The Comparison of Investment Portfolio Optimization Result of Mean-Variance Model Using Lagrange Multiplier and Genetic Algorithm

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Abstract

Investment portfolio optimization is carried out to find the optimal combination of each stock with the aim of maximizing returns while minimizing risk by diversification. However, the problem is how much proportion of funds should be invested in order to obtain the minimum risk. One approach that has proven effective in building an optimal investment portfolio is the Mean-Variance model. The purpose of this study is to compare the results of the Mean-Variance model investment portfolio optimization using Lagrange Multiplier method and Genetic Algorithm. The data used are stocks that are members of the LQ45 index for the period February 2020-July 2021. Based on the research results, there are five stocks that form the optimal portfolio, namely ADRO, AKRA, BBCA, CPIN, and EXCL stocks. The optimal portfolio generated by the Lagrange Multiplier method has a risk of 0.000606 and a return of 0.000726. Meanwhile, using the Genetic Algorithm resulted in a risk of 0.000455 and a return of 0.000471. Thus, the Genetic Algorithm method is more suitable for investors who prioritize lower risk. Meanwhile, the Lagrange Multiplier method produces a relatively higher risk, making it less suitable for investors who expect a small risk.

Keywords: Mean-Variance, Lagrange Multiplier, Genetic Algorithm.

1. Introduction

The capital market plays a crucial role in nation's economic sector, serving as a fundamental pillar. Market activities have grown over time, driven by the increasing needs of the population, leading to heightened investments awareness. Various industries and enterprises utilize the capital market to attract investments and strengthen their financial positions. Investment, defined by Holland (2003), involves deploying capital in assets with the expectation of future gains. Stocks are particularly popular financial instruments traded on the capital market, offering attractive returns through dividends and capital gains.

By November 2023, the Indonesia Stock Exchange (IDX) listed 901 stocks, showcasing the market's diversity. Among its 43 stock indices, the LQ45 index is notable for measuring the price performance of 45 highly liquid and well-capitalized stocks. As emphasized by Hill, (2010) the LQ45 index is often used as a basis for portfolio construction. In stock investments, investors seek suitable returns aligned with their invested funds, making capital market investment a preferred choice due to its potential for higher returns compared to real asset or money market investments.

However, investing in the capital market entails risks, with higher returns often associated with greater risks. Therefore, investors face uncertainties regarding their investment outcomes, highlighting the importance of accurately estimating risk involved. Diversification, spreading funds across various securities, offers a strategy to mitigate investment risks. Yet, determining the optimal allocation of funds to minimize risks remains challenging. Efficient portfolio analysis aims to select the best stocks for an optimal investment portfolio formation. The Mean-Variance model, developed by Harry Markowitz, is an effective approach in optimizing portfolio risk a return by considering asset correlations.

Heuristic methods provide solutions to optimization problems in complex search spaces. Genetic Algorithms, in particular, represent a heuristic optimization technique inspired by natural evolutionary processes. Compared to other

heuristics, Genetic Algorithms yield superior results for multi-objective optimization problems. In the context of portfolio formation, Genetic Algorithms assist investor in selecting optimal weights for portfolio construction.

Based on the above discussion, this study will compare the Lagrange Multiplier method and Genetic Algorithm in the formation of optimal portfolios. The aim is to provide investors with a comprehensive guide to selecting the portfolio optimization method that best suits their objectives and risk preferences.

2. Literature Review

2.1. Stocks

Stocks can be defined as a form of participation or ownership by an individual or entity in a limited company or individual (Endarto et al., 2021).
1) Return of Stocks

$$R_{it} = \frac{P_{it} - P_{i(t-1)}}{P_{i(t-1)}}$$ (1)

R_{it} : return of stock i at time t
P_{it} : price of stock i at time t
$P_{i(t-1)}$: price of stock i at time $t-1$

2) Expected Return of Stocks

$$E(R_{it}) = \frac{\sum_{t=1}^{m} R_{it}}{m}$$ (2)

$E(R_i)$: expected return of stock i
R_{it} : return of stock i at time t
m : number of observation periods

