Available online at https://journal.rescollacomm.com/index.php/ijqrm/index

International Journal of Quantitative Research and Modeling

e-ISSN 2721-477X
p-ISSN 2722-5046

Vol. 5, No. 2, pp. 135-139, 2024

Mathematical Model Analysis of Mosaik Disease Spread on Jatropha Plants: Article Review

Ayun Sri Rahmani[1*], Subiyanto[2], Sudradjat Supian[3]

[1] Master of Mathematics Student, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Jl. Raya Bandung-Sumedang KM 21, Jatinangor, Sumedang, West Java, 45363, Indonesia
[2]Department of Marine Science, Faculty of Fishery and Marine Science, Universitas Padjadjaran, Jl. Raya Bandung-Sumedang KM 21, Jatinangor, Sumedang, West Jav, 45363, Indonesia
[3] Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Jl. Raya Bandung-Sumedang KM 21, Jatinangor, Sumedang, West Java, 45363, Indonesia

**Corresponding author email: ayun20001@mail.unpad.co.id*

Abstract

Mosaic disease is one of the plant diseases that can be detrimental and cause crop failure, the disease is caused by Begomovirus. Begomovirus is spread by whitefly vectors. The whitefly as a vector can infect healthy plants because once the whitefly is infected, the whitefly body will forever contain the disease. Therefore, we need a mathematical model to prevent the spread of mosaic disease on Jatropha plants and make a strategy to prevent mosaic disease with optimal control and other factors. In this study, mathematical modeling of the spread of jatropha mosaic disease will be discussed, with the addition of various compartments, parameters, and optimal control. Several strategies that can be used to prevent mosaic disease in Jatropha are adding effect awareness, delay, insecticides, interventions, natural predators, yellow stick, rouging, and a combination of all strategies.

Keywords: Mosaic disease, mathematics model, stability, jatropha plants.

1. Introduction

The development of the times is getting faster, the need for fuel oil is increasing every time as human needs increase. However, the supply of fuel every day is decreasing. To prevent a shortage of fuel supply in the market, other alternative fuels are needed, such as the use of biofuels, namely using Jatropha plant (Syakir, 2010., Harimurti & Sumangat, 2011). Jatropha plant *(Jatropha curcas L.)* is a biofuel containing about 30-40% oil content in the seeds, and 40-50% in the seed coat (Harimurti & Sumangat, 2011,Supriyadi-Tirtosuprobo & Riajaya, 2015). In addition to having oil content, Jatropha is also an annual plant that can live and thrive in dry soil, so it can minimize the use of water and fertilizers (Supriyadi-Tirtosuprobo & Riajaya, 2015., Pambudi et al., 2019).

With the advantages of Jatropha, Jatropha can likely be used as an alternative fuel, in the future. However, it is not uncommon for jatropha plant farmers to experience losses, which are caused by various factors, one of which is mosaic disease. Mosaic disease is caused by begomovirus (*Begomovirus G),* which is spread by whiteflies (Venturino et al., 2016., Pambudi et al., 2019). Whiteflies are vectors of plant diseases. Jatropha plant infested with whiteflies can cause abnormal growth, the leaves curl, change color, wilt, and under the leaves are filled with a sticky white waxy substance (Venturino et al., 2016, Pambudi et al., 2019).

2. Literature Review

Mathematical models have been widely studied to analyze and control jatropha plant infectious diseases. The formulation of a mathematical model of the spread of Jatropha plant disease is based on basic assumptions, compartments, and parameters. Mathematical models of the spread of Jatropha plant disease can provide various basic information such as reproduction numbers (R_0), infection rate, cure rate, and numerical simulation. Model analysis and numerical simulations can provide specific answers to conjectures (Hethcote, 2000). Analysis of mathematical models provides quantitative insight into the spread of Jatropha plant diseases, while numerical simulations help understand the long-term behavior of disease spread (Ratti, 2018). Therefore, the model of the spread of Jatropha plant disease is very useful for preventing the spread and predicting the treatment of the disease.

Many studies have discussed the modeling of the spread of mosaic disease on Jatropha plants, including: (Venturino et al., 2016), discussing the model of the spread of mosaic disease with optimal control of insecticide administration, the results showed that spraying insecticides was not necessary for the first ten days of the outbreak, but spraying was carried out for the next three months to eradicate the disease. Next, Basir et al., (2017), modifying the mosaic disease spread model by providing a compartment of the awareness farmer population, the results showed that if the awareness farmer population increased then the jatropha plant disease population decreased. In the same year Al-Basir, Roy, et al., (2017), modifying the model of the spread of mosaic disease with roguing and insecticides, results showed that the potential impact of rouging and spraying insecticides should be considered by applying optimal control, to save costs.

