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Best Distribution Selection in Modeling the Interest Rate as a Random Modifier

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Abstract

The interest rate is seen as a random variable because the interest rate has an unpredictable nature or changes over time. This means that the interest rate cannot be anticipated in the future with a certain degree of certainty. Therefore, mathematical models are needed to predict the behavior and value of future interest rates. The models used in this study were interest rate, uniform distribution, and lognormal distribution. The data used in the study were interest rate data for 2014-2015 and sample data for uniform distribution. The resulting model in interest rate modeling as a random variable uses for uniform and lognormal distributions with the application of data $i_t \sim U(0{,}0214; 0{,}0226)$and $(1+i_t) \sim Lognormal(1{,}022; 0{,}00003)$. The interest rate model as a uniformly distributed random variable is considered better with a smaller standard deviation, k, and $1-\frac{1}{k^2}$values compared to the lognormal distribution based on the data used.

Keywords: Continuous probability distribution, independent, uniform, lognormal.

1. Introduction

The interest rate is described as the rate of interest in the form of a percentage of the amount of money lent or invested that is charged on loans or investments in an economy (Alafif, 2023; Ogundipe et al., 2020). Changes in interest rates have a significant impact on economic growth, inflation, investment, and others (Musarat et al., 2021). In general, it is important to remember that risk remains an important factor in the financial decision making process. This is because changes in interest rates can affect investment returns over a year, so the way to manage investment risk is to diversify by spreading investments across various types of assets.

According to financial econometric and mathematical analysis, the interest rate is treated as a random variable that follows a certain probability distribution. Research on interest rates treated as random variables with uniform and lognormal distributions is relevant for various reasons (Mitzenmacher, and Upfal, 2017). First, a good understanding of the distribution of interest rates can help market players to make the right decisions in managing assets and debt. Second, a significant increase in interest rates has a major impact on the stability of financial values and the economic condition of the capital market. Apart from that, interest rates also participate in developing high-risk market instruments (Engel, 2016).

The interest rate will influence the rise and fall of foreign exchange rates. In Algifari and Rohman's (2022) research, interest rates have a negative effect on foreign exchange rates. An increase in interest rates causes foreign exchange rates to fall. Conversely, a decrease in interest rates results in an increase in foreign exchange rates. Furthermore, research conducted by Wang et al. (2013) uses a constant interest rate in their modeling.

The interest rate is viewed as a random variable because interest rates are unpredictable and vary over time, so the value cannot be anticipated in the future (Cochrane, 2016). Therefore, mathematical models are needed to predict the behavior and value of interest rates in the future (Bauer and Rudebusch, 2020).

Based on the description above, the interest rate which is viewed as a random variable will be modeled with a uniform and lognormal distribution. The uniform distribution is used because the model is easier to understand, while the lognormal distribution can be a model that can represent interest rates. In addition, the model will be applied at the data level of interest and compared with the data equality test and Chebysev's theorem to see which model is the best.

2. Research Methods

Steps taken in do modeling level flower as variable random use uniform and lognormal distributions are as following:

A. Assumes level flower as variable random;
B. Modeling level flower with uniform distribution;
C. Carry out data testing using the KS test for know what is that data? is uniform distribution using SPSS. Following is steps used in SPSS:
   a) Entering data.
   b) Carrying out KS tests for uniform distribution with method click Analyze ➔Nonparametric Tests ➔ Legacy Dialogues➔ 1 Sample KS
   c) Enter variable data with name "i" to in box Test Variable List.
   d) Then tick "Uniform" column in Test Distribution.
   e) Then click OK.;
D. Do application of level data interest in the uniform distribution model;
E. Modeling level flower with lognormal distribution;
F. Change the original data become form natural logarithm (ln) to perform a normal distribution test;
G. Carry out data testing using the KS test for know what is that data? is normal distribution using SPSS. Following is steps used in SPSS:
   a) Entering data.
   b) Carrying out KS tests for uniform distribution with method click Analyze ➔Nonparametric Tests ➔ Legacy Dialogues➔ 1 Sample KS
   c) Enter variable data with name "ln_i" to in box Test Variable List.
   d) Then tick column "Normal" in Test Distribution.
   e) Then click OK.;
H. Estimate the mean (μ) and variance (σ^2) parameters for lognormal distribution using original data;
I. Do application of level data interest in the lognormal distribution model.
J. Determine the best model in level data usage flowers used.

