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Calculation of Motor Vehicle Insurance Premiums by using the Moment Method to Estimate the Aggregate Claim Model

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Abstract

The aggregate claim model is a model that can be used to determine the amount of premium billed to the insured by the insurance company. This model consists of a combination of two independent random variables, namely the number of claims that occur and the size of the claim for each claim submitted. In this study, many claims are Poisson distributed and the size of the claim is exponentially distributed. The method of moments is used to estimate the parameters of each distribution. Based on the calculation results, the amount of premium billed to the insured for one year if based on the pure premium principle is IDR112,500,000.00 and if based on the expected value principle, variance value principle, and standard deviation principle is IDR165,900,000.00.

Keywords: Vehicle insurance premiums, aggregate claim model, method of moments, poisson distribution, exponential distribution, premiums.

1. Introduction

Based on the Commercial Code (KUHD) of the Republic of Indonesia, Chapter 9 Article 246, insurance is an agreement by which an insurer binds itself to an insured by accepting a premium to compensate him for a loss, damage or loss of expected profits, which may occur due to an uncertain event (Fauzi, 2017; Simanjuntak et al., 2022). The number of motorized vehicles increases every year. Based on data from the Indonesian National Police in 2022, the number of motorized vehicles will reach 152.51 million units (Sadya, 2023). This number of motorized vehicles can pose a risk to motorized vehicles, especially if it is not balanced by widening or adding roads. This is supported by a statement from the National Police Traffic Corps (Korlantas) which stated that traffic accidents throughout Indonesia from January to September 2022 continued to increase with the number of cases reaching 94,617 and 19,054 people dying (Kompas, 2022).

Based on the magnitude of the risk that occurs in this incident, to avoid losses from damage to the motor vehicle, the insurance party or company offers motor vehicle insurance (Anggraini and Rahma, 2022). As an institution taking over and accepting risks, insurance companies must be able to anticipate if there are many claims that could result in losses and make the company bankrupt (Praja, 2023). Determining the right premium price is very important so that the company does not lose money but can still attract customers. In risk management, losses can be predicted if the insurance company knows the characteristics of the loss distribution (Dickson, 2005).

In determining the right premium price, the company must first determine the right aggregate claims model. The aggregate claims model is a model that can be used to determine the amount of premium charged to the insured by the insurance company (Wu et al., 2018). The aggregate claims model in loss insurance consists of two variables, namely the number of claims that occur per period and the size of the claim for each claim submitted (Bowers et al., 1997). Many claims can be modeled with a discrete distribution, while claim sizes can be modeled with a nonnegative continuous distribution. In general insurance, the aggregate claims model is an important concern for insurance companies, because it is very useful for determining the amount of premiums paid by consumers or insureds to insurance companies (Garrido et al., 2016).

Based on the explanation above, this research aims to calculate the amount of premium billed based on the results of applying the moment method to estimate the aggregate claims model.

2. Research Methods

This research uses methodology by combining literature study with the use of secondary data. The data used is motor vehicle insurance data at insurance companies for the period 2018 to 2023. Data analysis was carried out with the help of software, namely SPSS and Microsoft Excel . The research steps used are as follows:

testing the multiple claims model: Poisson distribution and large claims model: exponential distribution using SPSS software determine parameter estimates using the moment method. In this research, to determine the estimated parameter values, we use the moment method by calculating the value of the kth sample moment which is defined as:

$$M_k = \frac{1}{n}\sum_{i=1}^{n} X_i^k. \tag{1}$$

with

M_k : Population moment/kth sample

n : Sample size

calculate the expected value and variance of aggregate claims. In this research, the expected value is calculated using the following equation;

$$\begin{aligned} E[S] &= E\left[E\left(\sum_{i}^{N} X_i | N = n\right)\right] \\ &= E[E(X_1 + X_2 + X_3 + \cdots + X_n)] \\ &= E[E(X_1) + E(X_2) + E(X_3) + \cdots + E(X_n)] \\ E[S] &= E\big[N \cdot E[X]\big] = E[X]E[N]. \end{aligned} \tag{2}$$

Meanwhile, the variance value is calculated using the following equation;

$$\begin{aligned} \mathrm{Var}(S) &= E[\mathrm{Var}(S|N)] + \mathrm{Var}(E[S|N]) \\ &= E[N \cdot \mathrm{Var}(X)] + \mathrm{Var}(N \cdot E[X]) \\ &= E[N]\mathrm{Var}(X) + (E[X])^2\mathrm{Var}(N). \end{aligned} \tag{3}$$