In the following year Al-Basir & Roy (2018), modifying the mosaic disease spread model with rouging and delay, results showed that stability analysis in the absence of delay made the local endemic equilibrium stable for harvesting, $h < h^* = 0.086$. When stability is with the system delay, for $\tau > 0$ the equilibrium will be local asymptotic with $\tau < \tau * = 24{,}56$. While the loss of stability, caused by the Hopf Bifurcation, can occur if the delay increases, $\tau > \tau^*$ and supercritical stable periodic bifurcation solutions. To control the spread of mosaic disease, the impact of behavioral changes caused by rouging must be considered in the modeling process. In the same year Al Basir, Venturino, et al., (2018), Modifying the mosaic disease spread model with awareness and delay, the results showed that there were two equilibrium points, free of disease and endemic. Disease-free equilibrium is stable if $R_0 < 1$, and endemic stability shows Hopf bifurcation whenever present. Still, in the same year Al Basir, Blyuss, et al., (2018), modifying the model of the spread of mosaic disease in the jatropha plant with the effect of intervention in the form of nutrition and insecticide based on awareness for controlling the mosaic disease. The results showed that the number of infected plants would lead to increased awareness of disease prevention so that the use of nutrients and insecticides would increase.

In the following year, modifying the model for the spread of mosaic disease by increasing the delay effect, and controlling pests using farming awareness, the results showed that increasing farming awareness based on biocontrol with a tolerable delay in eradicating pests in crop fields. In the same year Pambudi et al., (2019), modifying the model of the spread of mosaic disease, with optimal control of the use of sticky yellow adhesive and natural predators, results showed that non-endemic stability analysis would be asymptotically stable if $R_0 < 1$, and endemic stability will be stable if $R_0 > 1$. Furthermore, optimal control is applied by applying u_1 prevention and eradication u_2, The results of the analysis show that the joint use of prevention and eradication will be more effective and reduce the spread of mosaic disease on Jatropha plants.

Next researcher Pratiwi et al., (2020), modifying the model of the spread of mosaic disease in Jatropha with optimal control using insecticides and interventions based on people's awareness. The results of the numerical simulation of the study show that the high infection rate of Jatropha curcas (γ_{max}), the greater the chance of being infected with the Jatropha population. The application of a large enough amount of insecticide can also reduce the whitefly population, thereby increasing the healthy jatropha population. Further research by Al Basir et al., (2021), investigated the spread of plant diseases with delay and transmission of Beddington-DeAngelis disease. The results showed that the Beddington-DeAngelis functional response type disease transmission rate was successfully installed in the model. Prediction of the dynamic regime in the model depends on parameters, so it can provide an effective method for disease monitoring, as well as for developing techniques for disease management.

3. Methods

The method in this study uses a systematic review. Systematic reviews have criteria where the review of articles is carried out in a structured and planned manner (Hariyati, 2010). This study conducted a systematic review of the model for the spread of mosaic disease in Jatropha curcas. Search using the Google Scholar database with the keywords “mathematical modeling”, “spread of mosaic disease”, and “jatropha plant”.

4. Discussion

4.1 Mathematical Modeling of Epidemiology

According to Meyer (2004), A mathematical model is said to be good if it always has the following characteristics:

a. Accuracy, a model is said to be accurate if the model can provide the correct answer or approach the truth.
b. Realistic, a model is said to be realistic if the model is made based on the correct assumptions

c. Exactly, a model is said to be appropriate if it has a definite estimate or number.
d. Strong, a model is said to be strong if there are errors that will remain (relatively immune to errors in data input).
e. General, a model is said to be general if it can be applied to various situations.
f. Useful, a model is said to be useful if the conclusions obtained can be used and can lead to good models.

Based on the steps above, the steps to create a mathematical model can be described as shown in Figure 1.

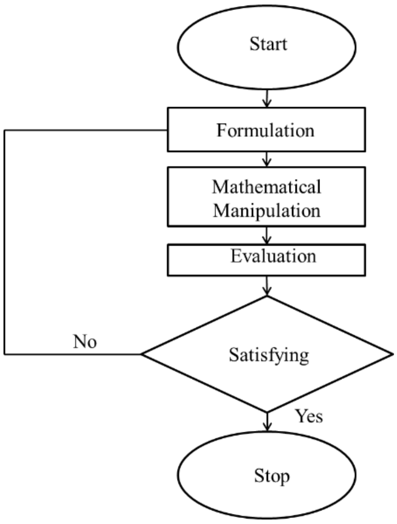

Figure 1. Mathematical modeling step diagram (Meyer, 2004)

