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Risk Measurement of Investment Portfolio Using VaR and CVaR from the Top 10 Traded Stocks on the IDX

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Abstract

Portfolio investment reflects a commitment to the allocation of funds or resources which is considered a strategic step in managing assets to achieve future profits. This research begins with a careful analysis of a portfolio consisting of the ten best stocks on the Indonesia Stock Exchange (IDX). Through in-depth processing and analysis of stock data, the dynamics of performance, risk and volatility involved in each investment commitment are revealed. The Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) methods at the 90%, 95%, and 99% confidence levels take centre stage, highlighting the potential for increased losses as confidence levels increase. In-depth analysis illustrates that CVaR, considering the extreme risks in the distribution, provides a more holistic picture than VaR. With a VaR (99%) value of IDR 84,973,959,424 and CVaR (99%) of IDR 471,795,822,064, this research provides a concrete picture of potential risks at the highest level of confidence. These results confirm that CVaR has a crucial role in identifying and measuring the potential for more significant losses, especially in the face of unexpected market uncertainty. As a guide for investment decision makers, this research forms an important basis for carefully considering the level of risk and potential return at various levels of confidence. This allows the development of smarter and more informed investment strategies.

Keywords: Value-at-Risk, Conditional Value-at-Risk, Portfolio, Investment, Risk Measurement

1. Introduction

Investment refers to the act of placing a certain amount of money or capital in an entity or financial instrument at present, with the hope of obtaining results or financial benefits in the future (Ines, 2016). In a business context, all forms of investment involve some level of risk. Portfolio is a field of science that specifically studies the strategies used by an investor to measure the risk in their investments. (Fahmi, 2009). Risk measures are real value functions that quantify the level of risk involved in random outcomes and several studies have established general properties for appropriate risk indicators.

Unfortunately, all risk measures commonly mentioned in the literature before 2000 do not satisfy some of the axiomatic properties necessary to be considered coherent. For example, the classic risk measure mean-variance (or mean-standard deviation), inspired by Markowitz H (1952), makes it economically meaningless because it is not monotonic. One of the weaknesses of this risk measure is that it applies profits and losses symmetrically, even though risk is asymmetric. Because of this, the popularity of these risk measures declined, and researchers began to show greater interest in quantile-based measures, such as value at risk (VaR).

Financial risk is divided into several main categories, such as credit risk, operational risk, and market risk, all of which are crucial elements in financial markets. While Value-at-Risk (VaR) is specifically related to market risk, this concept can also be applied to various other types of risk (Jorion, 2004). Although VaR has become very popular and has been integrated into industry regulations, it is incoherent and lacks subadditivity. Therefore, value at risk (VaR) can be interpreted as the maximum amount of loss that can be experienced in a financial position, considering a

certain time period and level of probability. (Dowd, 2002; Holton, 2002; Benninga & Wiener, 1998; Linsmeier & Pearson, 1996).

Conditional value at risk (CVaR) is a well-known measure of coherent risk according to Rockafellar and Uryasev, 2002; Rockafellar R.T (2007). Although Conditional Value at Risk (CVaR) is closely related to VaR, CVaR offers different advantages. In fact, according to Markowitz, when losses are normally distributed, looking for optimization through CVaR, VaR, and Minimum Variance produces identical optimal portfolios. CVaR becomes more clearly defined when the loss distribution does not follow a normal pattern or in situations where the optimization problem has high dimensions. CVaR is considered a coherent measure of risk for any type of loss distribution.

To understand the level of risk more deeply in investment portfolios, this research focuses on the ten best stocks on the Indonesia Stock Exchange (BEI). By applying the Value at Risk (VaR) and Conditional Value at Risk (CVaR) methods, this research attempts to provide a comprehensive picture of the potential losses that may occur in the portfolio. VaR is used to identify risk limits at a certain level of confidence, while CVaR provides additional information by detailing the average loss that may occur outside those limits. It is hoped that the results of this analysis will provide valuable insights for stakeholders in making smart and informed investment decisions.

