



Investment Portfolio Optimization in Infrastructure Stocks Using the Mean-VaR Risk Tolerance Model

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Abstract

Infrastructure a crucial role in economic development and the achievement of Sustainable Development Goals (SDGs), with investment being a key activity supporting this. Investment involves the allocation of assets with the expectation of gaining profit with minimal risk, making the selection of optimal investment portfolios crucial for investors. Therefore, the aim of this research is to identify the optimal portfolio in infrastructure stocks using the Mean-VaR model. Through portfolio analysis, this study addresses two main issues: determining the optimal allocation for each infrastructure stock and formulating an optimal stock investment portfolio while minimizing risk and maximizing return. The methodology employed in this research is the Mean-VaR approach, which combines the advantages of Value at Risk (VaR) in risk measurement with consideration of return expectations. The findings indicate that eight infrastructure stocks meet the criteria for forming an optimal portfolio. The proportion of each stock in the optimal portfolio is as follows: ISAT (2.74%), TLKM (33.894%), JSMR (3.343%), BALI (0.102%), IPCC (5.044%), KEEN (14.792%), PTPW (25.863%), and AKRA (14.219%). The results of this study can serve as a foundation for better investment decision-making.

Keywords: Investment; Optimal Portfolio; Mean-VaR; Infrastructure

1. Introduction

The presence of infrastructure, as a collection of physical and non-physical facilities serving society, plays a central role in supporting economic growth. In line with this view, adequate infrastructure development is seen as an essential strategy in improving the quality of human resources. Besides being a driver of economic growth, infrastructure also focuses on achieving Sustainable Development Goals (SDGs). SDGs set targets for building resilient infrastructure, promoting inclusive and sustainable industrialization, and fostering innovation. In this context, sustainable investment in the infrastructure and innovation sector plays a crucial role as an integral element of one of the 17 Global Goals in the 2030 Sustainable Development Agenda). The importance of adopting an integrated approach is key to the success of achieving all these goals.

Based on the previous explanation, public interest in investment is increasing as investment provides flexibility for use both in the short and long term. Investment activities involve the placement of money or capital in a company or project with the hope of gaining profit within a certain period. To achieve optimal investment goals, especially in facing market fluctuations, it is important for investors to form a portfolio that not only maximizes expected returns but also manages risks at an acceptable level (Asthana & Ahmed, 2023). Success in managing this portfolio requires a careful approach and consideration of factors such as asset diversity, balanced allocation, and appropriate risk management strategies. Portfolios are formed as a step to reduce investment risk by combining multiple assets (Deng et al., 2021). The principles of portfolio optimization and diversification play a significant role in the development and understanding of financial markets (Vereshchaka, 2021). Portfolio selection can create a combination that maximizes expected returns according to the accepted level of risk (Hu et al., 2021).

To achieve optimal investment goals, especially in the face of market fluctuations, it is crucial for investors to construct a portfolio that not only maximizes expected returns but also maintains risk at an acceptable level. Success in managing this portfolio requires a careful approach and consideration of factors such as asset diversification,

balanced allocation, and appropriate risk management strategies. Thus, the formation of an optimal investment portfolio becomes a crucial step toward achieving long-term financial success and minimizing potential risks.

Therefore, the formation of an optimal portfolio becomes a crucial step in designing an investment strategy. Portfolios are constructed as a measure to reduce risk in investments by combining multiple assets. The principles of portfolio optimization and diversification play a significant role in the development and understanding of financial markets. Portfolio selection can form a combination that maximizes expected returns according to the accepted level of risk (Hu et al., 2021).

The determination of weights to achieve an optimal portfolio has involved several researchers using Mean-Variance optimization. However, it is worth acknowledging that traditional approaches like Mean-Variance Optimization have shortcomings, particularly in the use of variance as a risk parameter, which is often questioned (Salsabilla et al., 2023). Therefore, research in this field is increasingly highlighting the use of alternative methods such as Value at Risk (VaR). VaR becomes a crucial risk measure, defined as the estimate of the maximum loss that can occur over a specific period at a certain confidence level. Although widely applied to estimate financial risk, VaR has the advantage of providing a more comprehensive overview of risk by identifying the percentiles of the distribution of losses or gains, without focusing on every loss that exceeds the level (Lesmana et al., 2019).

