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Model for Determining Earth's Gravitational Acceleration on a Mathematical Pendulum

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Abstract

Gravity is an accelerating property of the earth that causes objects to fall freely. The acceleration of gravity is not the same at every place on the Earth's surface. To measure the Earth's gravity (small g), scientists can use various techniques, such as dropping a mass from a certain height and measuring the time it takes to fall to the ground or using a mathematical pendulum to measure the period of oscillation and use it to calculate the acceleration due to gravity. In this paper, a study of the mathematical pendulum in the measurement of the Earth's gravitational acceleration is conducted, and the measurement experiment is illustrated. Method To measure the length of the pendulum, you must have a ruler, meter stick, or tape measure. At the top end of the string, start the measurement at the point where the string rotates out of place. Then, measure up to the center of the pendulum, which is the object hanging on the string. From the results of this study, it can be concluded that the value of the period of a pendulum is affected by several factors, including the length of the rope used and the angle of initial deviation, while the factors that do not affect the period are the mass and diameter of the pendulum.

Keywords: Acceleration, Earth's gravity, falling objects, mathematical pendulum, influential factors.

1. Introduction

Materials as a cornerstone of classical mechanics. This concept is closely associated with the work of Sir Isaac Newton, a pivotal figure in the history of science. Newton's exploration of gravitational forces and acceleration reportedly began with a simple yet profound observation (Kumar et al., 2024), the fall of a fruit, commonly believed to be an apple. This observation sparked Newton's curiosity, leading him to develop the universal law of gravitation, which describes how every mass exerts an attractive force on every other mass.

Newton's insight into gravitational acceleration was revolutionary. He proposed that the force of gravity causes objects to accelerate towards the Earth at a specific rate (Chaichian et al., 2021; Kembhavi et al., 2020), which he initially estimated through observations and mathematical reasoning. Newton's work laid the foundation for understanding that this value can vary depending on the geographical location due to differences in altitude, latitude, and local geological conditions. Despite the regional variations in gravitational acceleration, an average value is widely accepted for general calculations. This standard value is approximately 9.831302275 m/s^2 (Sebastian et al., 2023). The precision of this value reflects the advancements in measurement techniques and the importance of gravitational acceleration in scientific calculations, engineering, and various technological applications.

Gravitational acceleration is the acceleration experienced by an object when it falls freely to the surface of the earth from a certain height. Based on existing literature, the average value of gravitational acceleration is 9.8 m/s^2 (Yuningsih, 2021). The direction of gravitational acceleration is towards the center of the earth or perpendicular to the earth's surface. From the quote, gravitational acceleration is the gravitational acceleration experienced by an object on the earth's surface because it experiences a gravitational force towards the earth's core, and this gravitational force is equal to or smaller than the mass of each object that falls to the earth, we can conclude that it appears in shape. A force acts on it, so it becomes a surface. Pull them up to the earth's core.

Gravity is the relationship between objects that have mass in the form of an attractive force. The amount of gravity is influenced by the mass and distance of an object (Jarvis, 2020). The relationship between the two is the product of the two masses directly proportional, while the distance is inversely proportional to the square of gravity. Changes in gravity can also be calculated using Newton's gravity equation to calculate the period of a pendulum swing event. The pendulum swing phenomenon is closely related to the Earth's gravity. In addition, the longer the pendulum swing, the slower the pendulum swing will be due to the influence of damping forces (air resistance) (Wang et al., 2022).

There are three factors that affect the difference in gravitational acceleration. First, our Earth is actually not round (Micheli et al., 2018), and the acceleration of gravity depends on the distance from the center of the Earth Second, the acceleration of gravity depends on the distance to the surface. And third, the density of the Earth's mass is different.

The characteristic of a simple arithmetic pendulum is that it does not deviate much. A simple pendulum moves back and forth when it deviates from its equilibrium point as far as the center of the pendulum. Earth's gravitational acceleration is the acceleration experienced by an object due to the influence of the earth's gravitational field on the center of the earth.

2. Materials and Methods

This research is a literature study. The approach taken in this research is a literature study of previous research results related to Determining Earth's Gravity on a Mathematical Pendulum (Yuningsih et al., 2020). Through research with related literature study methods, it is hoped that this article can be developed and become a reference for subsequent authors.

3. Results and Discussion

Gravitational acceleration g is the acceleration that an object experiences due to its own weight. The weight of an object is the Earth's gravitational pull on it. This force is gravity, or the force of attraction between two masses. Because the Earth is not round, the magnitude of g is not the same across the Earth's surface (Kidd et al., 2017). Newton's law of gravitation. Formulated as:

$$F = G\frac{m_1 m_2}{r^2}, \tag{1}$$

with
F : Attractive force between masses m 1 and m 2,
m_1: mass of the first object,
m_2: mass of the second object,
r: the distance between its two centers of mass,
G: gravitational constant.

Another form of oscillatory motion is simple pendulum motion. The pendulum string has weights and weights attached to its bottom. The upper end of the string is tied to a fixed place (Slisko and Cruz, 2019). The weight is suspended freely and moves back and forth under the influence of gravity.