2. Mathematical Model

2.1. Value-at-Risk(VaR)

VaR (Value at Risk) is a statistical method used to measure financial risk. VaR measures the extent to which the value of a portfolio or investment can fluctuate over a certain period with a certain level of confidence. Thus, it is important to note that at any given time, investors are interested in assessing the risk of their financial position for the coming period *l* . Referring to Tsay (2005) and Tsai (2004), assume changes in asset values in financial positions from time *t* to $t+1$. This quantity is measured in currency and is a random variable over time (t) . The cumulative distribution function (CDF) of this variable is expressed as $F_l(x)$. Definition of VaR long position (selling one's own shares) over the time l horizon with α probability as follows.

$$
\alpha = P[\Delta V(l) \leq VaR] = F_l(VaR)
$$
\n(1)

Shareholders with long positions will experience losses when VaR is defined in (1) assuming a small α negative value. A negative sign indicates potential loss. From this definition, the probability that shareholders will face a loss of at least VaR over a certain period l is α . Alternatively, VaR can be interpreted as the probability that the potential loss faced by shareholders in their financial position over that period *l* will be less than or equal to VaR, given probability $1-\alpha$. (Tsay, 2005; Tsai, 2004).

Shareholders with a short position (selling what they do not own) will face losses if the asset price ($\Delta V(l) > 0$) increases. VaR for shareholder short positions is defined as $\alpha = P[\Delta V(l) \geq VaR] = 1 - P[\Delta V(l) \leq VaR] = 1 - F_l(VaR)$ increases. VaR for shareholder short positions is defined as

$$
\alpha = P[\Delta V(l) \geq VaR] = 1 - P[\Delta V(l) \leq VaR] = 1 - F_l(VaR)
$$

If VaR has a small value α for a short position, this reflects a positive value being assumed. Where, a positive sign indicates a loss.

2.1.1. VaR pendekatan Distribusi Normal Standar

The standard method assumes that asset returns can be modelled as a normal univariate distribution with two parameters: mean μ and standard deviation σ . The challenge in estimating Value at Risk (VaR) is determining the value at the α percentile of the standard normal distribution z_{α}

standard normal distribution
$$
z_{\alpha}
$$

\n
$$
\alpha = \int_{-\infty}^{q} f(r)dr = \int_{-\infty}^{z_{\alpha}} \Phi(z)dz = N(z_{\alpha}), \text{ quantile } q = z_{\alpha}\sigma + \mu
$$
\n(2)

where $\Phi(z)$ the standard normal distribution density function $N(z)$, cumulative normal distribution function r, random variables for portfolio returns $f(r)$, normal distribution density function for returns (log returns) with mean

 μ and standard deviation σ , as well as the smallest log return q at a certain level of confidence α are all involved in the estimation process Value at Risk (VaR). VaR is calculated using an equation involving these elements.
 $VaR = -W_0 \times q = -W_0(z_\alpha \sigma + \mu)$ (3)

$$
VaR = -W_0 \times q = -W_0(z_\alpha \sigma + \mu)
$$
\n(3)

The following explains that it refers to the initial investment, and the minus sign (-) indicates potential losses. (Kang & Yoon, 2007; Dokov & Stoyanov, 2007; Kindanova & Rachev, 2005; Jorion, 2004)

2.1.2. VaR Portfolio Investment

portfolio return r_{pt} at time t, it can be determined that.

$$
r_{pt} = \sum_{i=1}^{N} w_i r_{it} \text{ ; condition } \sum_{i=1}^{N} w_i = 1 \tag{4}
$$

 $t : t = 1, ..., T$ with period T observation data

 r_{it} : *return* from each asset *i* (*i* = 1,..., *N*) at time *t*

 w_i : w_i weight (proportion) of funds allocated to assets *i*

Equation (4) can also be expressed in terms of vector equation (5).

$$
r_{pt} = \mathbf{w}^T \mathbf{r} \; ; \text{ condition } \mathbf{e}^T \mathbf{w} = 1 \tag{5}
$$

 $\mathbf{w}^T = (w_1 \ w_2 \ ... \ w_N)$: transpose the weight vector $\mathbf{r}^T = (r_1 \ r_2 \ ... \ r_N)$: transpose the asset return vector $\mathbf{e}^T = (1 \ 1 \dots 1)$: transpose a vector whose elements are N as many as one

Also need to know $\mathbf{e}^T \mathbf{w} = \mathbf{w}^T \mathbf{e} = 1$. Based on equation (4), μ_{pt} can be determined using this equation.

$$
\mu_{pt} = \sum_{i=1}^{N} w_i \mu_{it} \text{ or } \mu_{pt} = \mathbf{w}^T \mathbf{\mu}, \qquad (6)
$$

