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Determining the Pure Premium at Jasa Raharja Insurance Company Purwakarta Branch using Fast Fourir Transform (FFT) through Estimated Aggregate Loss Distribution

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Abstract

Insurance is a contractual agreement between two parties, namely the insured party (customer) and the insurer (insurance company), in which the insured party pays a premium to the insurer, then in return, the insurer will provide compensation (claim) to the insured party if an insured event occurs. Each customer is required to pay a premium as an obligation stated in the insurance agreement by paying a premium, the customer fulfills his obligations and is entitled to the benefits stated in the policy. Therefore, the Insurance Company needs to carry out a scheme in the process of paying pure premiums for the sustainability of the insurance company. When determining the premium, it is done by estimating the aggregate loss distribution. This research will calculate thepure premium at the Purwakarta Branch of Jasa Raharja Insurance Company. The model used in this study is the distribution of aggregate loss with a compound distribution of claim frequency and claim size. Many claims follow the Poisson distribution and large claims follow the Lognormal distribution. In the process of estimating the probability of aggregate loss with the compound distribution model, the Inverse method with the Fast Fourier Transform (FFT) algorithm is used. This research will provide understanding and insight to insurance companies in determining the amount of premium that must be charged to customers.

Keywords: Premium, Compound Distribution, Aggregate Loss, Fast Fourier Transform (FFT)

1. Introduction

Insurance is a contractual agreement between two parties, namely the insured party (customer) and the insurer (insurance company), in which the insured party pays a premium to the insurer, then in return, the insurer will provide compensation (claim) to the insured party if an insured event occurs. In general, insurance products can be categorized into conventional insurance, where insurance payments are directly calculated from actual losses suffered by policyholders, and index-based insurance or also known as parametric insurance, where payments are based on a number of predetermined indices (Zhang, 2024). Insurance companies have concentrated market power to determine the price and design of insurance products, to set contract terms and to decide to whom and when to pay claims (Feng, Liu, & Zhang, 2024). Insurance plays an important role in achieving sustainable growth by protecting individuals and families from falling into poverty in the event of losses caused by insured risks (Aboul Ela, 2024).

The insurance market plays an important role in this by developing a claims settlement system (determining the size of the damage and the amount of compensation to be applied) and by directing funds to preventive measures (Pauch, Bera, & Walczak, 2024). Insurance companies should reduce the debt ratio as much as possible and maintain an optimal capital structure. Meanwhile, the positive relationship between the claims ratio and financial distress indicates that a high claims burden compared to the premium received increases the likelihood of financial distress (Kebede, Tesfaye, & Erana, 2024). Then insurance companies should place more emphasis on corporate responsibility, greater disclosure, and better data for risk quantification in the insurance process (Otavova, Glaserova, & Hasikova, 2023).

The premium calculation process in insurance companies is no less important, considering that the premium problem is very important for insurance companies in managing the company's finances or assets, where the company needs to determine the right premium charged to the insured (insured) and the size of the company's reserves in handling claims. In the process of estimating these two things for a certain period, the company needs to have data on claims (losses) for the previous period submitted by the insured to the company. Based on the claims data, the company can calculate the estimated distribution of total claims for the next period as the basis for determining the pure premium. This total loss is called the aggregate loss (Manurung & Mananohas, 2016). One estimation of aggregate loss can use the Inverse method. The purpose of the inverse method is to obtain the numerical distribution of the characteristic function. The inverse method consists of two algorithms, namely Fast Fourier Transform and Direct Numerical Inversion. However, in this research, the algorithm used in estimating the aggregate loss distribution is the Fast Fourier Transform (FFT).

There are studies that discuss pure premiums such as those conducted by Putra et. al. calculating motor vehicle insurance premiums with the aim of determining variables that can affect the amount of pure premiums using mixed distributions in determining the amount of premiums through Generalized Linear Models (GLM) and determining the appropriate premium pricing model based on the variables that influence it (Putra, Lesmana, & Purnaba, 2021). Rahma and Mutaqin applied the limited-fluctuation credibility method in predicting pure premiums based on motor vehicle insurance data (Rahmah & Mutaqin, 2021). Then Manurung and Mananohas (2016) conducted simulations in determining the estimated aggregate loss distribution to obtain pure premiums.