There are several relevant studies to this research, Liu et al. (2021), discussed portfolio selection with uncertain returns based on Value at Risk. Behera et al. (2023), discussed portfolio optimization using Mean-VaR and developed a Machine Learning model for predictive modeling. Gharaibeh, O. (2019), discussed portfolio optimization on infrastructure sub-index returns in Jordan using CVaR.

Based on the descriptions above, this research examines optimization conducted with the Mean Value-at-Risk (Mean-VaR) model approach to determine the optimal selection of company stocks for constructing a portfolio with minimal risk and maximum return. The results of this study are expected to provide considerations for investment decision-making for investors, especially in the stocks analyzed in this research.

2. Literature Review

2.1. Investment

Investment is an individual's commitment to allocate owned assets with the goal of obtaining benefits from the allocation in the future (Balamurugan & Sivanesan, 2022). Investment should have a specific goal, allowing the determination of a timeframe to align with suitable products. One of the benefits of investment is the potential for asset or capital growth, as it can generate higher profits. Investments are divided into two types: real assets and financial assets. Real assets are usually tangible assets, such as land, machinery, gold, or houses. Meanwhile, financial assets include stocks, deposits, and mutual funds (Feruza, 2023). The selection of capital placement to be invested can be in various types; therefore, sufficient knowledge is needed to analyze the risks and benefits of which investment type is good to buy or sell.

2.2. Stock Return

In investing, individuals aim to achieve rewards after allocating their capital to a particular stock. Return serves as the reward for investors who bear the risk of their investment. Stocks offer investors the potential for significant returns in a short period, but these returns are proportionate to the associated risks (Liu et al., 2021). Stock return can be calculated using the following formula,

$$r_{it} = \frac{P_{it} - P_{i(t-1)}}{P_{i(t-1)}} \quad (1)$$

with,

r_{it} : Return of stock i at time t ,
 P_{it} : Price of stock i at time t ,
 $P_{i(t-1)}$: Price of stock i at time $t - 1$.

Furthermore, the expected value of return can be determined from the stock return using the following formula,

$$\mu_i = E(r_{it}) \approx \frac{1}{k} \sum_{n=0}^{k-1} r_{it-j} \quad (2)$$

Where k is the number of periods used. The determination of variance and covariance can be calculated using the following mathematical equations,

$$\sigma_i^2 = \frac{\sum_{t=1}^m (r_{it} - \mu_i)^2}{m} \tag{3}$$

$$\sigma_{i,j} = \frac{\sum_{t=1}^m (r_{it} - \mu_i)(r_{jt} - \mu_j)}{m} \tag{4}$$

with,

- σ_i^2 : Variance of stock i ,
- $\sigma_{i,j}$: Covariance between stock i and j ,
- μ_i : Expected return of stock i ,
- μ_j : Expected return of stock j .

2.3. Burr (4P) Distribution

The Burr distribution is one of the significant non-negative continuous probability distributions with fat tails (M. Shakil & Kibria, 2020). The Burr distribution is typically used to depict statistical characteristics that are not uniform and is widely applied in financial risk assessment and insurance (Xia et al., 2023). The probability density function and cumulative distribution function of the Burr distribution with 4 parameters, they are as follows,

$$f(x) = \frac{\alpha k \left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}}{\beta \left(1 + \left(\frac{x-\gamma}{\beta}\right)^\alpha\right)^{k+1}} \tag{5}$$

$$F(x) = 1 - \left(1 + \left(\frac{x-\gamma}{\beta}\right)^\alpha\right)^{-k} \tag{6}$$

with $\alpha, \beta, k > 0$ and $-\infty \leq \gamma \leq \infty$.