3. Results and Discussion

3.1. Motivation establishment of interest rates as random variables

First of all will showed that mark expectation accumulation and value Now No The same with mark accumulation and value Now in expectation level flower. That matter can showed with exists unit investment for 10 years with level flower effective constant namely 8%, 9%, 10% and 11%.

Based on statistics basics, expectations level flower $\bar{i}$is given by

$$\bar{i} = E[i] = \frac{1}{4}[0.08 + 0.09 + 0.10 + 0.11] = 0.10$$

Then For expectation from mark accumulation is given by

$$E[(1+i)^{10}\ ] = \frac{1}{4}[\ (1.08)^{10} + (1.09)^{10} + (1.10)^{10} + (1.11)^{10}] = 2.49$$

However, for expectation mark accumulation from level flower is given by

$$\left(1+\bar{i}\right)^{10} = (1.10)^{10} = 2.60$$

With so, deep illustration calculation expectation from mark accumulation No The same with mark accumulated on expectations level flowers that can be illustrated in equation (1).

$$\left(1+\bar{i}\right)^{n} \neq E[(1+i)^{n}] \tag{1}$$

Temporary expected equality That own the same result and can obtained if level flower modeled as variable uniform random and lognormal are possible contained in equation (2).

$$\left(1+\bar{i}\right)^{n} = E[(1+i)^{n}] \tag{2}$$

3.2. Interest rate model as random variables with uniform distribution

Interest rate will walk throughout period $t-1$until t, with the notation by $i_t$for $t = 1,2,\dots,n$. Will be used mark accumulation $a(n)$ given in equation (3) (Kellison, 2009: 13) .

$$a(n) = (1+i_1)(1+i_2)\dots(1+i_n) = \prod_{t=1}^{n}(1+i_t) \tag{3}$$

Furthermore assumed that i_t nature independent and identical with mean$\bar{i}$ so that expectation mark accumulation given by equation (4).

$$E[a(n)] = E[(1+i_1)]\,E[(1+i_2)]\dots E[(1+i_n)] = \left(1+\bar{i}\right)^n \tag{4}$$

Mark$\bar{i}$ for level models flower as variable random in equation (4) is obtained from the mean of a given uniform distribution equation (5) (Sugiyarto, 2021: 248) .

$$\bar{i} = \frac{a+b}{2} \tag{5}$$

3.3. Application of uniform distribution model results

3.3.1. Application using Bank Interest Rate Data

By general , level flower considered constant , so expectation level flower is given by

$$E(i_t) = \bar{i} = \frac{2}{24}[0.0214] + \frac{2}{24}[0.0215] + \frac{1}{24}[0.0216] + \frac{2}{24}[0.0217] + \frac{2}{24}[0.0218] + \frac{2}{24}[0.0219] + \frac{1}{24}[0.0220] + \frac{2}{24}[0.0221] + \frac{2}{24}[0.0222] + \frac{2}{24}[0.0223] + \frac{2}{24}[0.0224] + \frac{3}{24}[0.0225] + \frac{1}{24}[0.0226] = 0.022$$

Furthermore For expectation from level considered interest constant is given by

$$\left(1+\bar{i}\right)^2 = 1.0445$$

Level data testing flower using the KS test with uniform distribution can be seen in Table 4.1.

Table 1: KS test results for uniformly distributed interest rates

One-Sample Kolmogorov-Smirnov Test		
		Interest rate (%)
N		24
Uniform Parameters[a,b]	Minimum	2.14
	Maximum	2.26
Most Extreme Differences	Absolute	0.083
	Positive	0.083
	Negative	-0.083
Kolmogorov-Smirnov Z		0.408
Asymp Sig. (2-tailed)		0.996

a. Test distribution in Uniform

b. Calculated from data

Based on Table 1, it is obtained mark $P_{value} = 0.996 > \alpha = 0.05$ using assumptions that H_0 state level data flowers used uniform distribution and criteria rejection accept H_0, If $\alpha < P_{value}$. So that 's the conclusion obtained is level data flowers used uniform distribution.