4.2 Epidemic Model

There are many models of disease spread, one of the simplest models is the SIR model. The Suspected-Infected-Removed (SIR) epidemic model is the basis of the infection development model. The model discussed is the SIR model without birth and death factors adopted from the Kermack and Mc Kendrick model. The following are SIR models:

$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dI}{dt} = \beta SI - \gamma I \tag{1}$$
$$\frac{dR}{dt} = \gamma I$$

It can be seen that the susceptible population R will decrease when interacting with the infected population I with the interaction rate. Then the infected population will increase if the susceptible population interacts with the infected population, and will decrease because it has recovered, with a recovery rate of. Meanwhile, the recovered population will increase because the infected population has recovered. The schematic diagram of a model (1) can be seen in Figure 2.

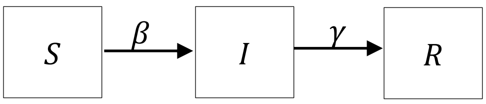

Figure 2. Suspected-Infected-Removed (SIR) model schematic diagram

In addition to the SIR model, many models can be used as references for the development of disease spread models, such as SI, SIRS, SRIR, SEI, SEIR, SEIRS, SEIS, SIS, MSEIR, and MSEIRS, and others (Ratti, 2018). Of course, the more compartments and parameters in the development of a model, the more complex the model analysis will be, but the model will provide a clear picture of the long-term behavior of a disease and how to prevent it from spreading effectively.

4.3 Dynamic System

A dynamic system is a system that changes or develops over time. One of the most important in dynamic systems is the stability point criteria because analyzing a stability point and visualizing the stability point, can give an idea of whether a system can be predicted easily or not. (Fischel & Gröller, 1995).

4.3.1 Equilibrium Point

The equilibrium point is the point obtained from a system that will never change over time. In general, it is defined as Definition 1. **Definition 1** *Given a system of differential equations* $\dot{x} = f(x)$. *Point* $\bar{x} \in R^n$ *is called the equilibrium point if* $f(\bar{x}) = 0$. (Perko, 1991).

4.3.2 Basic Reproduction Number

The spread of a disease within an area or population can cause serious problems. So it is necessary to prevent disease, by analyzing the stability of a system at a certain time. So we need an analysis of the equilibrium point of the system.

Basic Reproduction Number (R_0), is the threshold value of the disease-free equilibrium point, which is used to analyze the ability of the disease to spread through a disease-infected area or population. So, it is expected that R_0<1, so that the spread of the disease will not occur. If, $R_0 < 1$ hence indicating that the disease-free equilibrium point is locally asymptotically stable. On the other hand, if $R_0 > 1$, Then the disease-free equilibrium point is unstable and the spread of disease is inevitable (Van den Driessche & Watmough, 2002).

4.3.3 Routh-Hurwitz Stability Criteria

The Routh-Hurwitz stability criterion is one of the methods to analyze the stability of a system through the coefficients of the characteristic equation of the system without looking for characteristic roots. In addition, the number of sign changes indicates the number of roots of the characteristic equation which is positive.
For example:

$$P(\lambda) = a_0\lambda^n + a_1\lambda^{n-1} + \ldots + a_{n-1}\lambda + a_n = 0, \text{ where } a_0 \neq 0 \text{ and } a_n > 0.$$

Determination of stability is done by analyzing the coefficients of the polynomial equation. If the polynomial equation has zero or negative coefficients, then there is a positive root, so the system is unstable, and vice versa. If all the coefficients are positive in the polynomial equation, then arrange the polynomial coefficients in rows and columns according to the following pattern:

λ^n	a_0	a_2	a_4	a_6	...
λ^{n-1}	a_1	a_3	a_5	a_7	...
λ^{n-2}	b_1	b_2	b_3	b_4	...
λ^{n-3}	c_1	c_2	c_3	c_4	...
λ^{n-4}	d_1	d_2	d_3	d_4	...
...	...	...	...	...	...
λ^2	e_1	e_2			
λ^1	f_1				
λ^0	g_1				

With $b_1 = \frac{a_1 a_{2i} - a_0 a_{2i+1}}{a_1}$, $c_1 = \frac{b_1 a_{2i+1} - a_1 b_{i+1}}{b_1}$, and $d_1 = \frac{c_1 b_{i+1} - b_1 c_{i+1}}{c_1}$, where $i = 1, 2, \ldots, n$ with all elements in the first column in the table must be positive, then the system will be stable (Routh, 1877).