3) Risk of Stocks

$$\sigma_i^2 = \sum_{t=1}^{m} \frac{[R_{it} - E(R_{it})]^2}{m}$$ (3)

σ_i^2 : variance of stock i
R_{it} : return of stock i at time t
$E(R_{it})$: expected return of stock i at time t
m : number of observation periods

4) Covariance
Risk measurement can be expressed in the form of covariance, which is a measure indicating the tendency of movement of two variables. The equation is as follows (Hill, 2010):

$$\sigma_{i,j} = \sum_{t=1}^{m} \frac{[(R_{it} - E(R_{it})) \cdot (R_{jt} - E(R_{jt}))]}{m}$$ (4)

$\sigma_{i,j}$: covariance of stock i and j
R_{it} : return of stock i at time t
R_{jt} : return of stock j at time t
$E(R_i)$: expected return of stock i
$E(R_j)$: expected return of stock j
m : number of observation periods

5) Correlation Coefficient
Although covariance can be used to express the direction of stock movement, covariance figures are sensitive to measurement units. Therefore, it is necessary to calculate the correlation coefficient. The equation is as follows (Hill, 2010):

$$r_{i,j} = \frac{\sigma_{i,j}}{\sigma_i \sigma_j}$$ (5)

$r_{i,j}$: correlation coefficient of stock i and j
$\sigma_{i,j}$: covariance of stock i and j
σ_i : standard deviation of stock i
σ_j : standard deviation of stock j

2.2. Portfolio

A stock portfolio is a collection of various stocks, owned by individuals or companies with different weights.

1) Expected Return of Portfolio

$$\mu_P = \boldsymbol{\mu}^{\mathbf{T}}\mathbf{w} \tag{6}$$

$\boldsymbol{\mu}^{\mathbf{T}}$: transpose of the mean vector
$\mathbf{w}$: weight vector

2) Risk of Portfolio

$$\sigma_P^2 = \mathbf{w}^{\mathbf{T}}\boldsymbol{\Sigma}\mathbf{w} \tag{7}$$

$\mathbf{w}^{\mathbf{T}}$: transpose of weight vector
$\boldsymbol{\Sigma}$: covariance matrix

2.3. Mean-Variance Portfolio Model

Modern portfolio theory was first introduced by Harry Markowitz in 1952. This theory states that return and risk are important factors to consider when forming a portfolio. The objective function used to determine the optimal portfolio is written as follows:

$$\left. \begin{aligned} min. \ & \sum_{i=1}^{n} w_i^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} w_iw_j\sigma_{ij} \\ s.t. \ & \sum_{i=1}^{n} w_i = 1 \\ & w_i \geq 0 \\ & R_p = \sum_{i=1}^{n} w_iR_i \end{aligned} \right\} \tag{8}$$

2.4. Genetic Algorithm

The Genetic Algorithm was first introduced by John Holland in 1960 and further popularized by David Goldberg in 1980 (Vose, 1999). Tang et al. (2011) states that the Genetic Algorithm is a heuristic method based on genetic mechanism and natural selection processes according to Darwin's theory of evolution. The Genetic Algorithm is an optimization technique that can tackle many difficult problems that conventional techniques cannot handle (Tang et al., 1996). There are important definitions to be aware of in Genetic Algorithms (Lambora et al., 2019):
   a) Gene, a value that represents basic units forming a specific meaning.
   b) Chromosome, a combination of genes.
   c) Individual, representing a single value or state that represents one of the possible solutions.
   d) Population, a set of individuals that will be processed together in one evolutionary cycle.
   e) Generation, representing one evolutionary cycle.

2.5. Investment Portfolio Optimization of Mean-Variance Model with Lagrange Multiplier

The Mean-Variance portfolio optimization model is expressed as follows:

$$\left. \begin{aligned} max. \ & 2\tau\mathbf{w}^{\mathbf{T}}\boldsymbol{\mu} - \mathbf{w}^{\mathbf{T}}\boldsymbol{\Sigma}\mathbf{w} \\ s.t. \ & \mathbf{w}^{\mathbf{T}}\mathbf{e} = 1 \\ & w_i \geq 0 \end{aligned} \right\} \tag{9}$$

