



Application of Mathematical Model in Bioeconomic Analysis of Skipjack Fish in Pelabuhanratu, Sukabumi Regency, Jawa Barat

Fathimah Syifa Nurkasyifah^{1*}, Asep K. Supriatna², Herlina Napitupulu³

¹*Undergrad Program in Mathematics, Faculty of Mathematics and Sciences, Universitas Padjadjaran, Sumedang, Jawa Barat, Indonesia*

^{2,3}*Department of Mathematics, Faculty of Mathematics and Sciences, Universitas Padjadjaran, Sumedang, Jawa Barat, Indonesia*

**Corresponding author email: fathimah20001@mail.unpad.ac.id*

Abstract

Presently, sustainability has emerged as a crucial and compelling concern across diverse sectors, evolving into a long-term agenda championed by the United Nations through the implementation of the Sustainable Development Goals (SDGs). Within the SDGs, particularly under point 14 addressing life below water, emphasis is placed on ensuring sustainability in aquatic ecosystems, encompassing the fisheries sector. The concept of Maximum Sustainable Yield (MSY) holds significance in the bioeconomic analysis of fisheries, influencing decision-making processes aimed at preserving sustainability. Regrettably, several studies have identified inaccuracies in the determination of MSY, leading to instances of overfishing in various regions. Conversely, it is imperative to give due attention to Maximum Economic Yield (MEY) to ensure that economic considerations remain integral to decision-making processes. Consequently, a more comprehensive and detailed bioeconomic analysis, incorporating mathematical models, becomes essential. Among these models, the logistic growth rate model and the Gompertz growth rate model stand out as significant contributors.

Keywords: mathematical models; logistic growth rate model; Gompertz growth rate model.

1. Introduction

Global fish catches have exceeded sustainability limits, with exploitation rates reaching 34.2% in 2017 (Food and Agriculture Organization of the United Nations, 2020). In Indonesia, in 2017, around 75% of water areas experienced complete exploitation, causing cases of overfishing (Napitupulu et al., 2022, p. 4). Although the Indonesian Government is trying to establish a Maximum Sustainable Yield (MSY) for fish catch management, in 2020, 71.14% of the total permitted catch had been exceeded (Napitupulu et al., 2022, p. 17). It is necessary to improve management of fisheries sustainability by considering MSY and Maximum Economic Yield (MEY) (Alfarizi et al., 2019; Diop et al., 2018; Mayalibit et al., 2014).

The Sustainable Development Goals (SDGs) agenda by the United Nations (UN) includes fisheries sustainability, especially SDG 14.4 which is emphasized by Indonesia (Napitupulu et al., 2022, p. 17). Improving the quality of fisheries sector data and analysis is the key to achieving better sustainability management (Napitupulu et al., 2022, p. 17). In this context, MSY and MEY become important instruments, with mathematics playing a crucial role in calculations through various growth models such as logistics and Gompertz (Dichmont et al., 2010; Sholahuddin et al., 2015).

Parameter estimation methods, such as multiple linear regression with Ordinary Least Square (OLS), can be applied to support regression analysis in logistic and Gompertz growth rate models (Sholahuddin et al., 2015). This research will apply the logistic growth rate model with the Schaefer and Schnute discrete model, as well as the Gompertz growth rate model with the Fox and CY&P discrete models on Cakalang fish catch data in Pelabuhanratu, Sukabumi Regency, Jawa Barat. Parameter estimation will be carried out using multiple linear regression with the OLS estimation method. This approach is expected to provide better insight into the sustainability of the skipjack tuna fishery and support effective fisheries management decision making.

2. Literature Review

2.1. Logistic Growth Rate Model

The logistic growth model, established by Pierre-François Verhulst in 1838, serves as a widely used framework for comprehending population growth. Verhulst devised the logistic equation to depict the self-regulating growth evident in biological populations (Sholahuddin et al., 2015). The logistic differential equation stands out as the most widely recognized form of the model. If we let $y(t)$ denote the size or density of a population at time t , the growth rate in the logistic differential equation is described as follows (Supriatna et al., 2016):

$$\frac{dX}{dt} = rX \left(1 - \frac{X}{K}\right) \tag{1}$$

Where r is the growth rate, K is the environmental capacity, X is the population size. Harvesting can impact the growth rate of a population. Equation (1) is adjusted by introducing the harvest coefficient C , defined as follows (Husniah & Supriatna, 2015):