This research will calculate the net premium through the estimated distribution of aggregate loss in the insurance company PT Jasa Raharja (Persero) Purwakarta Representative with data on the number of claims and the amount of claims. In determining the estimated distribution of aggregate loss, and obtaining expectations (net premium), and obtaining the standard deviation of aggregate loss using the Fast Fourier Transform (FFT) method with the help of phyton software. This research will provide understanding and awareness to the Insurance Company PT Jasa Raharja (Persero) Purwakarta Branch in managing pure premiums in the future.

2. Materials and Methods

2.1. Compound Distribution

The compound distribution is a probability distribution that describes the total amount of loss over a certain period of time by considering two main elements, namely the number of claims which are usually modeled using discrete distributions, such as Poisson, Binomial, or Negative Binomial. While the magnitude of loss per occurrence (Claim Severity) is modeled using a continuous distribution such as Exponential, Gamma, or Normal.

The formulation of the Compound Distribution can be written as follows:

$$Pr\ (S = k) = g_k = \sum_{n=0}^{\infty} P_n\, f_k^{*n} \tag{1}$$

where $f_k^{*n}, k = 0,1,\dots,n$. The n fold convolution of f_k with k = 0,1, … , n which is the sum of the probability of n of identically distributed and mutually independent random variables with probability function f_k. with the following probability function.

2.2. Aggregate Loss Model

Aggregate loss is the total policyholder loss that must be borne by the insurance company in a certain period of time. The method used to obtain aggregate loss is to record each large claim and add up all the claims. Random variables S states the aggregate loss and the random variable N expresses the number of claims in one period of a portfolio. The amount of each claim can be expressed in random variables $X_1, X_2, \dots, X_N$ So that a collective risk model is obtained which is expressed by.

$$S = X_1 + X_2 + X_2 + \cdots, +X_N;\ \ N = 1,2,3\dots \tag{2}$$

According to Cartesian Number of claims that have a probability mass function $\Pr(N = n)$ with mean $\mathrm{E}(N)$ and variance $\mathrm{Var}(N)$ is expressed by random variable N (Kartikasari, 2017). Severity of claims with mean $\mathrm{E}(N)$ and variance $\mathrm{Var}(N)$ expressed by random variables $X_1, X_2, \dots, X_N$. The assumptions that must be considered in the aggregate loss for the collective risk model are as follows:

a. Given $N = n$ random variables $X_1, X_2, \dots, X_N$ is an identically distributed and mutually independent random variable.
b. Given $N = n$ joint distribution of random variables $X_1, X_2, \dots, X_N$ independent of the value n.
c. Distribution of a random variable N does not depend on the values of the random variable $XX_1, X_2, \dots, X_N$.

2.3. Model Compound for Aggregate Loss

The Compound Model for Aggregate Loss is a probability approach used to model the total loss in an insurance portfolio or risk over a period. This model, often used in actuarial to determine pure premiums, which are premiums that include expected losses without the addition of operating costs, profits, or safety margins. The Compound Aggregate Loss process involves two main components, namely claim frequency and claim severity, to calculate the expected aggregate loss.

Suppose S denotes aggregate loss and satisfies the assumptions of (1), then the random variable S has the following distribution function:

$$f_S(x) = \sum_{n=0}^{\infty} p_n f_x^{*n}(x) \tag{3}$$

Equation (2) is used to calculate the aggregate loss probability. If the equation is used directly to calculate the aggregate loss probability function, it will be inefficient and very complicated, especially for large interval sizes k size. It is necessary to use another method in solving it, namely Fast Fourier Transform.

2.4. Fast Fourier Transform

Fast Fourier Transform (FFT) is an algorithm that can be used to calculate the inverse of the characteristic function to obtain the probability function of a discrete random variable. In theory, the FFT is the discrete form of the Fourier transform or characteristic function. If the characteristic function maps a continuous probability density function to a continuous function of complex values, then the FFT maps a vector of probability values of size n to a vector of complex number probability values of size n. The FFT is a one-on-one mapping or function of n points to n point.

FFT can be applied to evaluate certain quantities of interest in classical risk theory. The FFT can be used to evaluate the probability of ultimate ruin and quantiles of the distribution of aggregate claim amounts. The approach used to evaluate the probability of ultimate ruin can be further extended to evaluate the first moment of time of ruin in classical risk models.

