Available online at https://journal.rescollacomm.com/index.php/ijqrm/index

International Journal of Quantitative Research and Modeling

e-ISSN 2721-477X
p-ISSN 2722-5046

Vol. 5, No. 3, pp. 262-272, 2024

Premium Sufficiency Reserve of Last Survivor Endowment Life Insurance Using Exponentiated Gumbel Distribution

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Abstract

Life insurance is a protection effort provided by the insurer against risks to the insured's life that will arise from an unpredictable event. Insurance companies are required to prepare reserves to fulfill the sum insured when a claim occurs. Premium sufficiency reserves are modified reserves whose calculations use gross premiums that contain administrative maintenance costs. The purpose of this study is to determine the amount of premium sufficiency reserves of endowment life insurance for two insurance participants aged x years and y years using the exponentiated Gumbel distribution. The parameters of the exponentiated Gumbel distribution are estimated using the maximum likelihood method and then determined by a Newton-Raphson iteration method. The solution of the problem is obtained by determining the initial life annuity term, single premium, and annual premium so as to obtain the reserve formula of the premium sufficiency of the last survivor status endowment life insurance using the exponentiated Gumbel distribution. The results of the calculation of reserves premium sufficiency of endowment life insurance last survivor status using the exponentiated Gumbel distribution is slightly smaller than premium sufficiency reserve for endowment life insurance using the Indonesian Mortality Table 2019.

Keywords: Life insurance, premium sufficiency reserve, last survivor status, exponentiated Gumbel distribution, maximum likelihood method, Newton-Raphson iteration method.

1. Introduction

Human life cannot be separated from the possibility of risks that occur in the future, one way to reduce the possibility of such risks is to participate in a life insurance program. Endowment life insurance is a combination of term insurance and pure endowment insurance where the sum insured must be paid by the insured party during the coverage period until the end of the policy coverage period, whether death or survival occurs (Futami, 1993). Last survivor life insurance is a combined life insurance in which premiums are paid as long as the participants are alive and payments will stop after all members die (Bowers, et al., 1997).

Premium is the amount of money that must be paid by the policyholder to the insurer under the insurance policy during the period of coverage (Dickson, et al., 2009). On the payment method, premiums can be divided into single premiums and annual premiums. Premium payments can be made periodically, namely premiums paid at once are called single premiums and premiums paid annually are called annual premiums (Futami, 1993). If the premium has been paid by the participant, the insurance company has the obligation to keep some of the money in the form of reserves.

Reserves are a reserve amount of money within the insurance period at the insurance company that can be used if at any time unexpected things happen such as claims outside the estimate (Futami, 1993). Based on the calculation method, reserves are divided into two types, namely retrospective reserves and prospective reserves. Prospective reserves are reserve calculations based on subtracting the present value of all future expenses from the present value of total future income for each policyholder (Futami, 1993). One modification of prospective reserves is the premium sufficiency reserve. Premium sufficiency reserves are reserves that are calculated based on gross premium assumptions.

The exponentiated Gumbel distribution was introduced in 2006 which is a modification of the exponential distribution and the Gumbel distribution. The application of the exponentiated Gumbel distribution can be found in various fields such as hydrology, meteorology, climatology, actuarial science, finance and geology. The exponentiated

Gumbel distribution can also be used in the field of insurance to estimate the risk of claims in insurance products. The assumption of the survival function is used in determining the chance of life and the chance of death obtained based on the exponentiated Gumbel distribution expressed in the probability density function.

There are several previous studies on last survivor endowment life insurance premium reserves, namely research by Riaman et. al (2019) regarding the determination of last survivor endowment premium reserves using the Gompertz assumption with retrospective principles where, the results of the reserves obtained are compared with the premium reserves using the 2011 Indonesian mortality table. In the research by Ananda et al. (2023) regarding the calculation of premium reserves from last survivor endowment life insurance using the New Jersey method where the premium reserve results will be compared with premium reserves in the 2019 Indonesian mortality table. Furthermore, research by Hasriati et al. (2023) regarding the calculation of premium reserves from last survivor endowment life insurance using the Zillmer method for 3 cases with the research results obtained, namely in each case, Zillmer reserves are increasing every year.

There are several previous studies on the calculation of premium reserves using the premium sufficiency method, namely research by Riaman et. al (2019) regarding the comparison of the Zillmer method and the premium sufficiency method with The Vasicek interest rate to determine the stochastic rate with the results obtained, namely the calculation of reserves with the Zillmer method produces a smaller value than the premium sufficiency method.

