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# Waiting Time Optimization at Traffic Light Intersection in Purbalingga by Using Compatible Graphs

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**Abstract**

Traffic congestion is a problem that often occurs at crossroads. One of the causes of congestion is the waiting time for traffic at a crossroad improper, so it can cause the accumulation of vehicles in several branches. The purpose of this paper is to determine the optimal waiting time for traffic lights at the Sudirman-Pujowiyoto intersection in Purbalingga by using a compatible graph. The traffic flow at the intersection can be modeled into a compatible graph, where a vertex represents the traffic flow to be managed and the edges indicate that the two flows are compatible. It means that they can run simultaneously without crossing. Based on secondary data from Dinas Perhubungan Kabupaten Purbalingga, the total waiting time applied to the Sudirman-Pujowiyoto intersection is 317 seconds. Meanwhile, according to the compatible graph calculation, by using the assumption of 60 seconds in a cycle, an optimal total waiting time is 120 seconds.

*Keywords:* compatible graph, optimal total waiting time, traffic light

## 1. Introduction

Congestion is one of the problems that often occurs at intersections. Congestion occurs as a result of unbalanced traffic management. Besides that, the number of vehicles in this country is also increasing every year. The accumulation of vehicles on a certain high lane is also the cause of congestion. Several policies to overcome congestion have been implemented, one of them is the regulation of traffic light waiting times at intersections. However, the policies implemented in traffic management are not always effective. So, an optimal solution is needed to solve those problems. By determining the traffic light optimal total waiting time at an intersection, the congestion can be reduced.

Traffic is the movement of vehicles and people in the traffic space of the road. While traffic lights are generally used to regulate traffic flow at an intersection so that traffic congestions can be avoided (Rouphail et al., 2001). Traffic control at intersections is very important to implement to determine the movement of vehicles from each lane. Vehicles can move alternately following the existing traffic lights. Many traffic light settings are found at intersections that have a longer red light duration and a very short green light. One of them is at the Sudirman-Pujowiyoto Intersection in Purbalingga.

In this paper, the optimal total waiting time for traffic lights at the Sudirman-Pujowiyoto intersection in Purbalingga will be determined using a compatible graph. This intersection is one of the intersections in Purbalingga with busy vehicles because the intersection is one of the routes used by public transportation. In addition, the intersection is also an access road to Purbalingga Square, shopping areas, and workplaces. Therefore, it is very necessary to optimize traffic lights so that congestion can be reduced.

Traffic flow at an intersection can be modeled into a compatible graph. A vertex in a compatible graph represents a traffic flow at an intersection and edges in the graph connect compatible flows. Two or more flows are compatible if and only if they can move simultaneously without causing conflict. By using a compatible graph to model traffic flow at an intersection, the output value will be obtained in the form of the optimal total waiting time at the intersection. Total waiting time is the time required for each traffic flow to move and the number of flows contained in a system. By using the assumption of 60 seconds in one cycle, the waiting time at an intersection can be obtained.

The data used in this paper is traffic cycle data at the Sudirman-Pujowiyoto intersection in Purbalingga. The calculation of the optimal total waiting time will be done using a compatible graph. The purpose of this paper is to obtain the optimal total waiting time at the Sudirman-Pujowiyoto Intersection in Purbalingga using a compatible graph.

## 2. Literature Review

### 2.1 Graph

A graph $G$ is a pair of two sets $G = (V, E)$ (Diestel, 2024), where $V$ is a finite or infinite non-empty set whose elements are called vertices, and $E$ is a finite set connecting two vertices, whose elements are called edges. Some basic terms related to graphs are as follows.

(a) Adjacent; two vertices in a graph $G$ are said to be adjacent if they are directly connected by an edge.
(b) Incident, the edge $e = (v_i, v_j)$ is said to be incident with points $v_i$ and $v_j$.
(c) Vertex degree; the degree of a vertex $v$ in a graph $G$, denoted by $deg(v)$, is the number of edges adjacent to that vertex.

#### 2.1.1. Types of graphs

Based on the orientation of the direction on the edge, graphs are divided into two types:

(a) Undirected graph, is a graph whose edges have no direction. In an undirected graph, edge $(u, v)$ is the same as edge $(v, u)$.
(b) Directed graph (directed graph or digraph), is a graph whose sides have direction. In a directed graph, edge $(u, v)$ is not the same as edge $(v, u)$.

Based on the presence or absence of loops or multiple edges, graphs can be classified into two types:

(a) Simple Graph, is a graph that does not contain loops or multiple edges.
(b) Unsimple Graph, is a graph that contains multiple edges or loops. There are two types of unsimple graphs, dual graphs (graphs that contain multiple edges) and pseudo graphs (graphs that contain loops).