Definition: Suppose f_x is a periodic function with period size n defined for all values of x non-negative integers ie, $f_{x+n} = f_x$ for all values x in the vector $f_0, f_1, f_2, \dots, f_{n-1}$, the discrete Fourier transform is a mapping $\tilde{f}_x, x = 0,1,2, \dots, n$ defined by

$$\tilde{f}_k = \sum_{j=0}^{n-1} f_j \exp\left(\frac{2\pi i}{n} jk\right), k = 0,1,2, \dots, n-1 \tag{4}$$

This mapping is bijective (one-one mapping) $\tilde{f}_k$ periodic with period size n. The inverse mapping is:

$$f_j = \frac{1}{n} \sum_{j=0}^{n-1} \tilde{f}_k \exp\left(-\frac{2\pi i}{n} kj\right), j = 0,1,2 \dots, n-1 \tag{5}$$

2.5. Algorithm Fast Fourier Transform in Calculating Aggregate Loss Distribution

In determining the Aggregate loss distribution, FFT is used to invert the characteristic function when discretizing the severity distribution. In the previous discussion, the Aggregate loss model was obtained as follows:

$$\mathrm{S} = X_1 + X_2 + X_2 + \cdots + X_N, \tag{6}$$

with the following probability function

$$f_S(x) = \sum_{n=0}^{\infty} p_n f_x^{*n}(x) \tag{7}$$

so the characteristic function can be determined as follows:

$$\begin{aligned} \varphi_S(t) &= E\left[e^{itS}\right] \\ &= E_N\left[E\left(e^{it(X_1+\cdots+X_N)} \mid N\right)\right] \\ &= E_N[\varphi_X(t)]^N \\ &= P_N[\varphi_X(t)] \end{aligned} \tag{8}$$

where P_N is the probability generating function N.

2.6. Data

The data used in this study are data on the number of claims and the amount of insurance with the distribution of many claims is Poisson distributed and for the distribution of large claims is Lognormal. This data is data on the number of claims and the amount of claims from the insurance company PT Jasa Raharja Purwakarta Branch from 2018 to 2020, the following data is used in **Table 1** as follows:

Table 1. Number of claims and amount of insurance claims of PT Jasa Raharja (Persero) Purwakarta

Month	Number of Claims	Amount of Claim
Jan 2018	111	2,216,679,464
Feb 2018	130	1,794,333,023
Mar 2018	134	1,922,520,382
Apr 2018	150	1,785,299,831
May 2018	103	2,149,836,245
June 2018	46	1,413,073,162
Jul 2018	104	2,399,922,689
Aug 2018	104	2,056,460,639
Sep 2018	192	2,491,872,852
Oct 2018	192	2,491,872,852
Nov 2018	157	2,247,211,358
Dec 2018	157	2,247,211,358
Jan 2019	108	2,252,319,425
Feb 2019	147	1,594,981,275
Mar 2019	90	2,191,961,899
Apr 2019	77	2,081,403,928
May 2019	70	2,180,864,762
June 2019	80	2,438,997,146
Jul 2019	114	2,397,146,892
Aug 2019	162	3,017,124,151
Sep 2019	112	2,395,009,492
Oct 2019	106	2,344,740,258
Nov 2019	105	1,935,616,654
Dec 2019	104	1,621,825,729
Jan 2020	86	1,599,146,007

3. Results and Discussion

3.1. Calculation of Estimated Aggregate Loss Distribution

The aggregate loss probability can be obtained as a random variable S where S is the sum of all claim sizes or can be written in equation (6) called the Collective Risk model with the distribution of random variables N which states many claims is Poisson and the distribution of the random variable X which expresses the size of individual claims is Lognormal, so that the random variable S which states the aggregate loss has a compound Poisson- Lognormal distribution.

Calculation of the Frequency Distribution of claims at the Purwakarta Representative of PT Jasa Raharja (Persero) Insurance Company based on the average Poisson Distribution:

$$\lambda = \frac{\sum_{i=1}^{T} N_i}{T} = \frac{4036}{36} = 112$$

Furthermore, the calculation of the claim size distribution parameter is as follows:

$$\mu = \frac{1}{n}\sum_{i=1}^{T} ln(X_i) = 21.4606$$

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{T} (\ln(X_i) - \mu)^2} = 0.1707$$