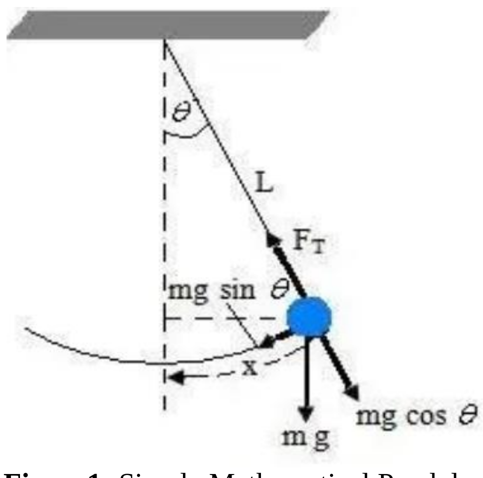

Figure1: Simple Mathematical Pendulum

Due to the simple mathematical nature of the pendulum, the deviation should not be too large. If the deviation is very large, the force acting on the body is no longer directly proportional to the deviation. Only for the small deviation in Figure 1, the force is directly proportional to the deviation. The force that pulls an object to an equilibrium position (the force tangent to the object's trajectory) can be formulated as follows:

$$F = -mg.\sin\theta, \tag{2}$$

with :
F : force (Newton),
m : mass (kg),
g : Gravitational acceleration ($\mathrm{m/s^2}$),
θ : deviation (degree).

The negative sign of the angle θ indicates that the direction of the restoring force is opposite to the small declination angle θ, so that $\sin\theta = \theta = \mathrm{s/L}$ (S is the arc) is the trajectory of the object and L is the length of the string. Since the mass is zero, the mass of the system is assumed to be only the weight of the pendulum. The pendulum is then deflected from its equilibrium point by applying a small deflection angle of θ. This is very important for estimation purposes (Ranreng, 1984).

So the equation becomes:

$$\mathrm{F} = -\frac{mg}{L}\ \mathrm{S}, \tag{3}$$

One way to determine the acceleration of gravity is to use the mathematical swing method. Mathematical swing here refers to the swinging motion of a simple pendulum. Vibration is the periodic motion that occurs in a system when the system moves from its equilibrium position. The period of oscillatory motion can be determined using Equation 1.

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{4}$$

The equation for determining the earth's gravity by the mathematical swing method is obtained by changing the form of Equation 1 as follows:

$$\mathrm{g} = \frac{4\pi 2\ \mathrm{L}}{T2} \tag{5}$$

equation 2 is linear with graph gradient m.

$$\mathrm{m} = \frac{4\pi 2}{T2} \tag{6}$$

Therefore, if we can measure L and T, we can determine the magnitude of the gravitational acceleration at a given location. To get accurate data, you must meet the following requirements: The hanging rope must be lighter than the mass of the object and the deviation must be small. (θ) must be small.

4. Trial Illustration

As an illustration, a simple experiment was conducted to measure the acceleration of the Earth's gravity using a mathematical pendulum, the measurement results of which are given in Table 1.

Table 1: Experimental Results of Earth Gravity Measurement

No	Length of String (cm) l	Number of Oscillations n	Time (seconds) t	Period (seconds) T	Frequency (Hz) f	T^2	g(m/s²)
1	30	10	11.55	1.15	1/1.15	1.322	8.7
2	40	10	13.36	1.33	1/1.33	1.77	8.95
3	50	10	14.80	1.48	1/1.48	2.19	8.73
4	30	10	16.14	1.61	1/1.61	2.6	8.82
						$\sum g$	35.24
						Average g	8.81 m/s²

Table 1 presents the experimental results of Earth gravity measurements using a mathematical pendulum. The experiment was conducted with four different string lengths: 30 cm, 40 cm, 50 cm, and 60 cm. For each length, the pendulum was set to oscillate 10 times, and the total time for these oscillations was recorded. The period of oscillation was then calculated by dividing the total time by the number of oscillations.

For the 30 cm string, the 10 oscillations took 11.55 seconds, resulting in a period of 1.15 seconds. The 40 cm string had a total time of 13.36 seconds and a period of 1.33 seconds. The 50 cm string oscillated for 14.80 seconds with a period of 1.48 seconds. Finally, the 60 cm string took 16.14 seconds for 10 oscillations, giving a period of 1.61 seconds.

Using these measurements, the gravitational acceleration (g) was calculated for each trial. The results were 8.7 m/s² for the 30 cm string, 8.95 m/s² for the 40 cm string, 8.73 m/s² for the 50 cm string, and 8.82 m/s² for the 60 cm string. The average of these four measurements gives a final experimental value for Earth's gravitational acceleration of 8.81 m/s².

Calculation:

a. Finding the Period

$$T = \frac{t}{n}$$

$$T = \frac{11.55}{10} = 1.155$$

$$T = \frac{13.36}{10} = 1.336$$

$$T = \frac{14.80}{10} = 1.480$$

$$T = \frac{16.14}{10} = 1.614$$

Searching for gravity

$$T = 2\pi\sqrt{\frac{t}{g}} \, or \, T^2 = 2^2\pi^2 \, \frac{1}{g}$$

$$1.32 = 4 \times \, 9.56 \, \frac{0.3}{g}$$

$$g = \frac{4 \times 9.56 \times 0.3}{1.32} \, 8.70 \, m/s^2$$

$$g = \frac{4 \times 9.56 \times 0.4}{1.77} \, 8.95 \, m/s^2$$

$$g = \frac{4 \times 9.56 \times 0.5}{2.19} \, 8.73 \, m/s^2$$

$$g = \frac{4 \times 9.56 \times 0.6}{2.6} \, 8.82 \, m/s^2$$

$$\sum g = 8.70 + 8.95 + 8.73 + 8.82 = 35.24 \, m/s^2$$

$$\text{Average g} \, \frac{35.24}{4} = 8.81 \, m/s^2$$

Thus, based on observations, it can be said that the Earth's gravitational force at the location where the experiment was conducted is 8.81 m/s^2.

5. Conclusion

Gravitational acceleration g is the acceleration that an object experiences due to its own weight. The weight of an object is the gravitational pull of the earth on it. Due to the simple mathematical nature of the pendulum, the deviation should not be too large. If the deviation is very large then the force acting on the object is no longer directly proportional to the deviation.

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