#### 2.1.2. Complete graph

A complete graph is a simple graph such that every pair of vertices is joined by an edge, denoted $K_n$ for any complete graph on $n$ vertices (Zhang & Chartrand, 2006). Figure 1 below is an example of a complete graph.

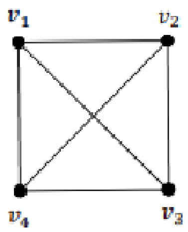


**Figure 1.** Complete Graph

#### 2.1.3. Subgraph

A graph $H$ is called a subgraph of graph $G$, written $H \subseteq G$, if graph $H$ contained in a graph $G$ or $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$ (Tutte, 2001). If $H \subseteq G$ and $V(H) = V(G)$, then $H$ is called a spanning subgraph.

#### 2.1.4. Graphs in traffic networks

In a traffic network, the vertices in the graph represent intersections, while the edges connecting the vertices represent roads, i.e.

a. An undirected edge represents a two-way road.
b. A directed edge represents a one-way road.
c. A double undirected edge represents a double two-way road connecting two identical intersections.
d. A double directed edge represents a double one-way road starting at one intersection and ending at the next intersection.

### 2.2 Clique

A clique $Q$ of a graph $G$ is a complete undirected subgraph of the graph $G$ in which every vertex is connected to every other vertex (Hosseini & Orooji, 2009). A maximal clique is a complete subgraph that no longer has any adjoinable vertices with the subgraph. Maximal Clique Algorithm. can be used to determine the maximal clique of a graph $G$. The steps that can be taken to detect maximum clique are as follows (Dharwadker, 2006).

Step 1:

(a) Take any simple graph $G$ with $n$ vertices, and take a clique $Q$ on $G$.
(b) Take any initial vertex $Q_i$, with $i = 1, 2, 3, \dots, n$.
(c) Find the vertices $v$ that are adjoinable to $Q_i$.
(d) Find the vertices that are adjoinable to $(Q_i \; U \; \{v\})$.
(e) Find the number of adjoinable vertices $\rho(Q_i \; U \; \{v\})$.
(f) Check the maximum location of $\rho(Q_i U \; \{v\})$ at $v_{max}$ on $Q_i$.
(g) Join $v_{max}$ to $Q_i$.
(h) Repeat steps 3 to 7 for further searches.
(i) The search stops when there are no more adjoinable vertices.

Step 2:

(a) Take any simple graph $G$ with $n$ vertices, and take a maximal clique $Q_k$ in $G$.
(b) If $Q_k$ is a maximal clique of $G$ with each $v \in G \backslash Q_k$ connected to at most $k - 1$ vertices in $Q_k$, then we ignore vertices that are not in $k - 1$.
(c) Determine the points that are adjoinable to $Q_k$ by following step 1.

### 2.3 Compatible Graph

Compatibility is the extent to which a performance is considered to be in accordance with the existing system, without changing the activities and components in the system. Traffic flows are said to be compatible if two traffic flows can run simultaneously without any conflict with each other so they are not intersect each other. In traffic problems, the compatibility relationship in traffic flows can be solved using a compatible graph. A compatible graph is a graph with vertices representing objects to be regulated and its edges indicating compatible pairs of vertices (Lusiani et al., 2020). To model traffic flows at an intersection into a compatible graph, it is necessary to first find compatible flows, flows that can run simultaneously without causing conflict.

## 3. Materials and Methods

Data used in this paper is the traffic cycle at the Sudirman-Pujowiyoto Intersection in Purbalingga, secondary data obtained from Dinas Perhubungan Kabupaten Purbalingga. Details of the Sudirman-Pujowiyoto Intersection traffic cycle data can be seen in Table 1.

**Table 1.** Traffic Cycle of Sudirman-Pujowiyoto Intersection

| **Street Name** | **Traffic Light** | | |
|---|---|---|---|
| | **Red** | **Yellow** | **Green** |
| Pujowiyoto | 81 | 6 | 15 |
| Jenderal Soedirman I | 76 | 6 | 20 |
| Kapten Serengat | 84 | 6 | 12 |
| Jenderal Soedirman II | 76 | 6 | 20 |
| Total | 317 | 24 | 67 |

The steps to model the traffic flow and determine the optimal total waiting time with compatible graphs are:

(a) Draw an illustration of an intersection.
(b) Represent the existing traffic cycle into a compatible graph, where the nodes represent the traffic flows and the edges indicate that the two flows are compatible.
(c) Determine the largest complete subgraph (maximal clique), which is a graph obtained from a simple graph where each vertex is connected to all vertices. The complete subgraph shows the traffic flow of one path that is compatible with the traffic flow of the other path.
(d) Determine the cycle time of each traffic flow by assigning a time period and dividing it by the number of complete subgraphs.
(e) Determine the optimal total waiting time. The total waiting time is obtained by multiplying the previously obtained cycle time by the number of traffic flows.