5. Conclusion

Based on the results of the review that has been carried out, there are several studies on the spread of mosaic disease in jatropha. In this review paper, the researcher provides a brief overview of the literature on the spread of mosaic disease in Jatropha plants with the addition of various compartments and parameters. Several compartments and parameters used are based on the literature, namely effect awareness, insecticide, intervention, yellow stick, natural predators, rouging, and a combination of several compartments and parameters.

In developing a model for the spread of mosaic disease in Jatropha, it is hoped that existing models can be considered, by adding new compartments and parameters or combining existing ones. Based on this literature review, it is hoped that it can provide a little picture of the formation of a model for the spread of mosaic disease in Jatropha. Thus, in the future, it is expected to formulate a strategy to prevent the spread of the disease optimally, at an optimal cost to increase the yield of Jatropha curcas.

References

Al-Basir, F. Al, Venturino, E., & Roy, P. K. (2017). Effects of awareness program for controlling mosaic disease in Jatropha curcas plantations. *Mathematical Methods in the Applied Sciences*, *40*(7), 2441–2453.

Al-Basir, F., & Roy, P. K. (2018). Dynamics of mosaic disease with roguing and delay in Jatropha curcas plantations. *Journal of Applied Mathematics and Computing*, *58*(1), 1–31.

Al-Basir, F., Roy, P. K., & Ray, S. (2017). Impact of roguing and insecticide spraying on mosaic disease in Jatropha curcas. *Control and Cybernetics*, *46*(4), 325–344.

Al Basir, F., Blyuss, K. B., & Ray, S. (2018). Modelling the effects of awareness-based interventions to control the mosaic disease of Jatropha curcas. *Ecological Complexity*, *36*, 92–100.

Al Basir, F., Takeuchi, Y., & Ray, S. (2021). Dynamics of a delayed plant disease model with Beddington-DeAngelis disease transmission. *Mathematical Biosciences and Engineering*, *18*(1), 583–599.

Al Basir, F., Venturino, E., Ray, S., & Roy, P. K. (2018). Impact of farming awareness and delay on the dynamics of mosaic disease in Jatropha curcas plantations. *Computational and Applied Mathematics*, *37*(5), 6108–6131.

Fischel, G., & Gröller, E. (1995). Visualization of local stability of dynamical systems. *Visualization in Scientific Computing'95*, 106–117.

Harimurti, N., & Sumangat, D. (2011). Pengolahan biji jarak pagar (Jatropha curcas L.) menjadi sumber bahan bakar nabati dan pemanfaatan produk samping. *Buletin Teknologi Pasca Panen*, *7*(1), 48–55.

Hariyati, R. T. S. (2010). Mengenal systematic review theory dan studi kasus. *Jurnal Keperawatan Indonesia*, *13*(2), 124–132.

Hethcote, H. W. (2000). The mathematics of infectious diseases. *SIAM Review*, *42*(4), 599–653.

Meyer, W. J. (2004). *Concepts of Mathematical Modeling*. Courier Corporation.

Pambudi, A. S., Fatmawati, F., & Windarto, W. (2019). Analisis Kontrol Optimal Model Matematika Penyebaran Penyakit Mosaic pada Tanaman Jarak Pagar. *Contemporary Mathematics and Applications (ConMathA)*, *1*(2), 104–120.

Perko, L. (1991). Differential Equations and Dynamical Systems. In *Differential Equations and Dynamical Systems* (pp. 1–63). Springer.

Pratiwi, N. P., Aldila, D., Handari, B. D., & Simorangkir, G. M. (2020). A mathematical model to control Mosaic disease of Jatropha curcas with insecticide and nutrition intervention. *AIP Conference Proceedings*, *2296*(1), 20096.

Ratti, I. (2018). *A review on mathematical modeling of infectious diseases*.

Routh, E. J. (1877). A Treatise on the Stability of Motion. *Macmillima, London.*

Supriyadi-Tirtosuprobo, S.-T., & Riajaya, P. D. (2015). *Analisis Usaha Tani Jarak Pagar (Jatropha curcas L.) Hasil Peremajaan*.

Syakir, M. (2010). Prospek dan kendala pengembangan jarak pagar (Jatropha curcas L.) sebagai bahan bakar nabati di Indonesia. *Perspektif*, *9*(2), 55–65.

Van den Driessche, P., & Watmough, J. (2002). Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Mathematical Biosciences*, *180*(1–2), 29–48.

Venturino, E., Roy, P. K., Al Basir, F., & Datta, A. (2016). A model for the control of the mosaic virus disease in Jatropha curcas plantations. *Energy, Ecology and Environment*, *1*(6), 360–369.