Calculate the premium size based on the principles of pure premium, expectation value, variance and standard deviation. In this research, the premium amount is determined based on the results of calculating expectations and aggregate claim variance. To determine the premium loading factor value from the principles of expectation value, variance and standard deviation, an approximate distribution of aggregate claims is used using the central limit theorem. Below we discuss 4 premium principles, namely the pure premium principle, the expected value principle, the variance principle, and the standard deviation principle taken in the book (Dickson, 2005), namely

For the calculation of the premium based on the pure premium principle, the equation is:

$$High\ premium = \Pi_S = \mathrm{E[S]}. \tag{4}$$

Premium loading factor value ψ calculated based on the approximation of aggregate claims using the Central Limit P theorem (Bowers et al., 1997: 39) at the level of significance α with Equation (5):

$$P(S \le \Pi_S) = P\left(\frac{S - E[S]}{\sqrt{\mathrm{Var}(S)}} \le \frac{\Pi_S - E[S]}{\sqrt{\mathrm{Var}(S)}}\right) = 1 - \alpha \tag{5}$$

To calculate the premium size based on the expected value principle, it is solved using the following equation:

$$High\ premium = \Pi_S = (1 + \psi)\mathrm{E[S]}. \tag{6}$$

To calculate the premium size based on the variance value principle, it is solved using the following equation:

$$High\ premium = \Pi_S = \mathrm{E[S]} + \psi Var(S) \tag{7}$$

To calculate the premium size based on the standard deviation principle, it is solved using the following equation:

$$High\ premium = \Pi_S = \mathrm{E}[S] + \psi\sqrt{Var(S)} \tag{8}$$

3. Results and Discussion

3.1. Data

The data used in this research is motor vehicle insurance data from insurance companies for the period 2018 to 2023 in the form of data on the number of claims and the size of the claims. Data on many claims is assumed to have a Poisson distribution, while data on large claims is assumed to have an exponential distribution.

3.2. Model testing

The initial step taken in calculating motor vehicle insurance premiums is to test the distribution of the number of claims and the size of the claims, to prove that the model for the number of claims has a Poisson distribution and the model for the size of the claims has an exponential distribution .

Testing of the multiple claim distribution model was carried out using the Kolmogorov-Smirnov test using SPSS software. The results of testing the multiple claim distribution model using SPSS software can be seen in Table 1 in output form SPSS software.

Figure 1: Test results of multiple claim distribution models.

One-Sample Kolmogorov-Smirnov Test

		Many claims
N		70
Uniform Parameters[a,b]	Mean	19.54
Most Extreme Differences	Absolute	0.078
	Positive	0.059
	Negative	-0.078
Kolmogorov-Smirnov Z		0.652
Asymp Sig. (2-tailed)		0.788

a. Test distribution is poisson

b. Calculated from data

The formula for hypothesis testing with the Kolmogrov-Smirnov test is

H_0 : Model of many claims with Poisson distribution;

H_1 : The multiple claims model does not have a Poisson distribution.

The null hypothesis (H_0) is not rejected if the asymptotic significance value (p-value) is greater than the value α. Based on Figure 1 The asymptotic significance value obtained for many claims was 0.788. Because the value of asymptotic significance = 0.788 > α= 0.05, it $H_0$is not rejected. This means that the results of the Kolmogrov-Smirnov test state that the claim size has a Poisson distribution.

Testing of the claim size distribution model was carried out using SPSS software. The results of testing the claim size distribution model using SPSS software can be seen in Table 2 in output form SPSS software.

Figure 2: Test results of claim size distribution model.

One-Sample Kolmogorov-Smirnov Test

		High claims
N		70
Exponential Parameters[a,b]	Mean	126618111.8
Most Extreme Differences	Absolute	0.150
	Positive	0.098
	Negative	-0.150
Kolmogorov-Smirnov Z		1.251
Asymp Sig. (2-tailed)		0.087

a. Test distribution is exponential

b. Calculated from data

The formula for hypothesis testing with the Kolmogrov-Smirnov test is:

H_0 : Exponentially distributed claim size model;
H_1 : The claim size model is not exponentially distributed.