The traffic system at the Sudirman-Pujowiyoto Intersection in Purbalingga can be illustrated as Figure 1 below.

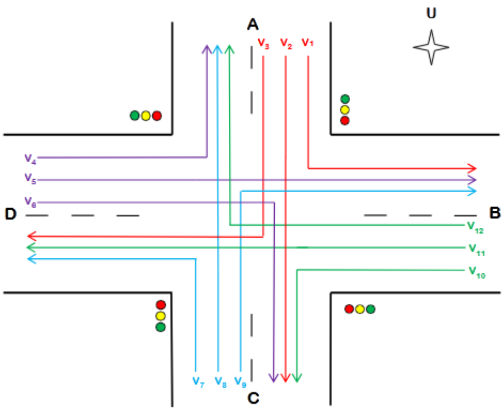


**Figure 2.** Traffic Illustration of Sudirman-Pujowiyoto Intersection

Description:
A : Pujowiyoto Street
B : Jenderal Sudirman I Street
C : Kapten Serengat Street
D : Jenderal Sudirman II Street
$v_1$ : Traffic Flow from Pujowiyoto to Jenderal Sudirman I Street
$v_2$ : Traffic Flow from Pujowiyoto to Kapten Serengat Street
$v_3$ : Traffic Flow from Pujowiyoto to Jenderal Sudirman II Street
$v_4$ : Traffic Flow from Jenderal Sudirman II to Pujowiyoto Street
$v_5$ : Traffic Flow from Jenderal Sudirman II to Jenderal Sudirman I Street
$v_6$ : Traffic Flow from Jenderal Sudirman II to Kapten Serengat Street
$v_7$ : Traffic Flow from Kapten Serengat to Jenderal Sudirman II Street
$v_8$ : Traffic Flow from Kapten Serengat to Pujowiyoto Street
$v_9$ : Traffic Flow from Kapten Serengat to Jenderal Sudirman I Street
$v_{10}$ : Traffic Flow from Jenderal Sudirman I to Kapten Serengat Street
$v_{11}$ : Traffic Flow from Jenderal Sudirman I to Jenderal Sudirman II Street
$v_{12}$ : Traffic Flow from Jenderal Sudirman I to Pujowiyoto Street

In Figure 1, flow $v_1$ is compatible with $v_2$, $v_3$, $v_4$, $v_5$, $v_6$, $v_7$, $v_8$, $v_9$, $v_{10}$, $v_{11}$, and $v_{12}$. It means that $v_1$ is compatible with all flows because it is a left-turning flow. Flow $v_2$ is compatible with $v_1$, $v_3$, $v_4$, $v_6$, $v_7$, $v_8$, and $v_{10}$, but it is not compatible with $v_5$, $v_9$, $v_{11}$, and $v_{12}$. Flow $v_3$ is compatible with $v_1$, $v_2$, $v_4$, $v_7$, $v_{10}$, $v_{11}$, but it is not compatible with $v_5$, $v_6$, $v_8$, $v_9$, and $v_{12}$. Table 2 shows the compatible flows at the Sudirman-Pujowiyoto Intersection.