Where τ denotes the risk tolerance.
The Lagrange Multiplier function of equation (…) with λ as the multiplier is as follows:

$$L(\mathbf{w}, \lambda) = 2\tau\mathbf{w}^{\mathbf{T}}\boldsymbol{\mu} - \mathbf{w}^{\mathbf{T}}\boldsymbol{\Sigma}\mathbf{w} + \lambda(\mathbf{w}^{\mathbf{T}}\mathbf{e} - 1) \tag{10}$$

Based on the Kuhn Tucker theorem, the necessary and sufficient condition for equation (12) is $\frac{\partial L(\mathbf{w},\lambda)}{\partial \mathbf{w}} = 0$ and $\frac{\partial L(\mathbf{w},\lambda)}{\partial \lambda} = 0$. thus yielding:

$$\frac{\partial L(\mathbf{w}, \lambda)}{\partial \mathbf{w}} = 2\tau\boldsymbol{\mu} - 2\boldsymbol{\Sigma}\mathbf{w} + \lambda\mathbf{e} = 0 \tag{11}$$

$$\frac{\partial L(\mathbf{w}, \lambda)}{\partial \lambda} = \mathbf{w}^T \mathbf{e} - 1 = 0 \tag{12}$$

expressed in the form of **w**, and multiplied by $\mathbf{e}^T$, resulting in:

$$\mathbf{e}^T \mathbf{w} = \frac{1}{2} \lambda \mathbf{e}^T \Sigma^{-1} \mathbf{e} + \tau \mathbf{e}^T \Sigma^{-1} \mu \tag{13}$$

Since $\mathbf{e}^T \mathbf{w} = 1$. we obtain:

$$\frac{1}{2} \lambda = \frac{1}{\mathbf{e}^T \Sigma^{-1} \mathbf{e}} - \frac{\tau \mathbf{e}^T \Sigma^{-1} \mu}{\mathbf{e}^T \Sigma^{-1} \mathbf{e}} \tag{14}$$

Thus, the equation for calculating **w** is as follows:

$$\mathbf{w} = \frac{1}{\mathbf{e}^T \Sigma^{-1} \mathbf{e}} \Sigma^{-1} \mathbf{e} + \tau \left(\Sigma^{-1} \mu - \frac{\mathbf{e}^T \Sigma^{-1} \mu}{\mathbf{e}^T \Sigma^{-1} \mathbf{e}} \Sigma^{-1} \mathbf{e} \right), \tau > 0 \tag{15}$$

$$\mathbf{w}^{min} = \frac{1}{\mathbf{e}^T \Sigma^{-1} \mathbf{e}} \Sigma^{-1} \mathbf{e}, \tau = 0 \tag{16}$$

2.6. Investment Portfolio Optimization of Mean-Variance Model with Genetic Algorithm

Here are the steps of optimization using Genetic Algorithm:
1) Population initialization
   Population initialization involves determining the representation of genes and chromosomes within the population. The initial population is formed by randomly generating chromosomes according to the population size.

2) Evaluation
   There are stages in evaluating the fitness value of an individual as follows:
   a. Determining the objective function
      The objective function represents the goal based on constrain to be sought for its solution.
   b. Calculating the fitness value
      The fitness value is a measure indicating the value of a function to achieve the goal. The fitness value is calculated using the following equation:

$$Fitness[i] = \frac{1}{1 + F_{obj}[i]} \tag{17}$$

3) Selection
   Selection is the process of selecting individuals that will remain in a population. The selected individuals in the selection process will become parents in the next generation. The method commonly used is the roulette wheel method. In this method, the higher the fitness value of an individual, the greater the chance of being selected as a parent (Sharma, 2024).
   There are several steps in the roulette wheel selection:
   a. Calculate the probability of each chromosome being selected as follows:

$$P[i] = \frac{Fitness[i]}{\sum_{i=1}^{n} Fitness[i]} \tag{18}$$

   b. Calculate the cumulative probability value as follows:

$$Q[i] = \sum_{i=1}^{n} P[i] \tag{19}$$

   c. Generate random numbers $0 < R[i\} < 1$ for each chromosome. If $Q[i] < R[i] < Q[i+1]$, then $Q[i+1]$ is selected as the new chromosome.