In this article, the premium sufficiency reserve is determined for endowment life insurance using the exponentiated Gumbel distribution function of x years old and years old insurance participants in a policy with sum assured paid by the insurance company to the insurance participants at the end of the agreed contract period.

2. Survival Function and Distribution Exponentiated Gumbel Distribution

The probability density function $f(x)$ is related to the cumulative distribution function denoted by $f(x)$. Suppose X is a continuous random variable, the cumulative distribution function of the continuous random variable X is defined as follows:

Definition 1 [10, h. 90] The cumulative distribution function $F(x)$ of a continuous random variable X with probability density function $f(x)$ is

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)dt \text{ for } -\infty \leq x \leq \infty.$$

In actuarial, the survival function can be interpreted as a life function denoted by $S(x)$ expressing the probability of a person surviving until time x years. Based on Definition 1, the relationship between the survival function and the cumulative distribution function of insurance participants is explained in (Bowers et al., 1997) as follows:

$$S(x) = 1 - F(x). \tag{1}$$

The cumulative distribution function of a continuous random variable $T(x)$ is expressed as follows (Dickson et al., 2009):

$$F_{T(x)}(t) = P(T(x) \leq t), t \geq 0. \tag{2}$$

The function $F_{T(x)}(t)$ is the probability of a person aged x years dying within t years, denoted by ${}_tq_x$ as follows (Bowers et al., 1997):

$${}_tq_x = P(T(x) \leq t), t \geq 0. \tag{3}$$

Based on equation (2) and equation (3), the relationship between the cumulative distribution function and the probability of death is obtained, namely

$$F_{T(x)}(t) = {}_tq_x. \tag{4}$$

The cumulative distribution function of a continuous random variable $T(x)$ can also be expressed as follows (Dickson et al., 2009):

$$F_{T(x)}(t) = \frac{F(x+t) - F(x)}{S(x)}. \quad (5)$$

The survival function of a continuous random variable $T(x)$ denoted by $S_{T(x)}(t)$ can be expressed as follows:

$$S_{T(x)}(t) = 1 - P(T(x) \leq t). \quad (6)$$

The relationship between the survival function of a continuous random variable $T(x)$ and the probability of dying based on equation (3), equation (6) can be expressed as follows:

$$S_{T(x)}(t) = 1 - {}_tq_x. \quad (7)$$

The function $S_{T(x)}(t)$ expressing the probability of a person aged x years surviving until the next t years is expressed as follows:

$$S_{T(x)}(t) = {}_tp_x. \quad (8)$$

Based on equation (7) and equation (8), the probability of life of a person aged x years can survive until the next t years, namely

$${}_tp_x = 1 - {}_tq_x. \quad (9)$$

In joint life insurance, the probability of living and the probability of dying of life insurance participants are expressed in terms of joint status. In this article,the combination is limited to two insurance participants aged x years and y years respectively, which is then expressed as the status of last survivor.

The distribution function for the joint state of the last survivor denoted by $F_{T(\bar{x})T(\bar{Y})}(t,t)$ with $t < 0$ is described in (Bowers et al., 1997), i.e.

$$F_{T(\bar{x})T(\bar{y})}(t,t) = P(T(x) \leq t)P(T(y) \leq t). \quad (10)$$

The life expectancy for the joint life status of participants aged x years and y years is described in (Bowers et al., 1997) as follows:

$${}_tp_{xy} = {}_tp_x \; {}_tp_y. \quad (11)$$

Based on equation (3), equation (9), equation (10) and equation (11) can express the probability of living the joint status last survivor of participants aged x years and y years as follows:

$${}_tp_{\overline{xy}} = {}_tp_x + {}_tp_y - {}_tp_{xy}. \quad (12)$$

In this article, the life expectancy and death expectancy of an insurance participant are determined using the exponentiated Gumbel distribution. The exponentiated Gumbel distribution with two parameters has a probability density function $f(x;\, \alpha, \theta)$ as follows (Qasim & Al-Dubaicy, 2022):

$$f(x;\, \alpha,\theta) = \alpha\theta\left[exp\left(-exp(-\alpha x)\right)\right]^{\theta} exp(-\alpha x); -\infty < x < \infty, \alpha > 0, \theta > 0. \quad (13)$$

where α is the scale parameter and θ is the shape parameter.

There are many ways to estimate parameter values such as the maximum likelihood estimator and the Newton-Raphson method.

