_{x:\overline{n}|}^1 + \frac{(P_x - c_x)}{a_{x:n-1}|} \quad (13)$$

$${}_tV_{x:\overline{n}|}^{can} = A_{x+t:\overline{n-t}|}^1 - \beta^{can} \cdot \ddot{a}_{x+t:\overline{n-t}|} \quad (14)$$

where  $\beta^{can}$ : annual net premium for subsequent years,  $P_{x:\overline{n}|}^1$ : annual net premium of term life insurance,  $P_x$ : annual net premium of whole life insurance,  $c_x$ : natural premium,  $a_{x:\overline{n-1}|}$ : end of  $(n-1)$  year term life annuity of policyholder aged  $x$  years,  ${}_tV_{x:\overline{n}|}^{can}$ : year  $t$  premium reserve for policyholder aged  $x$  in  $n$  years coverage period,  $A_{x+t:\overline{n-t}|}^1$ : single net premium of policyholder aged  $(x+t)$  years, with coverage period of  $(n-t)$ ,  $\ddot{a}_{x+t:\overline{n-t}|}$ : initial term life annuity of  $(n-t)$  years policyholder aged  $(x+t)$  years.

### 3. Materials and Methods

#### 3.1. Materials

The object used in this research is simulated policyholder data that will determine each premium reserve using the Fackler Method and the Canadian Method.

#### 3.2. Methods

The steps taken in this study were:

- 1) Calculating the term life annuity using equation (1),
- 2) Calculating the single net premium value of term life insurance using equation (9),
- 3) Calculating the annual net premium value of term life insurance using equation (10),
- 4) Calculating the premium reserve value of term life insurance using the Fackler method in accordance with equation (11) and (12),
- 5) Calculating the value of premium reserves on term life insurance using the Canadian method in accordance with equation (13) and (14).

### 4. Results and Discussion

This study uses simulated data, where the variables used in this study are the age of the policyholder, gender, payment period, interest rate, and compensation value. For example, the age of the policyholder is 45 years old, female gender, payment period of 30 years, interest rate of 5.75%, and compensation value of IDR 50,000,000.

The first step is to calculate the policyholder term life annuity using equation (1):

$$\begin{aligned} \ddot{a}_{x:\overline{n}|} &= \frac{N_x - N_{x+n}}{D_x} \\ \ddot{a}_{45:\overline{30}|} &= \frac{N_{45} - N_{75}}{D_{45}} \\ &= \frac{111185.551207}{7859.478523} \\ &= 14.14668 \end{aligned}$$

The next step is to calculate the policyholder net single premium using equation (9):

$$\begin{aligned} A_{x:\overline{n}|}^1 &= S \cdot \frac{M_x - M_{x+n}}{D_x} \\ A_{45:\overline{30}|}^1 &= 50,000,000 \cdot \frac{M_{45} - M_{75}}{D_{45}} \\ &= 50,000,000 \cdot \frac{761.2453173}{7859.478536} \\ &= 4,842,848.76 \end{aligned}$$

After that, the annual net premium of the policyholder is calculated using equation (10):

$$\begin{aligned} P_{x:\overline{n}|}^1 &= \frac{A_{x:\overline{n}|}^1}{\ddot{a}_{x:\overline{n}|}} \\ P_{45:\overline{30}|}^1 &= \frac{A_{45:\overline{30}|}^1}{\ddot{a}_{45:\overline{30}|}} \\ &= \frac{4,842,848.76}{14.14668} \\ &= 342,331.04 \end{aligned}$$

After obtaining the results of the term life annuity, single net premium, and annual net premium, then calculate the premium reserve using the Fackler method using equations (11) and (12):

$$\begin{aligned}
 {}_tV &= P \cdot {}_t u_x - {}_t k_x \\
 {}_tV &= P \frac{N_x - N_{x+t}}{D_{x+t}} - \frac{M_x - M_{x+t}}{D_{x+t}} \\
 {}_1V &= P \frac{N_{45} - N_{46}}{D_{46}} - \frac{M_{45} - M_{46}}{D_{46}} \\
 &= 342,331.04 \cdot 1.059544921 - 0.001933732 \\
 &= 362,715.12 \\
 {}_{t+1}V &= u_{x+t}({}_tV + P) - k_{x+t} \\
 {}_{1+1}V &= u_{45+1}({}_1V + P) - k_{45+1} \\
 &= \frac{D_{46}}{D_{47}}({}_1V + P) - \frac{C_{46}}{D_{47}} \\
 &= 1,05977(705,046.16) - 0.00214459 \\
 &= 747,185.29
 \end{aligned}$$