$$\frac{dX}{dt} = rX \left(1 - \frac{X}{K}\right) - C \tag{2}$$

$$\frac{dX}{dt} = rX \left(1 - \frac{X}{K}\right) - qEX \tag{3}$$

From this logistic growth rate model, the MSY and MEY are obtained (Husniah & Supriatna, 2014):

$$MSY_{logistic} = \frac{rK}{4} \tag{4}$$

$$MEY_{logistic} = \frac{rK}{4} - \frac{c^2r}{4Kp^2q^2} \tag{5}$$

In MEY there is E , which is effort, q , which is the catch coefficient, p , which is the producer's selling price, and c is the production cost MSY and MEY calculations can be carried out if the required parameters are known. Therefore, a discretization model is needed to estimate the parameters of the growth rate model. There are various types of discretization models that correspond to the logistic growth rate model, one of which is the Schaefer model (Clarke et al., 1992):

$$\frac{\bar{U}_{n+1} - \bar{U}_{n-1}}{2\bar{U}_n} = r - \left(\frac{r}{qK}\right) \bar{U}_n - q\bar{E}_n \tag{6}$$

$$\bar{U}_n = \frac{Catch_n}{Effort_n} \tag{7}$$

In the model \bar{U}_n is the catch per unit effort in n data. Another form of discretization model based on the logistic growth rate model is the Schnute model (Clarke et al., 1992):

$$\ln\left(\frac{\bar{U}_{n+1}}{\bar{U}_n}\right) = r - \left(\frac{r}{qk}\right) \left(\frac{\bar{U}_n + \bar{U}_{n+1}}{2}\right) - q\left(\frac{\bar{E}_n + \bar{E}_{n+1}}{2}\right) \tag{8}$$

2.2. Gompertz Growth Rate Model

Another growth model that employs the sigmoid curve is the Gompertz growth model, named after Benjamin Gompertz. There are notable distinctions between the Gompertz curve and the logistic curve. The primary difference is evident in the approach of their asymptotes. Utilizing a similar approach as seen in the logistic model, the Gompertz model is formulated as follows (Sholahuddin et al., 2015):

$$\frac{dX}{dt} = rX \ln\left(\frac{K}{X}\right) \tag{9}$$

Where r is the growth rate, K is the environmental capacity, X is the population size. Harvesting can impact the growth rate of a population. Equation (8) is adjusted by introducing the harvest coefficient C , defined as follows:

$$\frac{dX}{dt} = rX \ln\left(\frac{K}{X}\right) - C \tag{10}$$

$$\frac{dX}{dt} = rX \ln\left(\frac{K}{X}\right) - qEX \tag{11}$$

From this Gompertz growth rate model, the MSY and MEY are obtained (Supriatna et al., 2015a):

$$MSY_{Gompertz} = \frac{rK}{e} \tag{12}$$

$$MEY_{Gompertz} = r \left(1 - LambertW \left(\frac{ec}{Kpq} \right) \right) K e^{LambertW \left(\frac{ec}{Kpq} \right) - 1} \quad (13)$$

There are also various types of discretization models that correspond to the Gompertz growth rate model, one of which is the Fox model (Ramadhan et al., 2014):

$$\frac{\bar{U}_{n+1} - \bar{U}_{n-1}}{2\bar{U}_n} = r \ln(qK) - r \ln(\bar{U}_n) - q\bar{E}_n \quad (14)$$

Another form of discretization model based on the Gompertz growth rate model is the CY&P model (Supriatna et al., 2015b):

$$\ln(\bar{U}_{n+1}) = \left(\frac{2r}{2+r} \right) \ln(qK) + \left(\frac{2+r}{2-r} \right) \ln(\bar{U}_n) - \left(\frac{q}{2+r} \right) (\bar{E}_n + \bar{E}_{n+1}) \quad (15)$$

2.3. Multiple Regression

A linear regression model that consists of more than one regressor variable is called a multiple linear regression model. The multiple linear regression model with k regressors is (Muhammad et al, 2012):

$$y_k = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_k x_k + \varepsilon \quad (16)$$

| | |
|----------------------------------|--------------------------|
| y | : Dependent variable |
| $x_k, k = 1, 2, \dots, k$ | : Predictor variable |
| $\beta_k, k = 0, 1, 2, \dots, k$ | : Regression coefficient |
| ε | : Error |

In multiple regression, OLS is used to assess which of multiple predictors are more or less important in predicting the outcome variable or how one or more predictors interact with each other. The OLS method aims to minimize the sum of square differences between the observed and predicted values of the dependent variable. The coefficients β can be estimated using the following formula

$$\hat{\beta} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y} \quad (17)$$

where \mathbf{X} is the matrix of the explanatory variables and \mathbf{y} is the vector of the dependent variable.