Using Definition 1, we obtain the cumulative distribution function and exponentiated Gumbel distribution for an insurance participant aged x years, namely

$$F(x) = exp\left(-\theta\, exp(-\alpha x)\right). \quad (14)$$

Then based on equation (1) and equation (14), we obtain the survival function $S(x)$ from the distribution exponentiated Gumbel distribution, namely

$$S(x) = 1 - exp\big(-\theta\, exp(-\alpha x)\big). \tag{15}$$

Based on equation (14), the cumulative distribution function $F(x + t)$ of the exponentiated Gumbel distribution is obtained as follows:

$$F(x+t) = exp\left(-\theta\, exp\big(-\alpha(x+t)\big)\right). \tag{16}$$

Furthermore, by substituting equation (14), equation (16), and equation (15) into equation (5), we obtain the probability that a person aged x years will die at an interval of t years later with an exponentiated Gumbel distribution, namely

$$F_{T(X)}(t) = \frac{exp\Big(\theta\big[exp(-\alpha x) - exp\big(-\alpha(x+t)\big)\big]\Big) - 1}{exp\big(\theta\, exp(-\alpha x)\big) - 1}. \tag{17}$$

Then by using the equation (17) into the equation (4), the probability of a x year old surviving the next t year interval with an exponentiated Gumbel distribution is obtained, namely

$${}_tq_x = \frac{exp\Big(\theta\big[exp(-\alpha x) - exp\big(-\alpha(x+t)\big)\big]\Big) - 1}{exp\big(\theta\, exp(-\alpha x)\big) - 1}. \tag{18}$$

The parameter value of the exponentiated Gumbel distribution is influenced by the random variable X, in this case the age of the insurance participant. So the parameter values for insurance participants aged x years are denoted by α_x and θ_x, while for insurance participants aged y years are denoted by α_y and θ_y. The probability of death of an insurance participant aged x years using the exponentiated Gumbel distribution in the equation (18), namely

$${}_tq_x = \frac{exp\Big(\theta_x\big[exp(-\alpha_x x) - exp\big(-\alpha_x(x+t)\big)\big]\Big) - 1}{exp\big(\theta_x\, exp(-\alpha_x x)\big) - 1}. \tag{19}$$

The probability that an insurance participant aged x years can survive until the next t years using the exponentiated Gumbel distribution by substituting theequation (19) and the equation (9) namely

$${}_tp_x = 1 - \frac{exp\Big(\theta_x\big[exp(-\alpha_x x) - exp\big(-\alpha_x(x+t)\big)\big]\Big) - 1}{exp\big(\theta_x\, exp(-\alpha_x x)\big) - 1}. \tag{20}$$

Based on the equation (20) the life expectancy of a person aged y years can beexpressed as follows

$${}_tp_y = 1 - \frac{exp\left(\theta_y\left[exp(-\alpha_y y) - exp\left(-\alpha_y(y+t)\right)\right]\right) - 1}{exp\left(\theta_y\, exp(-\alpha_y y)\right) - 1}. \tag{21}$$

The life expectancy of a participant aged x years and y years to the next t yearsat last survivor status using the exponentiated Gumbel distribution is obtained by substituting the equation (20) and equation (21) into equation (12) with the conditions in equation (11), namely

$$\begin{aligned}{}_tp_{\overline{xy}} = 2 &- \frac{exp\Big(\theta_x\big[exp(-\alpha_x x) - exp\big(-\alpha_x(x+t)\big)\big]\Big) - 1}{exp\big(\theta_x\, exp(-\alpha_x x)\big) - 1} \\ &- \frac{exp\left(\theta_y\left[exp(-\alpha_y y) - exp\left(-\alpha_y(y+t)\right)\right]\right) - 1}{exp\left(\theta_y\, exp(-\alpha_y y)\right) - 1} \\ &- \left(\frac{exp\Big(\theta_x\big[exp(-\alpha_x x) - exp\big(-\alpha_x(x+t)\big)\big]\Big) - 1}{exp\big(\theta_x\, exp(-\alpha_x x)\big) - 1}\right) \\ &\cdot \left(\frac{exp\left(\theta_y\left[exp(-\alpha_y y) - exp\left(-\alpha_y(y+t)\right)\right]\right) - 1}{exp\left(\theta_y\, exp(-\alpha_y y)\right) - 1}\right).\end{aligned} \tag{22}$$

3. Early Life Annuities for Last Survivor Term Life Insurance using Exponentiated Gumbel Distribution

A life annuity is a series of payments made with a certain amount continuously paid to the insurance company by the insured while still alive. Life annuities based on their type are divided into two, namely annuities paid at the beginning of the annuity payment period called initial annuities, and annuities paid at the end of theannuity payment period called final annuities (Futami, 1993).