4) Crossover
   Crossover involves crossing two chromosomes to produce new individuals inheriting basic characteristics from their parents. In the case of real number encoding, the crossover technique used is arithmetic recombination. The equation is as follows (Thakur, 2014):

$$\left.\begin{array}{l} Child_1 = \alpha \cdot Parent_1 + (1 - \alpha) \cdot Parent_2 \\ Child_2 = \alpha \cdot Parent_2 + (1 - \alpha) \cdot Parent_1 \end{array}\right\} \tag{20}$$

Where α is a parameter controlling how much each parent contributes to the offspring's gene values.

5) Mutation
Mutation is the process of changing one or more genes within the same individual to produce a new individual. The number of chromosomes undergoing mutation is determined by mutation rate (ρm).
There are several steps in the mutation process:
   a. Calculate the total length of genes in the population
   b. Calculate the total genes in the population to undergo mutation as follows:
$$mutation\ count = \rho m \times total\ genes \tag{21}$$
   c. Generate random numbers equal to the mutation count. The random numbers generated represent the location of the genes to be mutated.
   d. Generate random numbers $[0.1]$ equal to the mutation count. The random numbers generated are added to the corresponding genes.

6) Termination criteria
The termination criteria function to determine when the Genetic Algorithm will stop.

3. Materials and Methods

3.1. Materials

In this study, the object used is daily stock closing proce from ten companies that are members of the LQ45 index. The data, sourced from yahoo finance with the period February 2020-July 2021. Data processing use Microsoft Excel tools and the Python programming language.

3.2. Methods

1) Calculating stock returns using equation (1) and expected stock returns using equation (2).
2) Calculating the risk of stocks with positive expected returns using equation (3).
3) Calculating stock covariance using equation (4) and forming them into the covariance matrix Σ.
4) Calculating the correlation coefficient using equation (5)
5) Calculating the Mean-Variance optimal portfolio from the Lagrange Multipiler method using equation (18) for $\tau = 0$ and equation (17) for $\tau > 0$.
6) Calculating the expected return portfolio return using equation (6) and portfolio risk using equation (7) for the Lagrange Multiplier method. The portfolio with the lagrest ratio is considered optimal.
7) Calculating the optimal portfolio using Genetic Algorithm method for 10 iterations.
8) Calculating the expected portfolio return using equation (6) and portfolio risk using equation (7) for the Genetic Algorithm method. The portfolio with the smallest risk is considered optimal.
9) Comparing the result.

4. Results and Discussion

1) Expected return of stocks
The calculation results of the expected return of return using Microsoft Excel are presented in Table 1.

Table 1: Table of expected return of stocks

Stock Code	Expected Return
ADRO	0.000794
AKRA	0.000883
ASII	-0.000420
BBCA	0.000019
BBNI	-0.000647
CPIN	0.000470
EXCL	0.000443
ICBP	-0.000711
UNVR	-0.001459

WIKA	-0.001138	

2) Risk of stocks

The risk calculation results of stocks with positive expected returns using Microsoft Excel are presented in Table 2.

Table 2: Table of risk of stocks

Stock Code	Column A (t)
ADRO	0.001112
AKRA	0.001062
BBCA	0.000467
CPIN	0.001015
EXCL	0.001224

3) Covariance value of stock returns

The calculation results of stock covariances using Microsoft Excel are presented in Table 3.

Table 3: Table of covariance value of stock returns

	ADRO	AKRA	BBCA	CPIN	EXCL
ADRO	0.001112	0.000409	0.000351	0.000435	0.000582
AKRA	0.000409	0.001062	0.000255	0.000347	0.000419
BBCA	0.000351	0.000255	0.000467	0.000316	0.000347
CPIN	0.000435	0.000347	0.000316	0.001015	0.000474
EXCL	0.000582	0.000419	0.000347	0.000474	0.001224

4) Correlation value of stocks

Based on the covariance values, correlations between stocks were then calculated using Microsoft Excel, with the results shown in Table 4.