Based on equation (5), the formulas for expectation and variance can be obtained as follows,

$$E(X) = \gamma + k\beta B\left(1 + \frac{1}{\alpha}, k - \frac{1}{\alpha}\right) \tag{7}$$

$$E[X^2] - (E[X])^2 = k\beta^2 B\left(1 + \frac{2}{\alpha}, k - \frac{2}{\alpha}\right) - k^2\beta^2 B^2\left(1 + \frac{1}{\alpha}, k - \frac{1}{\alpha}\right) \tag{8}$$

2.4. Log-Logistic (3P) Distribution

The Log-Logistic distribution is one of the significant continuous probability distributions with heavy tails determined by scale and shape parameters (Muse et al., 2021). The Log-Logistic distribution has a probability density function and a hazard function that resemble those of the Log-Normal distribution but with heavier tails, supporting more accurate inferences. As for the probability density function and cumulative distribution function of the Log-Logistic distribution with 3 parameters, they are as follows,

$$f(x) = \frac{\alpha}{\beta} \left(\frac{x-\gamma}{\beta}\right)^{\alpha-1} \left(1 + \left(\frac{x-\gamma}{\beta}\right)^\alpha\right)^{-2} \tag{9}$$

$$F(x) = \left(1 + \left(\frac{x-\gamma}{\beta}\right)^\alpha\right)^{-1} \tag{10}$$

Based on equation (5), the formulas for expectation and variance can be obtained as follows,

$$E(X) = \gamma + \beta B\left(1 + \frac{1}{\alpha}, 1 - \frac{1}{\alpha}\right) \tag{11}$$

$$E[X^2] - (E[X])^2 = \beta^2 B\left(1 + \frac{2}{\alpha}, 1 - \frac{2}{\alpha}\right) - \beta^2 B^2\left(1 + \frac{1}{\alpha}, 1 - \frac{1}{\alpha}\right) \tag{12}$$

2.5. Portfolio

Portfolios are collections of all the assets owned by an investor. While measuring the return and risk for individual assets is important, for portfolio managers, understanding the return and risk of the entire set of assets in the portfolio is crucial. In the modern era, the approach to portfolio construction has shifted from traditional portfolio approaches to modern portfolio approaches. Traditional portfolio approaches involve diversifying the portfolio by randomly

selecting assets, whereas modern portfolio approaches involve analytically forming portfolios using statistics and mathematics (Kumar & Shahid, 2023).

Algebraically, the return of an investment portfolio r_{pt} , consisting of N risky assets, is expressed as the weighted sum of the returns of each asset in the portfolio, as shown in Equation (13) (Sukono et al., 2017),

$$r_{pt} = \sum_{i=1}^N w_i r_{it}$$

$$\text{subject to } \sum_{i=1}^N w_i = 1$$
(13)

Using a mathematical approach, the return of an investment portfolio in Equation (13) can be expressed as shown in Equation (14),

$$r_{pt} = \mathbf{w}^T \mathbf{r}$$

$$\text{subject to } \mathbf{e}^T \mathbf{w} = 1$$
(14)

Based on Equation (13), the average return of an investment portfolio μ_{pt} can be determined as shown in Equation (15),

$$\mu_{pt} = \sum_{i=1}^N w_i \mu_{it} \text{ or } \mu_{pt} = \mathbf{w}^T \boldsymbol{\mu}$$
(15)

where $\mu_{it} = E(r_{it})$ is the average return of stock i ($i = 1, \dots, N$ with N being the number of analyzed stocks) at time t , and $\boldsymbol{\mu}^T = (\mu_{1t} \ \mu_{2t} \ \dots \ \mu_{Nt})$ is the transpose vector of the asset i returns ($i = 1, \dots, N$ with N being the number of analyzed stocks). If we denote \mathbf{r} , $\boldsymbol{\mu}$, \mathbf{w} , and \mathbf{e} , represent the vector of stock return, mean, vector of weights, and the unit vector, respectively, defined as,

$$\mathbf{r} = \begin{pmatrix} r_{1,t} \\ r_{2,t} \\ \vdots \\ r_{n,t} \end{pmatrix}, \boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_N \end{pmatrix}, \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{pmatrix}, \mathbf{e} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$
(16)

Referring to Equation (13), the algebraic expression for the variance of the investment portfolio return can be formulated in Equation (17),

$$\sigma_{pt}^2 = Var(r_{pt}) = \sum_{i=1}^N \sum_{j=1}^N w_i w_j \sigma_{ij} = \sum_{i=1}^N \sum_{j=1}^N w_i w_j \sigma_i \sigma_j \rho_{ij}$$
(17)

