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Markov Chain Method for Calculating Insurance Premiums

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Abstract

This study applies the Markov Chain method to calculate insurance premiums based on the dynamic health status of policyholders over time. The model considers three health states Healthy, Mild Illness, and Severe Illness each associated with a specific insurance premium. The transition probabilities between these states are represented in a transition matrix, capturing the likelihood of a policyholder remaining in their current health state or transitioning to another state in a given period. Using this approach, the steady-state distribution, which reflects the long-term probabilities of being in each health state, is calculated. This distribution is then used to determine the expected monthly premium by taking a weighted average of the premiums for each state. The methodology incorporates real-world scenarios where a policyholder's health condition may change over time, impacting the premiums they are required to pay. The Markov Chain model provides an effective framework for estimating these premiums by considering the "memoryless" nature of health state transitions, where future states depend only on the current state and not on prior health history. By solving the steady-state equations $\pi P = \pi$ and ensuring the total probabilities sum to one, the model yields a robust estimation of long-term health state distributions. These distributions, combined with the associated premiums, produce an accurate calculation of expected insurance costs. The results demonstrate the flexibility and accuracy of the Markov Chain method in assessing risks and setting premiums. Insurers benefit from this approach as it enables dynamic pricing strategies tailored to individual risk profiles. For policyholders, the model provides transparency in understanding how health status influences premiums. Overall, this study highlights the practicality of using Markov Chains in health insurance pricing and underscores their importance in creating equitable and sustainable insurance systems.

Keywords: Markov Chain, Insurance Premiums, Transition Matrix, Steady-State Distribution, Health Insurance

1. Introduction

The insurance industry plays a crucial role in managing financial risks by pooling resources to anticipate future uncertainties. Accurate insurance premium determination is essential to ensure fair pricing for policyholders while maintaining the profitability of insurance companies. Traditional methods for calculating premiums often rely on statistical models that assume independent risks, which may not always reflect the reality of correlated risks. As a result, new approaches, such as the use of stochastic processes like the Markov chain method, are gaining attention for improving premium calculation accuracy (Lieus et al., 2023).

Markov chains, a type of stochastic process, have been widely applied to model systems that undergo gradual state transitions over time. Their applications span various fields, including economics, biology, and actuarial science, due to their ability to capture temporal dependencies. In the context of insurance, this model enables the structured modeling of policyholder behavior, claim dynamics, and other risk factors. For instance, Markov chains have been employed to calculate premiums for long-term care insurance and specific diseases such as COVID-19 and dengue fever (Xu, et al., 2022).

Previous research has demonstrated that applying the Markov chain model can enhance the accuracy of insurance premium calculations by considering transitions between risk states. For example, Haryanto (2022) applied Markov chains to calculate premiums and reserves for endowment insurance, while Mucha et al. (2022) used Markov simulation for long-term care insurance. These findings highlight the importance of incorporating temporal dependencies into risk evaluations to achieve more reliable premium estimates.

In this study, we explore the application of the Markov chain method for calculating insurance premiums using simulated data. This approach effectively models transitions between risk states with greater accuracy, as demonstrated in prior studies on various insurance types, including unsecured credit insurance and cyber insurance (Hansen et al., 2023; Antonio et al., 2021). The results of this approach offer significant insights for risk management in the insurance industry.

The findings of this research indicate that the Markov chain model is not only effective for determining fair premiums but also relevant for supporting long-term risk management. This approach can be adapted for a wide range of insurance products. Its practical implications include improved premium estimation reliability, more efficient reserve management, and the strengthening of risk management strategies for insurance companies.

2. Literature Review

2.1 Markov Chain

A Markov chain is a stochastic process that satisfies the Markov property, meaning that the future state of the process depends only on its present state and not on the sequence of states that preceded it. Formally, if X_t represents the state of the process at time t, then the Markov property can be expressed as:

$$P(X_{t+1} = x_{t+1} \mid X_t = x_t, X_{t-1} = x_{t-1}, \dots, X_0 = x_0) = P(X_{t+1} = x_{t+1} \mid X_t = x_t), \tag{1}$$

where x_t denotes the state at time t.