**Table 2.** Compatible Flows at Sudirman-Pujowiyoto Intersection

| Flow | Compatible |
|---|---|
| $v_1$ | $v_2$, $v_3$, $v_4$, $v_5$, $v_6$, $v_7$, $v_8$, $v_9$, $v_{10}$, $v_{11}$, $v_{12}$ |
| $v_2$ | $v_1$, $v_3$, $v_4$, $v_6$, $v_7$, $v_8$, $v_{10}$ |
| $v_3$ | $v_1$, $v_2$, $v_4$, $v_7$, $v_{10}$, $v_{11}$ |
| $v_4$ | $v_1$, $v_2$, $v_3$, $v_5$, $v_6$, $v_7$, $v_8$, $v_9$, $v_{10}$, $v_{11}$, $v_{12}$ |
| $v_5$ | $v_1$, $v_4$, $v_6$, $v_7$, $v_9$, $v_{10}$, $v_{11}$ |
| $v_6$ | $v_1$, $v_2$, $v_4$, $v_5$, $v_7$, $v_{10}$ |
| $v_7$ | $v_1$, $v_2$, $v_3$, $v_4$, $v_5$, $v_6$, $v_8$, $v_9$, $v_{10}$, $v_{11}$, $v_{12}$ |
| $v_8$ | $v_1$, $v_2$, $v_4$, $v_7$, $v_9$, $v_{10}$, $v_{12}$ |
| $v_9$ | $v_1$, $v_4$, $v_5$, $v_7$, $v_8$, $v_{10}$ |
| $v_{10}$ | $v_1$, $v_2$, $v_3$, $v_4$, $v_5$, $v_6$, $v_7$, $v_8$, $v_9$, $v_{11}$, $v_{12}$ |
| $v_{11}$ | $v_1$, $v_3$, $v_4$, $v_5$, $v_7$, $v_{10}$, $v_{12}$ |
| $v_{12}$ | $v_1$, $v_4$, $v_7$, $v_8$, $v_{10}$, $v_{11}$ |

## 4. Results and Discussion

According to Table 2, the compatible flows can be modeled into a compatible graph as shown in Figure 2 below.

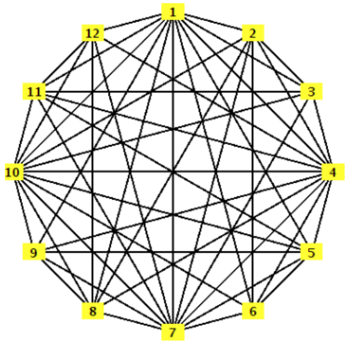


**Figure 3.** Compatible Graph of Sudirman-Pujowiyoto Intersection

In Figure 2, vertex 1 denotes the traffic flow in $v_1$, vertex 2 denotes the traffic flow in $v_2$, vertex 3 denotes the traffic flow in $v_3$, and similarly for vertices 4, 5, 6, 7, 8, 9, 10, 11, and 12. Furthermore, this graph has four vertices of degree 11, there are vertices 1, 4, 7, and 10. Thus, Sudirman-Pujowiyoto Intersection in Purbalingga has four flows that are compatible with all flows so they can move at any time with zero waiting time. The four flows are $v_1$, $v_4$, $v_7$, and $v_{10}$.

Assuming that $v_1$, $v_4$, $v_7$, and $v_{10}$ does not follow the traffic light, where the vehicles can move freely without having to stop when the traffic light is red, because those flows does not affect all the flows that follow the light. Thus, from the compatible graph in Figure 2, vertices 1, 4, 7, and 10 that represent the left-turning flows can be removed to obtain a simpler form of compatible graph. Figure 3 below shows the form of the compatible graph when the left turn is direct or does not follow the traffic light.

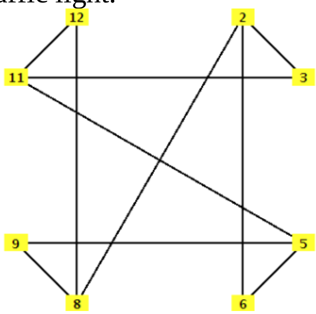


**Figure 4.** Simpler Compatible Graph of Sudirman-Pujowiyoto Intersection

From the compatible graph in Figure 3, we will find the largest complete subgraph (maximum clique) with the following search process.

1) First maximum clique
   a. The compatible graph has 8 vertices $V = \{2, 3, 5, 6, 8, 9, 11, 12\}$.
   b. Take an arbitrary starting point, let $i = 2$, to obtain clique $Q_2 = \{2\}$.
   c. Determine the adjacent points to $Q_2$, the adjacent points to $(Q_2\ U\ \{v\})$, and the number of adjacent points $\rho(Q_2\ U\ \{v\})$. The following table shows the adjacent points with $Q_2$.

**Table 3.** the adjacent points to $Q_2$

| $\boldsymbol{v}$ | $\boldsymbol{Q_2\ U}\ \{v\}$ | $\boldsymbol{\rho(Q_2\ U}\ \{v\})$ |
|---|---|---|
| 3 | None | 0 |
| 6 | None | 0 |
| 8 | None | 0 |

   From the table, it is known that the number of points adjacent to $Q_2$ is three points, there is points 3, 6, and 8. This means that the three points can be adjoined with $Q_2$. By choosing point 3, a new clique is obtained which is a combination of $Q_2$ and v = 3, and can be written $Q_{2,3} = Q_2\ U\ \{3\} = \{2, 3\}$.
   d. The search process is complete because there are no more points that are adjacent to the two points, so a complete subgraph with two points is obtained.