And the form of σ_{ij} can be expressed as,

$$\sigma_{ij} = Cov(r_{it}, r_{jt}) = E[(r_{it} - \mu_{it})(r_{jt} - \mu_{jt})] = \rho_{ij} \sigma_{it} \sigma_{jt}$$
(18)

where σ_{ij} is the covariance between stock i and j , with $\sigma_{it} = \sqrt{\sigma_{it}^2}$ ($i = 1, 2, \dots, N$) called standard deviation. Furthermore, let $\boldsymbol{\Sigma}$ dan \mathbf{I} respectively denote the covariance matrix and the identity matrix, as expressed in equation (18),

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \dots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & \dots & \sigma_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \dots & \sigma_N^2 \end{pmatrix} \text{ and } \mathbf{I} = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$
(19)

2.6. Optimization Portfolio Investment with Mean-VaR

The estimation of Value at Risk depends on the probability distribution of asset returns or investment portfolio. The Value at Risk (VaR) of an investment portfolio with weight vector \mathbf{w} , denoted as VaR_{pt} , is calculated using Equation (20)

$$VaR_{pt} = -W_0(\mu_{pt} + z_\alpha \sigma_{pt})$$

or

$$VaR_{pt} = -W_0 \left\{ \mathbf{w}^T \boldsymbol{\mu} + z_\alpha (\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w})^{\frac{1}{2}} \right\}$$
(20)

where W_0 is the allocated fund in the formation of the investment portfolio and z_α is the α percent quantile of the standard normal distribution when a significance level of α is given. Typically, the significance level $\alpha = 0,05$.

If the risk level is measured using Value at Risk (VaR), the optimization problem becomes the Mean Value at Risk portfolio optimization as expressed in equation (21) (Lesmana et al., 2019),

$$\begin{aligned} & \text{Max} \left\{ 2\tau \boldsymbol{\mu}^T \mathbf{w} + \left(\boldsymbol{\mu}^T \mathbf{w} + z_\alpha (\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w})^{\frac{1}{2}} \right) \right\} \\ & \text{subject to } \mathbf{e}^T \mathbf{w} = 1 \end{aligned} \tag{21}$$

In searching for a solution to the Mean Value at Risk portfolio optimization problem with a risk tolerance factor as in equation (20), there are approaches to determine the optimal weights, including: (a) Risk tolerance factor approach $\tau \geq 0$ and (b) Lagrange multiplier approach. The Lagrange function can be expressed as follows,

$$L(\mathbf{w}, \lambda) = (2\tau + 1) \boldsymbol{\mu}^T \mathbf{w} + z_\alpha (\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w})^{\frac{1}{2}} + \lambda (\mathbf{w}^T \mathbf{e} - 1) \tag{22}$$

The optimal weight values can be obtained using the risk tolerance factor approach ($\tau \geq 0$). The vector equation for the weights represents the solution to the Mean Value at Risk portfolio optimization problem, with the optimal weight solution for a specific risk tolerance factor denoted as \mathbf{w} as shown in equation (23),

$$\mathbf{w} = \frac{(2\tau + 1) \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} + \lambda \boldsymbol{\Sigma}^{-1} \mathbf{e}}{(2\tau + 1) \mathbf{e}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} + \lambda \mathbf{e}^T \boldsymbol{\Sigma}^{-1} \mathbf{e}} \tag{23}$$

In addition to utilizing the risk tolerance factor approach ($\tau \geq 0$), another method involves employing the Lagrange multiplier approach,

$$\begin{aligned} \lambda_{1,2} &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ & \text{subject to } \lambda > 0 \end{aligned} \tag{24}$$

with

$$a = \mathbf{e}^T \boldsymbol{\Sigma}^{-1} \mathbf{e}$$

$$b = (2\tau + 1) \mathbf{e}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}$$

$$c = (2\tau + 1)^2 \boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} - z_\alpha^2, \text{ with } \boldsymbol{\Sigma}^{-1} \text{ being the inverse of the covariance matrix } \boldsymbol{\Sigma}.$$

3. Materials and Methods

3.1. Materials

In this research, the objects used are daily stock closing data in the infrastructure sector listed on the Indonesia Stock Exchange. The period used is from December 1st, 2021, to December 1st, 2023. The data obtained is secondary data obtained from site yahoo finance. In the research, the tools used are Microsoft Excel and EasyFit.