Markov chains are characterized by their state space (the set of possible states), transition probabilities (the probabilities of moving from one state to another), and the initial state distribution. They are widely used in various fields, such as economics, biology, computer science, and physics, for modeling systems that evolve over time with uncertainty.

A transition probability matrix for a Markov chain is a square matrix that describes the probabilities of transitioning from one state to another in a stochastic process. Each entry in the matrix represents the probability of moving from one state to another in a single step, satisfying the following properties:

a). Non-negative entries: All elements of the matrix are non-negative, i.e., $P_{ij} \geq 0$, where P_{ij} is the probability of transitioning from state i to state j.

b). Row sums equal to 1: The sum of the probabilities in each row equals 1, i.e., $\sum_j P_{ij} = 1$.

In mathematical terms, for a Markov chain with n states, the transition probability matrix P is an $n \times n$ matrix where:

$$P = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{bmatrix}$$

where, P_{ij} represents the probability of moving from state i to state j in one time step.

2.2. Markov Chain in Insurance Premiums

Markov Chains are a mathematical framework used to model systems that undergo transitions between states in a probabilistic manner. In the context of insurance premiums, Markov Chains can be used to model the evolution of an insurance policyholder's risk profile over time, which in turn influences the premium they pay. For instance, a policyholder's risk classification may change from "low risk" to "high risk" depending on factors such as claims history, age, or driving record. These transitions can be captured using a Markov process, where the state represents a risk class, and the transition probabilities between states represent the likelihood of moving from one risk class to another in a given period. A typical Markov Chain model for insurance premiums would consist of several states, such as "low risk," "medium risk," and "high risk." The insurer can use the probabilities of transitioning between these states to predict future premiums.

State Vector (S):

$$S(t+1) = P \cdot S(t) \tag{2}$$

where:

$S(t)$ is the state vector at time tt, representing the probabilities of being in each state (e.g., low risk, medium risk, high risk).

$S(t+1)$ is the state vector at the next time step, which can be obtained by multiplying the current state vector $S(t)$ with the transition matrix P.

In the context of premiums, the insurer can apply the transition probabilities to adjust the premium rates based on the likelihood of the policyholder transitioning to a higher-risk state. The expected premium in the future can be computed by considering the transition probabilities and the corresponding premiums for each state.

3. Materials and Methods

3.1. Materials

The object of this study is the calculation of health insurance premiums using the Markov Chain method. The primary focus is on modeling transitions between three health states—Healthy, Mild Illness, and Severe Illness—and their associated premiums. The analysis aims to understand how the likelihood of transitioning between these states influences the premiums that policyholders are required to pay over time.

The data used in this study includes transition probabilities between the health states and the premiums assigned to each state. The key data elements are as follows:

- Transition Matrix: A matrix representing the probabilities of transitioning between the health states from one period to the next.
- Premium Information: The premiums for each health state:
  S1 (Healthy): Rp 100,000, S2 (Mild Illness): Rp 250,000, S3 (Severe Illness): Rp 500,000
- Steady-State Distribution: The long-term distribution of policyholders across the health states, derived from the transition matrix.
- Expected Premium Calculation: The weighted average premium based on the steady-state probabilities and the premiums for each health state.

3.2. Methods

The methods used in this study involve a sequential approach to applying the Markov Chain model for calculating insurance premiums. The analysis consists of several stages, each focusing on different aspects of the model, from defining the health states to calculating the expected premiums. The steps are as follows:

a). Defining Health States and Premiums
b). Creating the Transition Matrix
c). Solving for the Steady-State Distribution
d). The next step is to find the steady-state distribution, which represents the long-term proportion of individuals in each health state. This is done by solving the system of equations:

$$\pi P = \pi$$

where $\pi = [\pi_1, \pi_2, \pi_3]$ is the steady-state distribution vector.
e). Calculating the Expected Premium

$$Expected\ Premium = \pi_1 \times 100{,}000 + \pi_2 \times 250{,}000 + \pi_3 \times 500{,}000$$