Next level data owned flowers will assumed become i_t as variable random uniformly distributed in the interval $[0{,}0214; 0{,}0226]$with $t = 1,2, \ldots ,24$an accumulated value of 1 at the end year second so based on equation (5) on the uniform distribution is obtained

$$E(i_t) = \bar{i} = \frac{0.0214+0.0226}{2}$$

$$\bar{i} = 0.022$$

Furthermore application equation (4) can obtained mark $E[a(n)]$ as following

$$\begin{aligned} E[a(2)] &= (1+0.022)^2 \\ &= 1.044 \end{aligned} \quad (6)$$

Furthermore calculation $E[(1+i)^2]$done as follows

$$E[(1+i)^2] = \frac{2}{24}[(1+0.0214)^2] + \frac{2}{24}[(1+0.0215)^2] + \frac{1}{24}[(1+0.0216)^2] + \frac{2}{24}[(1+0.0217)^2] + \frac{2}{24}[(1+0.0218)^2] + \frac{2}{24}[(1+0.0219)^2] + \frac{1}{24}[(1+0.0220)^2] + \frac{2}{24}[(1+0.0221)^2] + \frac{2}{24}[(1+0.0222)^2] + \frac{2}{24}[(1+0.0223)^2] + \frac{2}{24}[(1+0.0224)^2] + \frac{3}{24}[(1+0.0225)^2] + \frac{1}{24}[(1+0.0226)^2 = 1.045 \quad (7)$$

Based on calculations (6) and (7) do not generated that equation $E[a(n)] = E[(1+i)^n]$. That matter caused Because respective $i_t$frequencies No appropriate the same.

3.3.2. Application use Example Data

After original data calculation level flower as variable random produce $[a(n)] \neq E[(1+i)^n]$ so implementation of the model will next with example data with the same frequency for every $i_t$one. Sample data will be tested using the KS test with a uniform distribution seen in Table 4.2.

Table 2: KS test results for example of uniformly distributed data

One-Sample Kolmogorov-Smirnov Test 2		
		Example data (%)
N		26
Uniform Parameters[a,b]	Minimum	2.14
	Maximum	2.26
Most Extreme Differences	Absolute	0.077
	Positive	0.077
	Negative	-0.077
Kolmogorov-Smirnov Z		0.392
Asymp Sig. (2-tailed)		0.998

a. Test distribution in Uniform

b. Calculated from data

Based on Table 2, it is obtained mark $P_{value} = 0.998 > \alpha = 0.05$ using assumptions that H_0 state level data flowers used uniform distribution and criteria rejection accept H_0, If $\alpha < P_{value}$. So that 's the conclusion obtained is level data flowers used uniform distribution.

Furthermore example data is assumed become $i_t$as a random variable that is uniformly distributed in the interval $[0.0214; 0.0226]$ with $t = 1,2, \ldots ,26$an accumulated value of 1 at the end year second so based on equation (5) on a uniform distribution so that obtained

$$E(i_t) = \bar{i} = 0.022$$

Value $\bar{i}$obtained the same with rate data calculation interest, so For application equation (4) value $E[a(n)]$what is obtained is also the same. The following are the calculation results for example data.

$$E[a(2)] = (1 + 0.022)^2 = 1.044 \tag{8}$$

Furthermore calculation $E[(1+i)^2]$done as follows

$$E[(1+i)^2] = \frac{2}{26}[(1+0.0214)^2] + \frac{2}{26}[(1+0.0215)^2] + \frac{2}{26}[(1+0.0216)^2] + \frac{2}{26}[(1+0.0217)^2] + \frac{2}{26}[(1+0.0218)^2] + \frac{2}{26}[(1+0.0219)^2] + \frac{2}{26}[(1+0.0220)^2] + \frac{2}{26}[(1+0.0221)^2] + \frac{2}{26}[(1+0.0222)^2] + \frac{2}{26}[(1+0.0223)^2] + \frac{2}{26}[(1+0.0224)^2] + \frac{2}{26}[(1+0.0225)^2] + \frac{2}{26}[(1+0.0226)^2] = 1.044 \tag{9}$$

Based on calculations (8) and (9) have generated that equation $E[a(n)] = [(1+i)^n]$. So, modeling level flower as variable random more in accordance compared to with calculation level considered interest constant.