3.2. Methods

- 1) Calculate the stock return values using equation (1). Next, conduct a test of the stock return distribution model using EasyFit. Stocks for which the hypothesis is rejected are excluded from the calculation.
- 2) Calculate the expected value and variance of stock returns based on their distributions using equations (7) and (8) for the Burr distribution (4P) and equations (11) and (12) for the Log-Logistic distribution (3P). Stocks with negative expected returns are excluded from the calculation. Next, calculate the covariance of returns using equation (4). And then, determine the expected value and covariance of the portfolio and represent them in vector form as in equation (16).
- 3) Optimizing the portfolio using Mean-VaR

4. Results and Discussion

4.1. Return of Stocks

The first step is to calculate the stock returns. Stock returns are calculated using equation (1). After obtaining the stock returns, the next step is to test the distribution model of the returns to examine the distribution of the return values for each stock. The distribution model test is conducted using the Anderson-Darling test with a significance level of 1%. Next, the distribution test process is carried out with the Anderson-Darling test with the following hypotheses:

H_0 : Stock returns follow the assumed distribution,

H_1 : Stock returns do not follow the assumed distribution.

The Anderson-Darling test is conducted using EasyFit. The results can be seen in Table 1.

Table 1: Anderson-Darling Test Return of Stock

Stock	Distribution	P_{value} ($\alpha = 0.01$)	Statistic	Rejected
CMNP	Burr	3.9074	6.7596	Yes
KARW	Burr	3.9074	4.6551	Yes
ISAT	Log-Logistic	3.9074	0.86928	No
⋮	⋮	⋮	⋮	⋮
AKRA	Log-Logistic	3.9074	1.6443	No

Based on Table 1, it is found that there are 21 stocks for which the assumption (H_0) is not rejected, namely ISAT, TLKM, JSMR, BALI, etc.

4.2. Expected, Variance, and Covariance of Stocks Return

Based on the previous results, 21 stocks follow either the Burr (4P) distribution or the Log-Logistic (3P) distribution. The first step is to calculate the expected and variance return values using the formulas for each distribution. Any non-positive expected return values are excluded from the calculation. The results can be seen in Table 2.

Table 2: Expected and Variance Return of Stock

Stock	Expected Return	Variance
ISAT	0.000482	0.000540
TLKM	0.000962	0.000213
SSIA	-0.000035	0.000696
⋮	⋮	⋮
AKRA	0.001260	0.000611

Based on Table 2, it is found that only 8 stocks meet the criteria for portfolio formation. The next step is to calculate the covariance, as presented in Table 3.

Table 3: Variance-Covariance Return of Stock

Stock	ISAT	TLKM	JSMR	BALI	IPCC	KEEN	PTPW	AKRA
ISAT	0.000540	0.000011	0.000052	0.000032	0.000050	0.000052	0.000023	0.000024
TLKM	0.000011	0.000213	0.000043	-0.000018	0.000027	0.000013	0.000010	-0.000001
JSMR	0.000052	0.000043	0.000345	0.000023	0.000032	0.000029	0.000020	0.000073
BALI	0.000032	-0.000018	0.000023	0.000558	0.000033	-0.000011	0.000026	0.000024
IPCC	0.000050	0.000027	0.000032	0.000033	0.000348	0.000053	0.000035	0.000031
KEEN	0.000052	0.000013	0.000029	-0.000011	0.000053	0.000872	0.000030	0.000031
PTPW	0.000023	0.000010	0.000020	0.000026	0.000035	0.000030	0.000199	0.000032
AKRA	0.000024	-0.000001	0.000073	0.000024	0.000031	0.000031	0.000032	0.000611