Table 4: Table of correlation value of stock returns

	ADRO	AKRA	BBCA	CPIN	EXCL
ADRO	1	0.376005	0.486511	0.409393	0.499181
AKRA	0.376005	1	0.362874	0.334447	0.36722
BBCA	0.486511	0.362874	1	0.459363	0.459095
CPIN	0.409393	0.334447	0.459363	1	0.424989
EXCL	0.499181	0.36722	0.459095	0.424989	1

5) Optimization using Lagrange Multiplier

The results of the optimal portfolio calculation using Microsoft Excel are shown in Table 5.

Table 5: Table of optimization results using Lagrange Multiplier

τ	ADRO	AKRA	BBCA	CPIN	EXCL	μ_P	σ_P^2	μ_P/σ_P^2
0	0.041592	0.162479	0.657153	0.109995	0.028781	0.000253	0.000408	0.621232
0.01	0.048112	0.168918	0.642782	0.111782	0.028406	0.000265	0.000408	0.64867
0.02	0.054632	0.175356	0.628412	0.113569	0.02803	0.000276	0.000408	0.675721
0.03	0.061152	0.181795	0.614041	0.115356	0.027655	0.000287	0.000409	0.702339
0.04	0.067672	0.188234	0.59967	0.117144	0.02728	0.000298	0.000409	0.728483
0.05	0.074192	0.194673	0.585299	0.118931	0.026904	0.00031	0.00041	0.754113
0.06	0.080712	0.201112	0.570929	0.120718	0.026529	0.000321	0.000412	0.779191
0.07	0.087232	0.207551	0.556558	0.122505	0.026154	0.000332	0.000413	0.80368
0.08	0.093752	0.21399	0.542187	0.124292	0.025778	0.000343	0.000415	0.827546
0.09	0.100272	0.220429	0.527816	0.126079	0.025403	0.000355	0.000417	0.850759
0.1	0.106792	0.226868	0.513446	0.127866	0.025028	0.000366	0.000419	0.87329
0.11	0.113312	0.233307	0.499075	0.129653	0.024653	0.000377	0.000421	0.895114

0.12	0.119832	0.239746	0.484704	0.131441	0.024277	0.000388	0.000424	0.916207
0.13	0.126352	0.246185	0.470333	0.133228	0.023902	0.0004	0.000427	0.936549
0.14	0.132872	0.252624	0.455963	0.135015	0.023527	0.000411	0.00043	0.956123
0.15	0.139392	0.259063	0.441592	0.136802	0.023151	0.000422	0.000433	0.974916
0.16	0.145912	0.265502	0.427221	0.138589	0.022776	0.000433	0.000436	0.992914
0.17	0.152432	0.271941	0.41285	0.140376	0.022401	0.000445	0.00044	1.01011
0.18	0.158952	0.27838	0.39848	0.142163	0.022025	0.000456	0.000444	1.026497
0.19	0.165472	0.284818	0.384109	0.14395	0.02165	0.000467	0.000448	1.042073
0.20	0.171992	0.291257	0.369738	0.145738	0.021275	0.000478	0.000453	1.056836
0.21	0.178512	0.297696	0.355367	0.147525	0.020899	0.00049	0.000457	1.070787
0.22	0.185032	0.304135	0.340997	0.149312	0.020524	0.000501	0.000462	1.083932
0.23	0.191552	0.310574	0.326626	0.151099	0.020149	0.000512	0.000467	1.096275
0.24	0.198072	0.317013	0.312255	0.152886	0.019773	0.000523	0.000472	1.107825
0.25	0.204592	0.323452	0.297884	0.154673	0.019398	0.000535	0.000478	1.118592
0.26	0.211112	0.329891	0.283514	0.15646	0.019023	0.000546	0.000484	1.128589
0.27	0.217632	0.33633	0.269143	0.158247	0.018647	0.000557	0.00049	1.137827
0.28	0.224152	0.342769	0.254772	0.160035	0.018272	0.000568	0.000496	1.146322
0.29	0.230672	0.349208	0.240401	0.161822	0.017897	0.00058	0.000502	1.15409
0.3	0.237192	0.355647	0.226031	0.163609	0.017521	0.000591	0.000509	1.161148
0.31	0.243712	0.362086	0.21166	0.165396	0.017146	0.000602	0.000516	1.167514
0.32	0.250232	0.368525	0.197289	0.167183	0.016771	0.000614	0.000523	1.173208
0.33	0.256752	0.374964	0.182918	0.16897	0.016395	0.000625	0.00053	1.178249
0.34	0.263272	0.381403	0.168548	0.170757	0.01602	0.000636	0.000538	1.182657
0.35	0.269792	0.387842	0.154177	0.172544	0.015645	0.000647	0.000546	1.186453
0.36	0.276312	0.394281	0.139806	0.174332	0.015269	0.000659	0.000554	1.189659
0.37	0.282832	0.400719	0.125435	0.176119	0.014894	0.00067	0.000562	1.192295
0.38	0.289352	0.407158	0.111065	0.177906	0.014519	0.000681	0.00057	1.194384
0.39	0.295872	0.413597	0.096694	0.179693	0.014143	0.000692	0.000579	1.195946
0.4	0.302392	0.420036	0.082323	0.18148	0.013768	0.000704	0.000588	1.197003
0.41	0.308912	0.426475	0.067952	0.183267	0.013393	0.000715	0.000597	1.197577
0.42	0.315432	0.432914	0.053582	0.185054	0.013018	0.000726	0.000606	1.197689
0.43	0.321952	0.439353	0.039211	0.186841	0.012642	0.000737	0.000616	1.197359
0.44	0.328472	0.445792	0.02484	0.188628	0.012267	0.000749	0.000626	1.196608
0.45	0.334992	0.452231	0.010469	0.190416	0.011892	0.00076	0.000636	1.195458
0.46	0.341512	0.45867	**-0.0039**	0.192203	0.011516	0.000771	0.000646	1.193926