The life annuity calculation is influenced by the interest rate denoted by i. The interest rate used in this thesis is the compound interest rate. In compound interestthere is a function v called the discount factor, which is the present value of a payment of 1 unit of payment made one year later (Futami, 1993), namely

$$v = \frac{1}{1+i}. \tag{23}$$

In addition to the discount factor, there is a function d called the discount ratewhich is the amount of interest lost if the payment is made one year sooner and can be expressed as follows:

$$d = 1 - v. \tag{24}$$

The cash value of an initial life annuity for a person aged x years and y yearswith a coverage period of n years in last survivor status can be expressed as follows:

$$\ddot{a}_{\overline{xy}:\overline{n|}} = \sum_{t=0}^{n-1} v^t \ {}_tp_{\overline{xy}}. \tag{25}$$

The cash value of the initial life annuity for participants aged x years and y years with a premium payment period of m years for $m < n$ last survivor status can be expressed as follows:

$$\ddot{a}_{\overline{xy}:\overline{m|}} = \sum_{t=0}^{m-1} v^t \ {}_tp_{\overline{xy}}. \tag{26}$$

The cash value of the initial term last survivor life annuity for participants aged$x$ years and y years with an insured time of n years using the exponentiated Gumbel distribution is obtained by substituting the equation (22) into the equation (25), namely

$$\begin{aligned}\ddot{a}_{\overline{xy}:\overline{n|}} = \sum_{t=0}^{n-1} v^t \Bigg[2 &- \frac{exp\big(\theta_x\big[exp(-\alpha_x x) - exp\big(-\alpha_x(x+t)\big)\big]\big) - 1}{exp\big(\theta_x\, exp(-\alpha_x x)\big) - 1} \\ &- \frac{exp\Big(\theta_y\Big[exp(-\alpha_y y) - exp\Big(-\alpha_y(y+t)\Big)\Big]\Big) - 1}{exp\Big(\theta_y\, exp(-\alpha_y y)\Big) - 1} \\ &- \left(\frac{exp\big(\theta_x\big[exp(-\alpha_x x) - exp\big(-\alpha_x(x+t)\big)\big]\big) - 1}{exp\big(\theta_x\, exp(-\alpha_x x)\big) - 1}\right) \\ &\cdot \left(\frac{exp\Big(\theta_y\Big[exp(-\alpha_y y) - exp\Big(-\alpha_y(y+t)\Big)\Big]\Big) - 1}{exp\Big(\theta_y\, exp(-\alpha_y y)\Big) - 1}\right)\Bigg].\end{aligned} \tag{27}$$

The right-hand side of equation (27) is denoted by B, so that

$$\ddot{a}_{\overline{xy}:\overline{n|}} = B. \tag{28}$$

Furthermore, the cash value of a $(n - t)$ year term early life annuity based on the equation (27), namely

$$\ddot{a}_{\overline{x+t,y+t:n-t|}} = \sum_{k=0}^{n-t-1} v^k \left[2 - \frac{exp\left(\theta_x\left[exp\left(-\alpha_x(x+t)\right) - exp\left(-\alpha_x(x+t+k)\right)\right]\right)-1}{exp\left(\theta_x\, exp\left(-\alpha_x(x+t)\right)\right)-1} - \frac{exp\left(\theta_y\left[exp\left(-\alpha_y(y+t)\right) - exp\left(-\alpha_y(y+t+k)\right)\right]\right)-1}{exp\left(\theta_y\, exp\left(-\alpha_y(y+t)\right)\right)-1} - \left(\frac{exp\left(\theta_x\left[exp(-\alpha_x x) - exp\left(-\alpha_x(x+t)\right)\right]\right)-1}{exp\left(\theta_x\, exp(-\alpha_x x)\right)-1}\right) \cdot \left(\frac{exp\left(\theta_y\left[exp\left(-\alpha_y(y+t)\right) - exp\left(-\alpha_y(y+t+)\right)\right]\right)-1}{exp\left(\theta_y\, exp\left(-\alpha_y(y+t)\right)\right)-1}\right)\right]. \tag{29}$$