3. Materials and Methods

3.1. Materials

The data used to calculate MSY and MEY involves information regarding month, year, number of catches, effort in the form of number of trips, size of catch vessel, producer selling price, and production costs for skipjack tuna catch data in Republic of Indonesia State Fisheries Management Area (WPPNRI) 573 with catch reporting port of Pelabuhanratu, Sukabumi Regency, West Java during the period January 2021 – September 2023. The work process will be assisted by Microsoft Excel to make it faster and more efficient

3.2. Methods

1. CPUE Calculation,
For monthly catch and effort data, CPUE will be calculated using Equation (7).
2. Data Transformation,
After obtaining the CPUE, then the data will be transformed into a discretization model in Equations (6), (8), (14), (15).
3. Multiple Linear Regression with Ordinary Least Squares (OLS),
Next, the data will be processed using multiple linear regression with the Ordinary Least Square (OLS) method to obtain regression coefficients that will be used in the next stage.
4. Parameter Estimation,
The results of the regression coefficients are substituted back into the discretization model to obtain the growth parameters r , q , and K . For negative parameter values, the calculation process is not continued, for positive parameter values, the calculation process continues to the next stage.
5. Calculation of MSY and MEY,
The next step involves calculating the MSY and MEY using the results of the estimated growth parameters r , q , and K according to the growth rate model. In the logistic growth rate model, Equation (4) is used for MSY and Equation (5) for MEY. Meanwhile, in the selected Gompertz growth rate model, Equation (12) is used for MSY and Equation (13) for MEY.

4. Results and Discussion

Data that has been transformed with a total of four different data will be processed for parameter estimation using a multiple linear regression model with the ordinary least square method according to Equation (17). So the following results are obtained:

Table 1: Coefficient regression and coefficient determination result from multiple regression linear

| Data | b_0 | b_1 | b_2 | R^2 |
|----------------|------------|------------|-----------|-----------|
| Model Schaefer | -0.2746251 | -0.0000141 | 0.0368477 | 0.2004303 |
| Model Schnute | -0.5697116 | -0.0000010 | 0.0543489 | 0.2143008 |
| Model Fox | 0.0483078 | -0.0050759 | 0.0015519 | 0.0003574 |
| Model CY&P | 1.8684391 | 0.7185055 | 0.0295229 | 0.6446479 |

After obtaining the regression coefficients, substitute these coefficients into the initial discretization model in Equations (6), (8), (14), (15) to obtain the parameter values:

Table 2: Parameter estimation results for each model

| Data | b_0 | b_1 | b_2 |
|----------------|-------------|-------------|--------------------|
| Model Schaefer | -0.27462511 | -0.03684773 | 529,619.57633 |
| Model Schnute | -0.56971156 | -0.05434893 | 10,104,061.25880 |
| Model Fox | 0.00507587 | -0.00155192 | -0,000008757518411 |
| Model CY&P | 0.32760380 | 0.06871773 | 11,106.85303 |

First, the results of parameter estimation using regression show that the highest coefficient of determination was obtained by the CY&P model, with 0.6446479 coefficient determination. The parameter estimation results show that in several models and data, the values of r , q , and K are negative. The parameters r , q , and K should be positive. Therefore, only the parameters from the CY&P model will be continued to calculate the MSY and MEY using Equations (12) and (13).

Table 3: MSY and MEY results

| Data | MSY | MEY |
|------------|-------------|-------------|
| Model CY&P | 1,338.58353 | 1,338.57978 |

The derived MSY and MEY values are remarkably similar, differing only in decimal places. Unfortunately, the outcomes of this assessment reveal overexploitation of skipjack tuna in Pelabuhanratu. Urgent attention from the government is needed to implement enhanced fisheries management and address this issue effectively.

5. Conclusion

Calculation of MSY and MEY on Skipjack Fish data in the Republic of Indonesia State Fisheries Management Area (WPPNRI) 573 with the catch reporting port of Pelabuhanratu, Sukabumi Regency, West Java for the period January 2021 – September 2023 can only be obtained through the Gompertz growth rate model and its discretization model. The process of calculating the MSY and MEY values using the logistic growth rate model cannot be continued because the parameter estimation results obtained to carry out the calculations are negative, meaning that the logistic model and the discretization model are not good at describing population growth in this data. This could also be caused by the parameter estimation method which cannot limit the parameters to positive values.

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