Next, the expected values are formed into vector form as follows,

$$\mu = \begin{bmatrix} 0.00048268 \\ 0.00096206 \\ 0.00059179 \\ 0.00007404 \\ 0.00061014 \\ 0.00182000 \\ 0.00080859 \\ 0.00126000 \end{bmatrix}$$

and covariances are formed into vector form as follows,

$$\Sigma = \begin{bmatrix} 0.000540 & 0.000011 & 0.000052 & 0.000032 & 0.000050 & 0.000052 & 0.000023 & 0.000024 \\ 0.000011 & 0.000213 & 0.000043 & -0.000018 & 0.000027 & 0.000013 & 0.000010 & -0.000001 \\ 0.000052 & 0.000043 & 0.000345 & 0.000023 & 0.000032 & 0.000029 & 0.000020 & 0.000073 \\ 0.000032 & -0.000018 & 0.000023 & 0.000558 & 0.000033 & -0.000011 & 0.000026 & 0.000024 \\ 0.000050 & 0.000027 & 0.000032 & 0.000033 & 0.000348 & 0.000053 & 0.000035 & 0.000031 \\ 0.000052 & 0.000013 & 0.000029 & -0.000011 & 0.000053 & 0.000872 & 0.000030 & 0.000031 \\ 0.000023 & 0.000010 & 0.000020 & 0.000026 & 0.000035 & 0.000030 & 0.000199 & 0.000032 \\ 0.000024 & -0.000001 & 0.000073 & 0.000024 & 0.000031 & 0.000031 & 0.000032 & 0.000611 \end{bmatrix}$$

4.3. Portfolio Optimization using Mean-VaR

The computation of the weight for each stock (w_i), the anticipated return, and the portfolio's VaR using Microsoft Excel yielded outcomes detailed in Table 4.

Table 4: Portfolio Optimization Results

τ	λ	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	μ_{pt}	$\mathbf{w}^T \mathbf{e}$	VaR_{pt}
0	0.01252	0.07015	0.27028	0.09312	0.09248	0.09748	0.05386	0.24950	0.07314	0.00081	1	0.012516
0.3	0.01225	0.06859	0.27278	0.09094	0.08913	0.09576	0.05730	0.24983	0.07566	0.00082	1	0.012519
0.6	0.01196	0.06707	0.27523	0.08881	0.08588	0.09408	0.06065	0.25016	0.07812	0.00082	1	0.012526
0.9	0.01165	0.06558	0.27763	0.08673	0.08269	0.09244	0.06393	0.25048	0.08053	0.00083	1	0.012538
1.2	0.01132	0.06411	0.27998	0.08468	0.07955	0.09083	0.06716	0.25079	0.08290	0.00084	1	0.012554
1.5	0.01098	0.06266	0.28231	0.08266	0.07644	0.08923	0.07035	0.25110	0.08525	0.00085	1	0.012574
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
6.3	0.00178	0.03377	0.32874	0.04230	0.01461	0.05743	0.13395	0.25727	0.13193	0.00101	1	0.013805
6.6	0.00077	0.03024	0.33441	0.03737	0.00706	0.05355	0.14172	0.25803	0.13764	0.00103	1	0.014051
6.8	0.00003	0.02742	0.33894	0.03343	0.00102	0.05044	0.14792	0.25863	0.14219	0.00105	1	0.014261

The relationship between the expected return portfolio and the VaR_{pt} risk level, or the efficient frontier graph, is depicted in graphical form in Figure 1.