The right-hand side of equation (29) is denoted by C, so that

$$\ddot{a}_{\overline{x+t,y+t:n-t|}} = C. \tag{30}$$

The initial life annuity of last survivor term life insurance for participants aged$x$ years and y years with a premium payment time of m years for $m < n$ based on the equation (26), namely

$$\ddot{a}_{\overline{xy:m|}} = \sum_{t=0}^{m-1} v^t \left[2 - \frac{exp\left(\theta_x\left[exp(-\alpha_x x) - exp\left(-\alpha_x(x+t)\right)\right]\right)-1}{exp\left(\theta_x\, exp(-\alpha_x x)\right)-1} - \frac{exp\left(\theta_y\left[exp(-\alpha_y y) - exp\left(-\alpha_y(y+t)\right)\right]\right)-1}{exp\left(\theta_y\, exp(-\alpha_y y)\right)-1} - \left(\frac{exp\left(\theta_x\left[exp(-\alpha_x x) - exp\left(-\alpha_x(x+t)\right)\right]\right)-1}{exp\left(\theta_x\, exp(-\alpha_x x)\right)-1}\right) \cdot \left(\frac{exp\left(\theta_y\left[exp(-\alpha_y y) - exp\left(-\alpha_y(y+t)\right)\right]\right)-1}{exp\left(\theta_y\, exp(-\alpha_y y)\right)-1}\right)\right]. \tag{31}$$

The right-hand side of the equation (31) is denoted by D, thus

$$\ddot{a}_{\overline{xy:m|}} = D. \tag{32}$$

Furthermore, the cash value of the initial life annuity of the last survivor term with a premium payment period of $(m-t)$ years based on the equation (26) namely

$$\ddot{a}_{\overline{x+t,y+t:m-t|}} = \sum_{k=0}^{m-t-1} v^k \left[2 - \frac{exp\left(\theta_x\left[exp\left(-\alpha_x(x+t)\right) - exp\left(-\alpha_x(x+t+k)\right)\right]\right)-1}{exp\left(\theta_x\, exp\left(-\alpha_x(x+t)\right)\right)-1} - \frac{exp\left(\theta_y\left[exp\left(-\alpha_y(y+t)\right) - exp\left(-\alpha_y(y+t+k)\right)\right]\right)-1}{exp\left(\theta_y\, exp\left(-\alpha_y(y+t)\right)\right)-1} - \left(\frac{exp\left(\theta_x\left[exp(-\alpha_x x) - exp\left(-\alpha_x(x+t)\right)\right]\right)-1}{exp\left(\theta_x\, exp(-\alpha_x x)\right)-1}\right) \cdot \left(\frac{exp\left(\theta_y\left[exp\left(-\alpha_y(y+t)\right) - exp\left(-\alpha_y(y+t+)\right)\right]\right)-1}{exp\left(\theta_y\, exp\left(-\alpha_y(y+t)\right)\right)-1}\right)\right]. \tag{33}$$

The right-hand side of the equation (33) is denoted by E, thus

$$\ddot{a}_{\overline{x+t,y+t:m-t}|} = E. \tag{34}$$

4. Last Survivor Endowment Life Insurance Premium using Exponentiated Gumbel Distribution

Premiums are a series of payments made by insurance participants (insured) to the insurance company (insurer) over a certain period of time with a predetermined payment amount. A single premium is an insurance premium payment that has been agreed at the beginning of the insurance contract and then there is no further payment (Futami, 1993). While the annual premium is the payment of premiums at the beginning of each year, the amount can be the same or different (Futami, 1993).

Single premium last survivor endowment life insurance is a combination of single premium last survivor term life insurance and single premium last survivor whole life insurance. The single premium of last survivor term life insurance of 1 payment unit is expressed as follows:

$$A^{1}_{\overline{xy}:\overline{n}|} = \sum_{t=0}^{n-1} v^{t+1}\,{}_{t|1}q_{\overline{xy}}. \tag{35}$$

The single premium of last survivor pure endowment life insurance with sumassured of n years and sum assured of 1 unit is

$$A_{\overline{xy}:\overline{n}|}^{\;\;1} = v^{n}\,{}_{n}p_{\overline{xy}}. \tag{36}$$

The single premium of last survivor endowment life insurance is denoted by $A_{\overline{xy}:\overline{n}|}$ as follows:

$$A_{\overline{xy}:\overline{n}|} = A^{1}_{\overline{xy}:\overline{n}|} + A_{\overline{xy}:\overline{n}|}^{\;\;1}. \tag{37}$$