 $\mu_{it} = E(r_{it})$: average stock return *i* (*i* = 1,..., *N*) at time *t*

 $\boldsymbol{\mu}^T = (\mu_{1t} \ \mu_{2t} \ ... \ \mu_{Nt})$: transpose the asset return vector *i* (*i* = 1,..., *N*) $\mu_{pt} = \mathbf{w}^T \mathbf{\mu} = \mathbf{\mu}^T \mathbf{w}$: average portfolio return

Portfolio return variance σ_{pt}^2 can be determined by

$$
\sigma_{pt}^2 = E[(r_{pt} - \mu_{pt})^2]
$$

=
$$
\sum_{i=1}^N w_i^2 \sigma_{it}^2 + \sum_{i=1}^N \sum_{j=1}^N \rho_{ij} \sigma_{it} \sigma_{jt} ; \text{condition } i \neq j
$$
 (7)

 $\sigma_{it}^2 = Var(r_{it})$: variance of asset *i*

 ρ_{ii} : correlation coefficient between assets *i* dan assets *j* $(i, j = 1, ..., N$ and $i \neq j$.

As well as σ_{it} and σ_{jt} respectively standard deviation of assets *i* and standard deviation of assets *j* at time *t*. Dapat also determined the covariance between assets *i* dan assets *j*, $\sigma_{ij} = Cov(r_{it}, r_{jt})$ using this equation $\sigma_{ij} = E[(r_{it} - \mu_{it})(r_{jt} - \mu_{jt})] = \rho_{ij}\sigma_{it}\sigma_{jt}$; $i \neq j$

$$
\sigma_{ij} = E[(r_{it} - \mu_{it})(r_{jt} - \mu_{jt})] = \rho_{ij}\sigma_{it}\sigma_{jt}; i \neq j
$$
\n(8)

For example, given the following covariance matrix **Σ**

$$
\Sigma = \begin{pmatrix}\n\sigma_{11} & \sigma_{12} & \sigma_{13} & \dots & \sigma_{1N} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} & \dots & \sigma_{2N} \\
\sigma_{31} & \sigma_{32} & \sigma_{33} & \dots & \sigma_{3N} \\
\dots & \dots & \dots & \dots & \dots \\
\sigma_{N1} & \sigma_{N2} & \sigma_{N3} & \dots & \sigma_{NN}\n\end{pmatrix},
$$
\n(9)

where $\sigma_{ii} = \sigma_i^2$ with $i = 1,..., N$. Portofolio variance σ_{pt}^2 given by equation (7) can be expressed as the product of the covariance matrix and the weight vector, namely

$$
\sigma_{pt}^2 = \mathbf{w}^T \Sigma \mathbf{w} \tag{10}
$$

Value-at-Risk (*VaR*) portfolio investasi with weight **w** vector, *VaRpt* ,

VaR_{pt} = -W₀(
$$
\mu_{pt} + z_{\alpha} \sigma_{pt}
$$
) atau VaR_{pt} = -W₀{ $\mathbf{w}^T \mathbf{\mu} + z_{\alpha} (\mathbf{w}^T \Sigma \mathbf{w})^{1/2}$ }. (11)

Where W_0 is the amount of funds allocated in forming an investment portfolio, and the normal distribution percentile z_{α} when given a significance level α . Usually, a significance level is $\alpha = 0.05$ (Gaivoronski & Pflug, 2005; Huisman *et al*., 1999)

2.2. Conditional Value-at-Risk (CvaR)

It is known that Value at Risk (VaR) does not fulfill the nature of subadditivity, thus causing a lack of coherence as a measure of risk. As a solution to this weakness, Acerbi and Tasche (2002) proposed the use of Expected Shortfall as an alternative. Expected Shortfall is defined as the conditional expectation of a loss more than value VaR_{α} . In fact, Conditional Value at Risk (CVaR) is a stronger measure of coherent risk compared to VaR, because CVaR maintains least legal invariant coherence even though VaR is unable to do so. CVaR at a confidence level $1 - \alpha$ (or at a risk level α) is defined as the average of losses that exceed the VaR value, as seen in the following formula.

$$
\text{CVaR}_{1-\alpha}(X) := \inf_{t \in \mathbb{R}} \left(t + \frac{1}{\alpha} E[X - t]_{+} \right), \qquad X \in \mathbf{L}_1, \alpha \in (0,1]
$$

where $[s]_+$ = max{0, s}. This measure can be represented by VaR as follows.

$$
CVaR_{1-\alpha} (X) = \frac{1}{\alpha} \int_0^{\alpha} VaR_{1-t} (X) dt,
$$

 $CVaR_{1-\alpha}(X)$ describes the average value of α % of the worst events in the distribution X. for more in-depth comparison between VaR and CVaR in risk management and optimization processes, see Sarykalin dkk. (2008)