Based on the calculation of premium reserves using the Fackler method above, the results of the calculation of premium reserves for 30 years are obtained using the same method:

**Table 1:** Premium Reserve Value using Fackler Method

t	premium reserve (IDR)	t	premium reserve (IDR)
1	747,185.29	16	10,660,001.31
2	1,154,923.79	17	11,752,966.24
3	1,587,601.76	18	12,933,563.41
4	2,047,024.54	19	14,211,789.50
5	2,535,211.13	20	15,599,074.11
6	3,054,424.39	21	17,108,851.70
7	3,607,291.64	22	18,756,988.38
8	4,196,828.79	23	20,561,677.58
9	4,826,320.18	24	22,544,939.08
10	5,499,228.98	25	24,727,763.99
11	6,219,055.20	26	27,141,025.87
12	6,989,338.65	27	29,820,800.04
13	7,813,876.37	28	32,808,943.53
14	8,697,026.78	29	36,157,380.36
15	9,643,696.10	30	39,928,048.80

Then the last one is calculating premium reserves with the Canadian method using equations (13) and (14):

$$\begin{aligned}
 {}_tV_{x:\overline{n}|}^{can} &= A^1_{x+t:\overline{n-t}|} - \beta^{can} \cdot \ddot{a}_{x+t:\overline{n-t}|} \\
 {}_1V_{x:\overline{n}|}^{can} &= A^1_{46:\overline{29}|} - \beta^{can} \cdot \ddot{a}_{46:\overline{29}|} \\
 &= 5,034,529.21 - 377430.03 \cdot 13.92950119 \\
 &= -222,882.84
 \end{aligned}$$

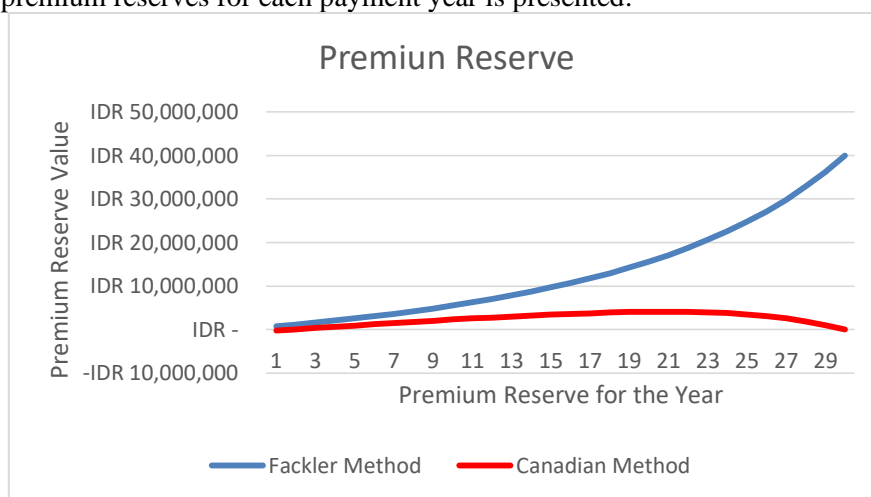
Based on the calculation of premium reserves using the Canadian method above, the results of the calculation of premium reserves for 30 years are obtained using the same method:

**Table 2:** Premium Reserve Value using Canadian Method

t	premium reserve (IDR)	t	premium reserve (IDR)
1	- 222,882.84	16	3,585,983.81
2	56,554.70	17	3,758,489.58
3	340,252.06	18	3,911,001.45
4	626,628.13	19	4,027,479.66
5	915,027.47	20	4,100,977.87
6	1,203,794.48	21	4,123,929.56
7	1,490,720.19	22	4,087,104.73

8	1,772,046.90	23	3,979,393.35
9	2,043,359.55	24	3,788,942.90
10	2,301,458.87	25	3,500,595.95
11	2,544,872.42	26	3,106,399.25
12	2,774,395.51	27	2,584,585.99
13	2,992,249.36	28	1,911,361.39
14	3,200,270.05	29	1,060,867.86
15	3,398,489.35	30	0.00

In order to see more clearly the calculation of premium reserves using the Fackler and Canadian methods, a comparison graph of premium reserves for each payment year is presented:



**Figure 1:** Comparison chart of policyholder premium reserves

## 5. Conclusion

It can be seen from the results of the calculation of the premium reserve value in term life insurance using the Fackler method, namely the first reserve always increases until the end of the policyholder's insurance period, while the results of calculating the premium reserve value using the Canadian method, namely the first year reserve is negative and increases in the next insurance period, then will decrease until the end of the period, namely IDR 0.00.

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