The single premium of last survivor endowment life insurance with an insurance period of n years based on equation (24), equation (36), and equation (37) can be expressed as follows:

$$A_{\overline{xy}:\overline{n}|} = 1 - d\ddot{a}_{\overline{xy}:\overline{n}|}. \tag{38}$$

The single premium of last survivor endowment life insurance using the expo- nentiated Gumbel distribution with an insurance period of n years is obtained by substituting the equation (28) into the equation (38) is obtained

$$A_{\overline{xy}:\overline{n}|} = 1 - dB. \tag{39}$$

Based on the equation (38), if the single premium with sum assured of R is paid at the end of the policy year, the amount of the last survivor endowment life insurancepremium will be $RA_{\overline{xy}:\overline{n}|}$.

The single premium of last survivor endowment life insurance using the expo- nentiated Gumbel distribution based on the equation (38) is obtained as follows:

$$A_{\overline{x+t,y+t:n-t}|} = 1 - dC. \tag{40}$$

Based on the equation (38), if the single premium with sum assured of R is paidat the end of the policy year, then the amount of the last survivor endowment life insurance premium for $(n-t)$ years becomes $RA_{\overline{x+t,y+t:n-t}|}$.

The annual premium of last survivor endowment life insurance with a premium payment period of m years is expressed as follows (Futami, 1993):

$$ {}_{m}P_{\overline{xy:n}|} = \frac{A_{\overline{xy:n}|}}{\ddot{a}_{\overline{xy:m}|}}. \tag{41} $$

Annual premium of last survivor endowment life insurance with a premium pay-ment period of m years by substituting equation (39) and equation (32) into equation (41), namely

$$ {}_{m}P_{\overline{xy:n}|} = \frac{1-dB}{D}. \tag{42} $$

5. Premium Sufficiency Reserve of Last Survivor Endowment Life Insurance using Exponentiated Distribution

Premium sufficiency reserve is a reserve calculation that relies on the gross premium assumption. The gross premium is an annuity, but the gross premium has a greater value than the net premium. This is because the determination of the gross premium is influenced by administrative costs. The gross premium of last survivor endowment life insurance is stated as follows (Futami, 1994):

$$ {}_{m}P^{*}_{\overline{xy:n}|} = \frac{1}{1-\beta}\left({}_{m}P_{\overline{xy:n}|} + \frac{\alpha}{\ddot{a}_{\overline{xy:m}|}} + \gamma + \gamma' \frac{\ddot{a}_{\overline{xy:n}|} - \ddot{a}_{\overline{xy:m}|}}{\ddot{a}_{\overline{xy:m}|}}\right). \tag{43} $$

Premium suffciency reserves are a modification of prospective reserves. Prospective reserves denoted by ${}^{m}_{t}V_{\overline{xy:n}|}$ can be expressed as follows (Futami, 1993):

$$ {}^{m}_{t}V_{\overline{xy:n}|} = A_{\overline{x+t,y+t:n-t}|} - {}_{m}P_{\overline{xy:n}|}\ddot{a}_{\overline{x+t,y+t:m-t}|}. \tag{44} $$

In prospective reserves, the calculation of reserves can be obtained from the difference between the present value of future payments denoted by A, and the present value of future receipts denoted by Pa. Therefore, the equation (44) of prospective reserves can be expressed as follows:

$$ {}^{m}_{t}V_{\overline{xy:n}|} = A - Pa. \tag{45} $$

The calculation of premium sufficiency reserves is based on future expenses plus insurance company management costs in the form of agent commission fees for each premium collection, premium maintenance costs during the payment period, and premium maintenance costs after the payment period until the end of the coverage period. So that the present value of future payments based on the prospective reserve method changes to (Futami, 1994):

$$ \begin{aligned} A = {} & A_{\overline{x+t,y+t:n-t}|} \\ & +\beta\, {}_{m}P^{*}_{\overline{xy:n}|}\, \ddot{a}_{\overline{x+t,y+t:m-t}|} + \gamma\, \ddot{a}_{\overline{x+t,y+t:m-t}|} + \gamma'\left(\ddot{a}_{\overline{x+t,y+t:n-t}|} - \ddot{a}_{\overline{x+t,y+t:m-t}|}\right) \end{aligned} \tag{46} $$

Meanwhile, the present value of future revenue uses gross premium instead of net premium. So the present value of future receipts in the equation (44) based on the prospective method changes to (Futami, 1994):

$$ P = {}_{m}\quad {}^{*}_{\overline{xy:n}|}\, \ddot{a}_{\overline{x+t,y+t:m-t}|}. \tag{47} $$