2.2.1. CvaR untuk Distribusi Kerugian Normal

For example, a normal distribution *Y* with a mean μ dan standard deviation σ . For $\alpha \in (0, 1)$, CvaR is given by
 $CVaR$ $(V) = u + \sigma \phi(\Phi^{-1}(\alpha))$

$$
CVaR_{\alpha}(Y) = \mu + \sigma \frac{\phi(\Phi^{-1}(\alpha))}{1-\alpha}
$$

Where ϕ represents the probability density function(pdf) of the standard normal distribution For comparison VaR and CVAR for normal distribution, $X \sim N(\mu, \sigma^2)$ are as follows: $V \cap D$ $(Y) = u + z \sigma$

$$
\operatorname{CVaR}_{1-\alpha}(X) = \mu + \frac{\phi z_{\alpha}}{\alpha}
$$

$$
\operatorname{CVaR}_{1-\alpha}(X) = \mu + \frac{\phi z_{\alpha}}{\alpha}
$$

In this situation, $\phi(.)$ refers to the density function of the standard normal distribution, and z_α is the α percentile of the standard normal distribution. If the data follows a normal distribution, all of these risk metrics can be interpreted in terms of mean and variance. Furthermore, this risk metric can be considered equivalent to the mean-standarddeviation risk metric for varying values of W with different λ .

3. Materials and Methods

3.1. Materials

In this section, we will explore the optimal weight (proportion) of funds invested in each stock to form a portfolio. The main goal is to achieve an optimal balance between the level of profit and the level of risk of the investment portfolio. This portfolio discussion focuses on measuring the investment portfolio for the ten best shares on the Indonesia Stock Exchange (IDX) in the period January 2023 to November 2023. The shares considered in this portfolio involve the best companies on the IDX such as ICBP, BBNI, TPIA, UNVR, ASII, BMRI, TLKM, BYAN, BBRI, and BBCA. The data used for analysis comes from Yahoo Finance sources. By exploring these optimal weights, it is hoped that we can achieve profitable portfolio results with a manageable level of risk.

3.2. Methods

The process of processing and analyzing stock data from the ten best companies on the Indonesia Stock Exchange (\overrightarrow{IDX}) is the main foundation for exploring portfolio characteristics. This process aims to explore the trend, volatility and historical performance of each stock, providing a deep understanding of performance dynamics that can form a smart investment strategy.

In analyzing stock returns, focus is given to significant variations between stocks, providing a clearer picture of the growth or decline in asset value. Furthermore, the estimation of the average value and the formation of the weight vector through the use of the average value estimator are key steps to summarize the overall performance of the portfolio. The formation of a unit vector as the weight of each stock creates a balanced approach in fund allocation, forming a strategic basis for portfolio management.

Followed by calculating the variance and standard deviation of the portfolio using the covariance matrix and weight vector, this analysis provides in-depth insight into the overall level of fluctuation and volatility of the portfolio. The next step is to enter the realm of risk analysis, using Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) methods at different levels of confidence. Discussion of the results and implications highlights the comparison between VaR and CVaR, reinforcing CVaR's role in providing a holistic understanding of portfolio risk.

Finally, risk policy recommendations are a crucial stage in anticipating and managing the risks contained in the portfolio. A holistic understanding of each step of this analysis is the basis for optimizing returns while minimizing risk, creating adaptive and effective investment strategies in dealing with market dynamics.

4. Results and Discussion

In this stage, the data that has been collected will be processed and analyzed in depth. The results of this analysis will form the basis for identifying investment patterns that can optimize profit levels and minimize risk.

4.1. Results

In analyzing an investment portfolio consisting of the ten best stocks on the Indonesia Stock Exchange (IDX), we can understand the performance dynamics and risks involved to form a comprehensive picture of potential investment returns. Each stock in a portfolio has its own characteristics, and the steps taken to manage risk and optimize returns are critical. The individual returns of each stock show significant variations, with some stocks experiencing growth, while others experience a decline.

ICBP stocks, for example, showed a return of -0.1066, indicating a decline in value. However, the interesting thing is that the weight remains at 1.00, providing an equal contribution to the portfolio formation. The average ICBP stock return is -0.00049, indicating that there is fluctuation but at a low level. On the other hand, UNVR shares recorded a quite positive return of 0.2657, showing solid performance. Even so, the average UNVR stock return is only around 0.00121, depicting more stable and consistent growth.