4. Results and Discussion

The transition matrix represents the probabilities of moving from one state to another in the next period. These probabilities are estimated from observed data or assumptions about the model. We will define the following transition matrix P:

$$P = \begin{bmatrix} 0.8 & 0.15 & 0.05 \\ 0.1 & 0.7 & 0.2 \\ 0.05 & 0.25 & 0.7 \end{bmatrix}$$

The transition matrix in this study is designed to model the movement of a policyholder between three distinct health states, each of which has an associated premium level. The first state, S1 (Healthy), represents a policyholder who is in good health, and thus the probability of remaining healthy in the next period is relatively high. As a result, the premium for this state is the lowest, reflecting minimal risk for the insurer. The second state, S2 (Mild Illness), occurs when a policyholder experiences mild health issues. Here, the transition probabilities indicate the likelihood of improving back to the Healthy state or deteriorating further into a more severe condition. The premium for this state is higher than in the Healthy state, as the insurer faces a moderate level of risk. S3 (Severe Illness) refers to a policyholder who has significant health problems. In this state, the probability of either recovering or remaining in the same condition is modeled, and the premium is the highest due to the greater financial risk posed to the insurer. Together, the transition matrix captures the dynamics of how policyholders might move between these states over time and helps calculate the expected premiums based on their health transitions.

Each health state is associated with a different premium, reflecting the level of risk and medical costs incurred by the policyholder in each state. The premium for a policyholder in the Healthy state (S1) is Rp 100,000, which represents the lowest premium, as this state carries the least risk for the insurer. The Mild Illness state (S2) has a higher premium of Rp 250,000, as the policyholder requires additional care but is not severely ill. The Severe Illness state (S3) has the highest premium, set at Rp 500,000, due to the significant health risks and high medical costs associated with this condition. To calculate the expected monthly premium, it is necessary to determine the steady-state distribution, which describes the long-term proportion of time a policyholder is expected to spend in each health state. The steady-state distribution is crucial, as it reflects the probabilities of being in each state over time, enabling the calculation of a weighted average of premiums based on the likelihood of each health condition occurring.

The calculated premiums over three periods demonstrate how the Markov Chain method dynamically adjusts insurance premiums based on changes in health status. In the first period $t = 1$, the expected premium is Rp 142,500, primarily influenced by the high probability (80%) of the policyholder remaining in the Healthy state, which carries the lowest premium. By the second period $t = 2$, the probabilities shift slightly, with an increased likelihood of transitioning to Mild Illness (18.5%) or Severe Illness (7.5%), raising the expected premium to Rp 157,750. In the third period $t = 3$, the probabilities stabilize at approximately 74.6% for Healthy, 17.55% for Mild Illness, and 7.85% for Severe Illness, resulting in a similar expected premium of Rp 157,725. This stabilization reflects the steady-state behavior of the Markov Chain, where probabilities and premiums converge over time. These results highlight the model's ability to account for health status transitions and provide dynamically adjusted premiums. For insurers, this approach offers a more realistic risk assessment and enables tailored pricing strategies, while for policyholders, it underscores the financial benefits of maintaining good health to avoid higher premiums associated with deteriorating health states.

5. Conclussion

This study demonstrates the effectiveness of the Markov Chain method in calculating insurance premiums based on the policyholder's health status over time. By using a transition matrix to model the movement between three health states—Healthy, Mild Illness, and Severe Illness—we can dynamically estimate the expected premium for each period. The results show how the premium adjusts according to the likelihood of a policyholder transitioning to a more severe health state, with the premiums increasing as health conditions worsen. The steady-state distribution, which represents the long-term probabilities of being in each health state, plays a critical role in calculating the expected premium. This method provides a more accurate and flexible approach to premium calculation, allowing insurers to better assess risk and adjust premiums accordingly. Additionally, it offers policyholders a clearer understanding of how their health status can influence their insurance costs over time. Overall, the Markov Chain method presents a valuable tool for modeling health insurance premiums, enhancing both pricing accuracy and long-term financial planning.

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