Premium sufficiency reserve denoted by ${}^{m}_{t}V^{ps}_{\overline{xy:n}|}$ with premium payment length m years and reserve calculation time t years as follows (Futami, 1994):

$$ \begin{aligned} {}^{m}_{t}V^{ps}_{\overline{xy:n}|} = {} & A_{\overline{x+t,y+t:n-t}|} \\ & +(-1)\, {}_{m}P^{*}_{\overline{xy:n}|}\, \ddot{a}_{\overline{x+t,y+t:m-t}|} + \gamma\, \ddot{a}_{\overline{x+t,y+t:m-t}|} + \gamma'\left(\ddot{a}_{\overline{x+t,y+t:n-t}|} - \ddot{a}_{\overline{x+t,y+t:m-t}|}\right). \end{aligned} \tag{48} $$

Furthermore, the premium sufficiency reserve using gross premium is obtained by substituting the equation (43) into the equation (48) as follows:

$$
\begin{aligned}
{}_{t}^{m}V_{\overline{xy:n}|}^{ps} = A_{\overline{x+t,y+t:n-t}|} - \left({}_{m}P_{\overline{xy:n}|} + \frac{a'}{\ddot{a}_{\overline{xy:m}|}}\right)\ddot{a}_{\overline{x+t,y+t:m-t}|} \\
+ \gamma'\left(\ddot{a}_{\overline{x+t,y+t:n-t}|} - \frac{\ddot{a}_{\overline{xy:n}|}}{\ddot{a}_{\overline{xy:m}|}}\ddot{a}_{\overline{x+t,y+t:m-t}|}\right).
\end{aligned} \tag{49}
$$

The premium sufficiency reserve of last survivor endowment life insurance with annual premium for m years paid in advance at time t years using exponentiated Gumbel distribution with sum assured R is expressed as follows:

$$
{}_{t}^{m}V_{\overline{xy:n}|}^{ps} = R(1 - d\,C) - R\left(\frac{R(1-d\,B) + a'}{D}\right)E + \gamma'\left(C - \frac{B}{D}E\right). \tag{50}
$$

Example: A couple aged 35 years and 30 years participated in a last survivor endowment life insurance with a coverage period of 20 years and a premium payment period of 18 years. The sum insured that the beneficiary will receive is $Rp100.000.000$ with an interest rate of 5%. The insurance participant is required to pay a premium that has been subject to a new closing fee denoted by a' of $0,8\%$ of the sum insured and an insurance company management fee denoted by $\gamma'$of 6% of the sum insured. Determine:

(i) Premium sufficiency reserve of last survivor endowment life insurance using exponentiated Gumbel distribution.

(ii) Premium sufficiency reserve for last survivor endowment life insurance using the 2019 Indonesian Mortality Table.

Based on the problem, $x = 35$, $y = 30$, $n = 20$, $m = 18$, $i = 0,05$, $a' = 0,008$, $\gamma' = 0,06$ and $R = 100.000.000$ are known. Based on equation (23), the discount factor is obtained, namely $v = 0,9523809524$ and the discount rate based on equation (24), namely $d = 0,0476190476$. Before calculating the reserves, first determine the estimated values of the parameters of the exponentiated Gumbel distribution, with the help of Maple 13 software obtained the values of $\alpha_x = 0,0442979158$, $\alpha_y = 0,0433937037$, $\theta_x = 15,5703650000$, and $\theta_y = 12,3234240800$. Furthermore, the calculation at $t = 1$ for each case is carried out as follows:

(i) Premium sufficiency reserve of last survivor endowment life insurance using exponentiated Gumbel distribution.

The premium sufficiency reserve of last survivor endowment life insurance using the exponentiated Gumbel distribution at the time of the first year of the contract based on the equation (50) is obtained

$$
{}_{1}^{18}V_{\overline{35,30:20}|}^{ps} = Rp3.222.121{,}774
$$

(ii) Premium sufficiency reserve for last survivor endowment life insurance using the 2019 Indonesian Mortality Table

Premium sufficiency reserves for last survivor endowment life insurance using the 2019 Indonesian Mortality Table at the time of the first year of the contract basedon the equation (50) is obtained

$$
{}_{1}^{18}V_{\overline{35,30:20}|}^{ps} = Rp3.224.176{,}364
$$

The complete calculation of the premium sufficiency reserve for last survivorendowment life insurance using the exponentiated Gumbel distribution and the 2019 Indonesian Mortality Table for a couple aged 35 years and 30 years in year t ispresented in Table 1 and Figure 1.