Using an estimator of average values, arranged into an average vector form, $\mu^T = (-0.00049 - 0.00053 - 0.00053)$ 0.00056 0.00121 0.00035 0.00155 0.00013 0.00073 -0.00028 0.00016). Because the number of shares analyzed was 10, a unit vector was formed = $(1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1$). For portfolio variance σ_{pt}^2 given by equation (10) can be expressed as the product of the covariance matrix and the weight vector. The covarianc expressed as the product of the covariance matrix and the weight vector. The covariance matrix is given as follows;

Therefore, another important aspect in this analysis is the level of volatility or risk contained in the portfolio. The variance and standard deviation of the portfolio can be seen in Table 3 where, respectively, they reach 5.58 and 2.36, highlighting a high level of fluctuation and volatility. This indicates that this portfolio has a fairly significant level of risk.

The weight given to each stock in the portfolio is very even, namely 1.00, indicating an equal approach in fund allocation. The average stock return reflects average growth, and all contribute to the formation of an overall portfolio return of 43.11%, which can be seen in Table 3.

The results of the estimated VaR and CVaR values can be seen in Table 4. At a 90% confidence level, the VaR value is IDR 46,810,931,394, while the CVaR value reaches 0.00224 or the equivalent of 0.224% of total assets at this time. This means that as much as 10% of the risk not covered by VaR can be identified through CVaR, indicating that the potential loss that may occur in the next day is around 0.224% of the total asset value.

For a 95% confidence level, the VaR value reaches IDR 60,081,179,996, with a CVaR value of 0.00256 or 0.256%. This illustrates that 5% of the risk that is not covered by VaR can be estimated through CVaR, indicating a potential loss of 0.256% of the total asset value in the next day.

At a 99% confidence level, the VaR value is IDR 84,973,959,424, with a CVaR value of 0.00328 or 0.328%. This result reflects that 1% of the risk not covered by VaR can be calculated through CVaR, indicating a potential loss of 0.328% of the total asset value in the next day.

Overall, this analysis implies that with higher levels of confidence, CVaR values indicate greater loss potential, providing a more comprehensive understanding of the risks to an asset portfolio. As an illustration, by having capital of IDR 100,000,000, the losses that can occur at 90%, 95% and 99% confidence levels are IDR 224,000, IDR 256,000, and IDR 328,000, respectively.

4.2. Discussion

Based on the results of this research showing the Value-at-Risk and Conditional Value-at-Risk values at the 90%, 95% and 99% confidence levels, it can be concluded that the higher the level of confidence, the VaR and CVaR values also increase. This means that as the level of trust increases, we see an increase in the potential losses that can occur. Increasing VaR values at higher confidence levels reflect increased risk to the portfolio, however, this information is not enough to provide an overall picture of potential losses.

Therefore, an in-depth analysis of Conditional Value-at-Risk (CVaR) reveals a more comprehensive view of portfolio risk. CVaR not only considers the extremes of the distribution, which Value-at-Risk (VaR) often ignores, but also provides a more in-depth look at how much potential loss may occur within the tails of the distribution. The importance of CVaR is increasingly evident at the highest confidence level, namely 99%. At this level of confidence, CVaR reflects a higher value compared to VaR, indicating that the potential loss at extreme risk is much greater than can be measured by VaR. This fact strengthens the argument that CVaR is an important instrument in identifying and measuring risk, especially in highly unpredictable market conditions.

This discussion highlights the essentiality of including CVaR in a portfolio risk evaluation framework, especially at high levels of confidence. Awareness of these extreme risks provides the holistic view necessary for decision makers in managing investment portfolios. In addition, this analysis confirms the need to adjust risk policies in accordance with established risk tolerance and investment objectives, making CVaR an important tool in making smart investment decisions.

5. Conclussion

In conclusion, the results of this research show that the investment portfolio analysis of the ten best shares on the Indonesia Stock Exchange (BEI) has provided in-depth insight regarding historical performance and the risks involved. As the level of confidence increases, the Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) values show an increase, indicating the potential for greater losses. In-depth analysis of CVaR provides a holistic picture of portfolio risk, especially at high confidence levels (99%).

This emphasizes the importance of CVaR as a risk evaluation tool that provides a more comprehensive perspective than VaR. In this context, it is recommended that the risk policy implemented must be in accordance with the risk tolerance and investment objectives that have been set, so as to optimize the level of profit while minimizing potential risks in managing the investment portfolio.

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