2) Second maximum clique
   a. The compatible graph has 6 vertices $V = \{5, 6, 8, 9, 11, 12\}$.
   b. Take an arbitrary starting point, let $i = 5$, to obtain clique $Q_5 = \{5\}$.

c. Determine the adjacent points to $Q_5$, the adjacent points to $(Q_5\ U\ \{v\})$, and the number of adjacent points $\rho(Q_5\ U\ \{v\})$. The following table shows the adjacent points with $Q_5$.

**Table 4.** the adjacent points to $Q_2$

| $v$ | $Q_5\ U\ \{v\}$ | $\rho(Q_5\ U\ \{v\})$ |
|---|---|---|
| 6 | None | 0 |
| 9 | None | 0 |
| 11 | None | 0 |

From the table, it is known that the number of points adjacent to $Q_5$ is three points, there is points 6, 9, and 11. This means that the three points can be adjoined with $Q_5$. By choosing point 6, a new clique is obtained which is a combination of $Q_5$ and v = 6, and can be written $Q_{5,6} = Q_5\ U\ \{6\} = \{5, 6\}$.

d. The search process is complete because there are no more points that are adjacent to the two points, so a complete subgraph with two points is obtained.

3) Third maximum clique

a. The compatible graph has 4 vertices $V = \{8, 9, 11, 12\}$.

b. Take an arbitrary starting point, let $i = 8$, to obtain clique $Q_8 = \{8\}$.

c. Determine the adjacent points to $Q_8$, the adjacent points to $(Q_8\ U\ \{v\})$, and the number of adjacent points $\rho(Q_8\ U\ \{v\})$. The following table shows the adjacent points with $Q_8$.

**Table 5.** the adjacent points to $Q_2$

| $v$ | $Q_8\ U\ \{v\}$ | $\rho(Q_8\ U\ \{v\})$ |
|---|---|---|
| 9 | None | 0 |
| 12 | None | 0 |

From the table, it is known that the number of points adjacent to $Q_8$ is three points, there is points 9 and 12. This means that the three points can be adjoined with $Q_8$. By choosing point 9, a new clique is obtained which is a combination of $Q_8$ and v = 9, and can be written $Q_{8,9} = Q_8\ U\ \{9\} = \{8, 9\}$.

d. The search process is complete because there are no more points that are adjacent to the two points, so a complete subgraph with two points is obtained.

4) Fourth maximum clique

After obtaining cliques $Q_{2,3}$, $Q_{5,6}$ and $Q_{8,9}$ of the compatible graph, then to find a different clique, points 2, 3, 5, 6, 8, and 9 can be ignored. The number of vertices in the compatible graph is left with 2 vertices, $V = \{11, 12\}$, where both vertices are connected by an edge. Thus, it can be confirmed that the two points can be adjoined, so that a complete subgraph is obtained again with two points $Q_{11.12} = \{11, 12\}$.

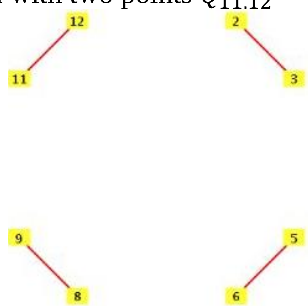


**Figure 5.** Complete Subgraph of Sudirman-Pujowiyoto Intersection

The complete subgraph has four cliques and can be written as a set of points $\{v_2v_3, v_5v_6, v_8v_9, v_{11}v_{12}\}$, where $v_2$ is point 2, $v_3$ is point 3, and so on through $v_{12}$.

Using the assumption of 60 seconds per cycle, the time required for each traffic flow to move is 60 seconds divided by 4 complete subgraphs equal to 15 seconds. Furthermore, because there are 8 flows, the optimal waiting time at the Sudirman-Pujowiyoto Intersection is $15 \times 8 = 120$ seconds.

When compared with the total waiting time applied to the intersection at that time, which was 317 seconds, it was found that the waiting time obtained by using the compatible graph was 37.9% of the existing time. Thus, it is clear that the waiting time is more optimal to be applied to the Sudirman-Pujowiyoto Fourth Intersection.

## 5. Conclussion

The optimal total waiting time at the Sudirman-Pujowiyoto intersection in Purbalingga by using a compatible graph with the assumption that turn left flow does not follow the traffic light is 120 seconds. Meanwhile, the total waiting time of the traffic light applied at the intersection is 317 seconds. It means that the waiting time obtained is 37.9% of the existing waiting time. The total waiting time obtained from the compatible graph calculation is clearly more optimal than the waiting time currently applied at the intersection.

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