The null hypothesis (H_0) is not rejected if the asymptotic significance value (p-value) is greater than the value α. Based on Figure 2, the asymptotic significance value for many claims is 0.087. Because the value of asymptotic significance = 0.087 > α= 0.05, it $H_0$is not rejected. This means that the results of the Kolmogrov-Smirnov test state that the claim size has an exponential distribution.

3.3. Aggregate claims model estimation

To determine aggregate claim model estimates or parameter value estimates for multiple claim distribution models (Poisson distribution) and claim size distribution models (exponential distribution) using the method of moments.
For multiple claim parameters (poisson distribution).

$$\hat{\lambda} = \frac{1}{70}\sum_{i=1}^{70} x_i = \frac{1}{70}(8 + 6 + \cdots + 19) = 19.54$$

For large claim parameters (exponential distribution).

$$\hat{\theta} = \frac{1}{70}\sum_{i=1}^{70} x_i = \frac{1}{70}(27546720 + 19234680 + \cdots + 103949667) = 1.27 \times 10^8$$

The results of estimating parameter values for the distribution model for the number of claims and the size of claims are shown in Table 3.

Table 3: Results of estimation of parameter values using the moment method.

Distribution	Parameter	Mark
Poisson	λ	19,.54
Exponential	θ	1.27×10^8

3.4. Calculation of expected value and variance of aggregate claims

Based on the results of estimating parameter values using the moment method, it can be concluded that many claims have a Poisson distribution with parameter $\lambda = 19{,}54$, while the number of claims has an exponential distribution with parameter $\theta = 1{,}27 \times 10^8$. Next, the expectations and variances of the number of claims and the size of the claim will be determined.
The expected value and variance of the Poisson distribution are:

$$E(X) = \lambda = 19.54$$
$$Var(N) = \lambda = 19.54$$

The expected value and variance of the exponential distribution are:

$$E(X) = \theta = 1.27 \times 10^8$$
$$Var(N) = \theta^2 = (1.27 \times 10^8)^2 = 1.60 \times 10^{16}$$

Based on Equations (2) and (3), the expected value and variance of the aggregate claims model are:

$$E[S] = (19.54)(1.27 \times 10^8) = 2.47 \times 10^9$$

$$\mathrm{Var}(S) = (1.27 \times 10^8)(19.54) + (19.54)^2(1.60 \times 10^{16}) = 6.12 \times 10^{18}$$

By $E[S]$stating the average amount of loss received by the insurance company each month, and $\mathrm{Var}(S)$stating the size of the distribution of losses received by the insurance company each month. The expected value and variance of the aggregate claims model received by the insurance company each year are:

$$E[S] = (2.47 \times 10^9) \times 12 = 2.97 \times 10^{10}.$$

$$\mathrm{Var}(S) = (6.12 \times 10^{18}) \times 12 = 7.35 \times 10^{19}.$$

3.5. Premium Calculation

The results of calculations using equations (4) to (8) are given in Table 2. The premium loading factor values (ψ)and the amount of premium per person charged based on each principle if there are 264 portfolios are given in Table 3. The figures are in Table 3. Obtained from equations (4) to (8). The premium amount for the pure premium principle is calculated without involving the premium loading factor so that in Table 2 there is no premium loading factor value.

Table 1: Motor vehicle insurance premium calculation results.

No	Premium calculation principles	Premium loading factor	Premium calculation results (Rp)
1	Pure premium principle	$-$	112.5×10^6
2	Expectation value principle	0.475	165.9×10^6
3	Principle of Variance Value	1.92×10^{-10}	165.9×10^6
4	Standard deviation principle	1.645	165.9×10^6

4. Conclusions and Recommendations

Based on the results and discussion the distribution of the number of claims is Poisson with the estimated parameter value λ = 19.54, while the size of the claim is distributed exponentially with parameter θ= 1.27×10^8. Furthermore, from the estimation results, the expected value of aggregate claims for a year is obtained, namely IDR2.97×10^{10} d with variance 7.35×10^{19}. The premium amount per person (insured) based on the pure premium principle is IDR 112,500,000.00, the expected value principle is IDR165,900,000.00, the variance value principle is IDR 165,900,000.00, and the standard deviation principle is IDR165,900,000,00.

Suggestions for further research are to use assumptions about the distribution of claim sizes and many different claims. Apart from that, it is also suggested that this can be done using methods other than the moment method.

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