The following are the results of calculations to determine the chances of Aggregate Loss at the Purwakarta Representative of PT Jasa Raharja (Persero) Insurance Company, with the help of Python software as follows:

```
=== Aggregate Loss Opportunity ===
Amount of Claim: Rp 0.00, Opportunities: 0.000927
Amount of Claim: Rp 302,014,429.53, Opportunities: 0.000247
Amount of Claim: Rp 604,028,859.06, Opportunities: 0.000000
Amount of Claim: Rp 906,043,288.59, Opportunities: 0.000000
Amount of Claim: Rp 1,208,057,718.12, Opportunities: 0.000000
Amount of Claim: Rp 1,510,072,147.65, Opportunities: 0.000022
Amount of Claim: Rp 1,812,086,577.18, Opportunities: 0.000607
Amount of Claim: Rp 2,114,101,006.71, Opportunities: 0.002549
Amount of Claim: Rp 2,416,115,436.24, Opportunities: 0.003436
Amount of Claim: Rp 2,718,129,865.77, Opportunities: 0.002260
```

Figure 1. Aggregate Loss Opportunity

Then the following is the Aggregate Distribution of Loss at the Purwakarta Representative of PT Jasa Raharja (Persero) Insurance Company, with the help of Python software as follows:

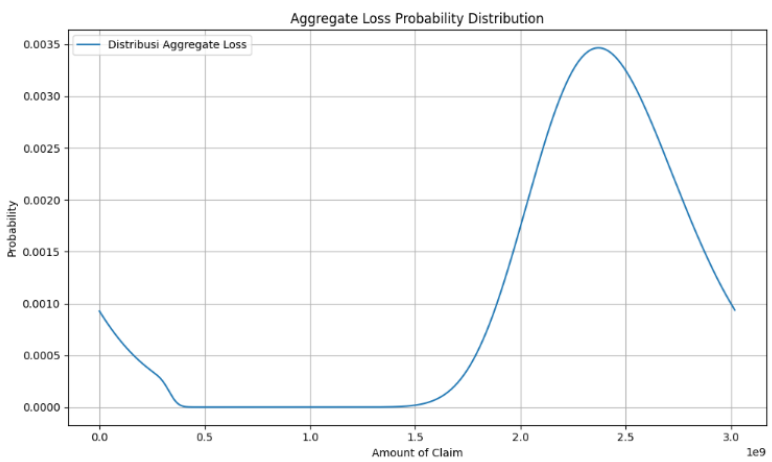

Figure 2. Aggregate Probability Distribution of Loss of Insurance Company PT Jasa Raharja (Persero) Purwakarta Representative

3.2. Calculation of Pure Premium and Standard Deviation of Aggregate Loss

After calculating the probability and distribution of aggregate loss, then the calculation of pure premium and standard deviation at the Insurance Company PT Jasa Raharja (Persero) Purwakarta Representative. Based on these results, the estimated expectation value (pure premium/pure premium) and the standard deviation of aggregate loss can be determined. The pure premium or aggregate loss expectation is as follows:

$$E(S) = \int S.f_s(S)\, dS = 2{,}265{,}558{,}872.93.$$

Then, the variance and standard deviation of aggregate loss are as follows:

$$Var(S) = E[S^2] - (E[S])^2 = 367{,}957{,}445{,}744{,}524{,}672.00$$

$$Standard\ Deviation(S) = \sqrt{Var(S)} = 606{,}594{,}960.20$$

Thus the value of the pure premium or expected aggregate loss is equal to 2,265,558,872.93, with a variance value of 367,957,445,744,524,672.00 and standard deviation of 606,594,960.20.

4. Conclussion

This research is a study that discusses the calculation of the value of pure premium in the insurance company PT Jasa Raharja (Persero) Purwakarta Branch in 2018 to 2020 with data on the number of claims and the amount of claims. This research uses the Fast Fourier Transform (FFT) method in determining the estimated distribution of aggregate loss and also obtaining the expectation (net premium), as well as the standard deviation of aggregate loss. Calculation of estimated aggregate loss distribution using FFT method with the help of Phyton software. The results showed that the net premium or expected aggregate loss value was 2,265,558,872.93. Then the variance value of this study is 367,957,445,744,524,672.00, with a new deviation value of 606,594,960.20. Research provides understanding and insight into the insurance company PT. Jasa Raharja (Persero) Purwakarta Branch, that the pure premium value in 2018 to 2020. So that it can become a reference for the Purwakarta Branch of PT Jasa Raharja (Persero) Insurance company in managing pure premiums in the future.

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