Table 1: Premium sufficiency reserves using the exponentiated Gumbeldistribution and the 2019 Indonesian Mortality Table

Year t	Exponentiated Gumbel Distribution	2019 Indonesian Mortality Table
	${}_{1}^{18}V_{\overline{35,30:20}|}^{ps}$ (1)	${}_{1}^{18}V_{\overline{35,30:20}|}^{ps}$ (2)
1	3.222.121,82	3.224.176,36
2	6.600.086,35	6.609.561,13
3	10.142.322,12	10.164.213,80

4	13.857.883,79	13.896.598,11
5	17.756.482,20	17.815.600,02
6	21.848.506,53	21.930.550,09
7	26.145.039,59	26.251.245,37
8	30.657.860,71	30.787.972,58
9	35.399.439,82	35.551.532,83
10	40.382.914,62	40.553.266,93

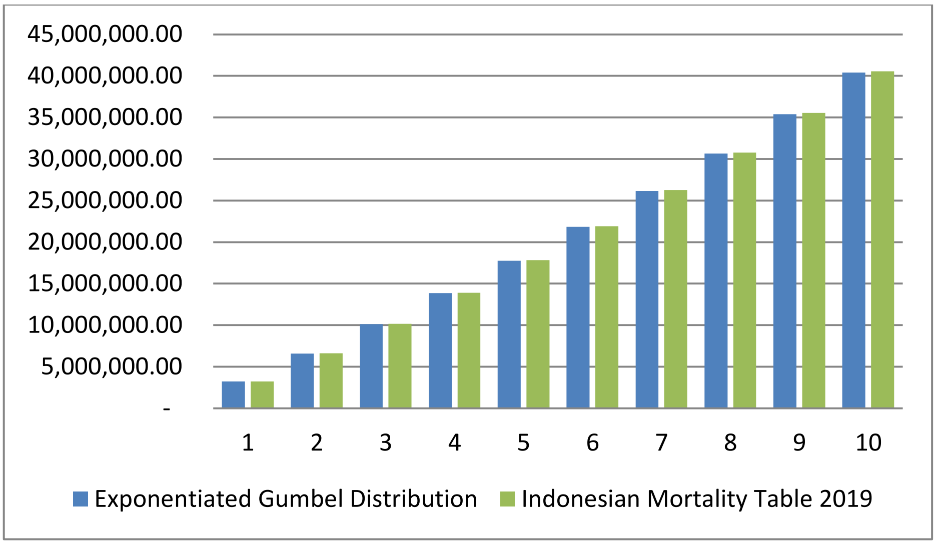

Figure 1: Premium sufficiency reserves using the exponentiated Gumbel distribution and the 2019 Indonesian Mortality Table

Based on the illustration of Figure 1 and Table 1, it is obtained that the amount of premium sufficiency reserve of last survivor endowment life insurance using the exponentiated Gumbel distribution is slightly smaller than the amount of premium sufficiency reserve of last survivor endowment life insurance using the Indonesian Mortality Table in 2019. The results obtained in Figure 1 only differ by tens of thousands so that the difference is not too visible. However, the difference is slightly visible in Figure 1 starting from the 7th year onwards.

6. Conclussion

Based on the discussion, it is found that the last survivor endowment life insurance premium reserve using the exponentiated Gumbel distribution produces a slightly smaller value than the last survivor endowment life insurance premium reserve using the 2019 Indonesian Mortality Table. This is because the probability of life in the exponentiated Gumbel distribution provides a lower value because it is influenced by the parameters α and θ that make up the distribution. In comparison, in the 2019 Indonesian Mortality Table, the value of life expectancy is higher so that the premium reserves obtained tend to be greater.

Furthermore, the amount of premium sufficiency reserves for last survivor endowment life insurance obtained using the exponentiated Gumbel distribution and the Indonesian Mortality Table in 2019 increases every year and at the end of the coverage period produces the same reserve value. In other words, at the end of the coverage period, the company has adequate reserves to ensure that the promised sum insured can be fully given to the policyholder in accordance with the agreement that has been made. This indicates that the premium reserve calculation is effective in maintaining the company's financial stability and ensuring appropriate protection for policyholders.

Acknowledgments

We would like to thank Dr. Haposan Sirait, M.Si. for guiding and providing direction in writing this article.

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