3.4. Interest rate model as random variables with lognormal distribution

Level model assumed interest as variable random $\ln(1+i_t)$ will follow normal distribution with mean $\mu$and variance σ^2. Therefore that, level flower follow lognormal distribution as a random variable $(1+i_t)$with parameters $\mu$and σ^2assumed to be a random variable $1+i_t$. Mean given by equation (10) (Balakrishnan and Lai, 2009: 8) :

$$Mean = e^{\mu + \frac{1}{2}\sigma^2} \tag{10}$$

Based on equation (3) value accumulation $a(n)$ is given by

$$a(n) = (1+i_1)(1+i_2)\ldots(1+i_n) = \prod_{t=1}^{n}(1+i_t)$$

Furthermore for $\ln a(n)$ given by equation (11)

$$\ln a(n) = \ln[(1+i_1)(1+i_2)\ldots(1+i_n)] = \sum_{t=1}^{n}\ln(1+i_t) \tag{11}$$

Temporary the mean and variance can added up as well as own characteristic independent or each other free, so can seen in equations (12) and (13)

$$E[\ln a(n)] = n\mu \tag{12}$$

$$Var[\ln a(n)] = n\sigma^2(\tag{13}$$

Mark$a(n)$ will follow resulting lognormal distribution the mean and variance can added up, so with application equations (12) and (13) in (10) for $a(n)$obtained.

$$E[a(n)] = e^{n\mu + \frac{1}{2}n\sigma^2} \tag{14}$$

3.5. Application of lognormal distribution model results

Lognormal distribution will follow normal distribution so level data testing interest 2014-2015 will be done using the KS test with normal distribution contained in Table 3.

Table 3: KS test results for normally distributed interest rates

One-Sample Kolmogorov-Smirnov Test		
		Interest rate (%)
N		24
Uniform Parameters[a,b]	Minimum	3.9238
	Maximum	15.35219
Most Extreme Differences	Absolute	0.538
	Positive	0.538
	Negative	-0.418
Kolmogorov-Smirnov Z		0.538
Asymp Sig. (2-tailed)		0.169c

a. Test distribution in Uniform

b. Calculated from data

c. Lilliefors significance correction

Based on Table 3, it is obtained mark $P_{value} = 0.169 > \alpha = 0.05$ using assumptions that H_0 state level data flowers used uniform distribution and criteria rejection accept H_0, If $\alpha < P_{value}$. So that 's the conclusion obtained is No Enough proof for reject H_0which means the interest rate data lnused is normally distributed

Assume that $1 + i_t$is the interest rate that follows a lognormal distribution with $\mu = 1.022$ and $\sigma^2 = 0.00003$. Next we will calculate the values $E[a(n)]$for $n = 2$ the following.

Based on equation (10) in the lognormal distribution then obtained

$$E[1 + i_t] = 2.77879$$

Furthermore calculation will next For $E[(1 + i)^2]$so that the calculation results can be obtained (15)

$$E[(1 + i)^2] = 7.7217 \tag{15}$$

Then calculation furthermore is application of the lognormal model using equation (14)

$$E[a(n)] = 7.7217 \tag{16}$$

The calculations produced by (15) and (16) are appropriate calculation Because get same result namely 7.7217, so the model used Already in accordance.

3.6. Determining the interest rate model as the best random variable

Determination of level models flower as variable random is the best can using data equality tests and theorems Chebysev in equations (17) and (18)

$$\bar{x} \pm k\ s = a \tag{17}$$

$$P(\mu - ks < X < \mu + ks) \geq 1 - \frac{1}{k^2} \tag{18}$$

Model calculation results with using data equality tests and theorems Chebysev served in Table 4.

Table 4: Comparison of uniform and lognormal distribution model analysis

	Uniform Distribution	Lognormal Distribution
$\bar{x}$	0.022	1.022
k	1.7341	1000.8
s	0.000346	0.001
$1 - \frac{1}{k^2}$	0.6675	0.99999

Based on Table 4, the uniform distribution has mark s, k, And$1-\frac{1}{k^2}$ which are more small compared to lognormal distribution. Uniform distribution that has mark $1-\frac{1}{k^2}$which are more small signify that distribution the will bring up more fluctuation small and characteristic homogeneous, in other words probable happen more outliers small. Therefore that, based on Table 4 as reference hence the level model flower as variable random with uniform distribution with level data usage flower 2014-2015 is considered best.

4. Conclusions and Recommendations

Based on results and discussion of the research This , model is generated in modeling level flower as variable random is $i_t \sim U(0.0214; 0.0216)$ and $(1+i_t) \sim$ Lognormal $(1.022; 0.00003)$. Based on level model determination the best flowers hence the level model flower as variable A uniformly distributed random is considered best with mark standard deviation , k, and value $\frac{1}{k^2}$ which are more small compared to with lognormal distribution .

Possible advice given for study furthermore is create a level model flower as variable random for expectations and variance others related with level flower. Furthermore for the bank is expected can consider level flower as variable random No only see level considered interest constant.

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