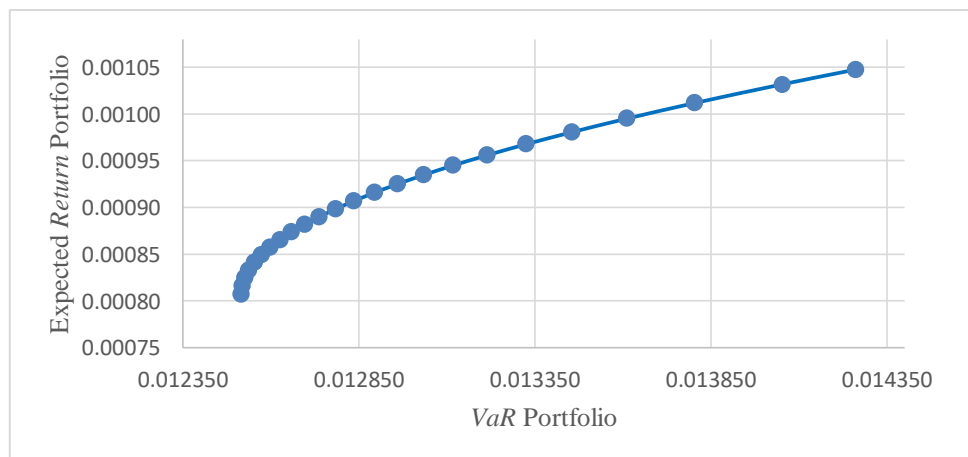


Figure 1: Efficient Frontier Portfolio

Next, calculate the ratio between the expected return and VaR, and the results are presented in Table 5.

Table 5: Ratio Portfolio

τ	μ_{pt}	VaR_{pt}	Ratio
0	0.00081	0.012516	0,064495
0.3	0.00082	0.012519	0,065183
0.6	0.00082	0.012526	0,065828
0.9	0.00083	0.012538	0,066435
1.2	0.00084	0.012554	0,067007
1.5	0.00085	0.012574	0,067548
⋮	⋮	⋮	⋮
6.3	0.00101	0.013805	0,073291
6.6	0.00103	0.014051	0,073416
6.8	0.00105	0.014261	0.073448

The optimization portfolio graph between the ratio and VaR is presented in Figure 2.

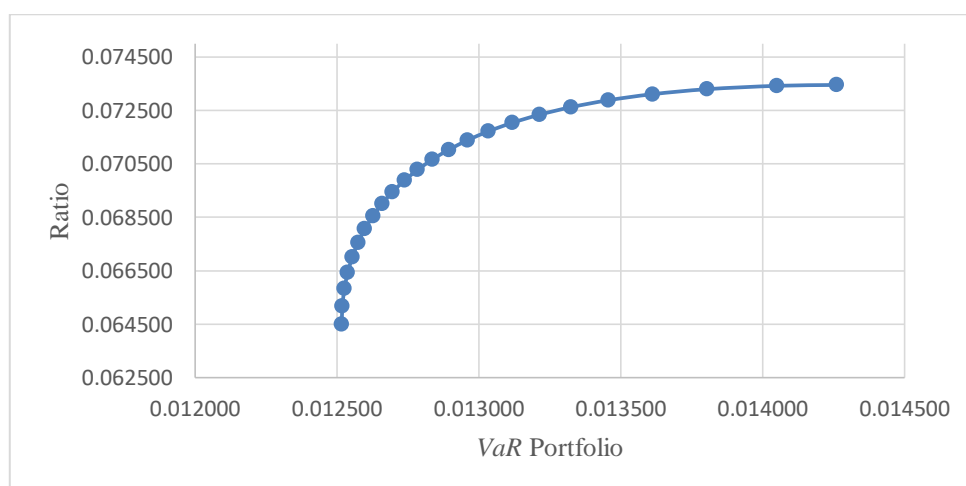


Figure 2: Optimal portfolio

Based on Figure 2, it can be observed that the ratio value between the expected return and portfolio VaR continues to increase within the risk tolerance interval of $0 \leq \tau \leq 6.8$. Subsequently, the highest ratio value between the expected return and portfolio VaR is 0.073448, with a τ value of 6.8. Thus, the optimal portfolio using the Mean-VaR model is obtained when $\tau = 6.8$.

5. Conclusion

There are 8 stocks that meet the criteria for forming an optimal portfolio with the proportion of each stock as follows: ISAT at 2.74%. TLKM at 33.894%. JSRM at 3.343%. BALI at 0.102%. IPCC at 5.044%. KEEN at 14.792%. PTPW at 25.863%. and AKRA at 14.219%. The optimal portfolio with Mean-VaR is obtained when the highest ratio value is achieved. In this study, the optimal portfolio is generated when the risk tolerance (τ) is 6.8. where the VaR risk level is 0.014261 and the expected return is 0.00105.

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