The risk tolerance value used ranges $0 \leq \tau \leq 0.45$ because these values result in positive stock weights. However, for values of $\tau > 0.45$, negative weights are obtained. Based on Table 5, it can be observed that the highest ratio is 1.197689. This value is obtained when the risk tolerance value $\tau = 0.42$.

Based on the calculations, the optimal portfolio generated by Lagrange Multiplier method yield weight for ADRO 31.5432%, AKRA 43.2914%, BBCA 5.3582%, CPIN 18.5054%, and EXCL 1.3018% with a risk of 0.000606 and a return of 0.000726.

6) Optimization using Genetic Algorithm

The result of the optimal portfolio calculation using Genetic Algorithm method implemented in Python programming language are shown in Table 6. Based on Table 6, it can be observed that the variance generated by each optimal portfolio candidate has values that are not significantly different. The portfolio with the smallest variance is 0.000455 and the return value is 0.000471. The combination of stock weights forming the optimal portfolio using the Genetic Algorithm method yields weights for ADRO 14.8720%, AKRA 27.5384%, BBCA 42.7878%, CPIN 1.9474%, and EXCL 10.4777%.

Table 6: Table of optimization results using Genetic Algorithm

Iteration	ADRO	AKRA	BBCA	CPIN	EXCL	σ_P^2	μ_P
1	0.17559	0.022832	0.261	0.2121	0.12298	0.000479	0.000499
2	0.13609	0.29742	0.28355	0.25857	0.02436	0.000476	0.000508
3	0.12568	0.42993	0.32599	0.07143	0.04696	0.000499	0.000540
4	0.0963	0.28352	0.28553	0.28292	0.05172	0.000475	0.000488
5	0.14872	0.275384	0.427878	0.019474	0.104777	0.000455	0.000471
6	0.23491	0.22641	0.36178	0.05063	0.12627	0.000471	0.000473
7	0.2332	0.23339	0.36502	0.11761	0.05078	0.000459	0.000476
8	0.11486	0.32174	0.18626	0.31021	0.06694	0.000513	0.000503
9	0.20899	0.23111	0.28182	0.17267	0.10541	0.000475	0.000503
10	0.23859	0.173	0.24871	0.26601	0.07369	0.000493	0.000504

5. Conclussion

The use of Genetic Algorithm method is more suitable for investors who prioritize lower risk. Meanwhile, the Lagrange Multiplier method yields relatively higher risk, making it less suitable for investors seeking lower risk.

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