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The Application of Compound Interest in Investment Portfolios

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Abstract

Effective long-term investment requires a well-structured strategy supported by detailed analysis. The compound interest model serves as a pivotal tool in assessing potential returns on investments by illustrating how interest accumulates on both the initial capital and previously accrued interest. This study delves into the application of compound interest within investment portfolios, aiming to elucidate its impact on long-term growth trajectories. By investigating various factors, such as investment duration and compounding frequency, the research highlights the intricate mechanisms driving investment expansion. A robust understanding of these elements is crucial for making informed financial decisions. The insights gained from this research are intended to equip investors and financial advisors with practical strategies for optimizing portfolio performance and achieving superior investment results, ultimately contributing to the advancement of more sustainable long-term investment practices.

Keywords: Investment, interest, strategy, growth

1. Introduction

In the realm of finance, the principle of compound interest stands as a cornerstone for wealth accumulation and investment growth. Unlike simple interest, which is calculated solely on the principal amount, compound interest involves earning returns on both the initial principal and the accumulated interest from previous periods (Bierwag, 1977; Lewin, 2019). This self-reinforcing cycle can significantly enhance the value of an investment over time, making it a critical concept for investors aiming to maximize portfolio performance.

The efficacy of compound interest is profoundly influenced by factors such as the duration of the investment and the frequency of compounding. Longer investment horizons allow more periods for interest to compound, leading to exponential growth. Similarly, more frequent compounding intervals result in interest being calculated and added to the principal more often, accelerating the accumulation process. Understanding these dynamics is essential for investors and financial advisors when devising strategies to achieve sustainable long-term growth.

This paper explores the application of compound interest within investment portfolios, focusing on its impact on long-term growth trajectories. By analyzing various factors, including investment duration and compounding frequency, we aim to elucidate the mechanisms that drive investment expansion. A comprehensive understanding of these elements is crucial for making informed financial decisions and optimizing portfolio performance.

2. Literature Review

An investment portfolio is defined as a collection of financial assets such as stocks, bonds, currencies, cash equivalents, and commodities that an investor owns. The primary aim of constructing a portfolio is to generate profit while also preserving capital (Ramsinghani, 2021). Additionally, portfolios are designed to balance risk and return, aligning with the investor's financial goals and risk tolerance, as highlighted by the CFI.

In the literature, compound interest is widely regarded as a powerful mechanism for long-term wealth accumulation. Kremer et al. (2019) argue that understanding compound interest is crucial for comprehending how investments grow over time, particularly when the investment horizon is extended. Similarly, Hue et al. (2019) emphasizes the importance of compounding in retirement planning, noting that even small, regular contributions to investments can compound into substantial sums by the time the investor reaches retirement age.

Hue et al. (2019) underscores the power of compounding in retirement planning, emphasizing how regular contributions can lead to substantial growth over time. Studies have shown that investors who utilize compound interest through reinvestment strategies, such as dividend reinvestment plans (DRIPs), often experience significant returns. For instance, Ho et al. (2019) demonstrated that portfolios employing compound interest strategies achieved higher cumulative returns compared to those relying solely on interest income. Similarly, Kremer et al. (2019) argue that compound interest is essential for maximizing the growth potential of investment portfolios, particularly for products that capitalize on the reinvestment of earnings. These findings underscore the critical role of compound interest in long-term wealth accumulation, illustrating how it can substantially enhance the growth of investment portfolios.

3. Materials and Methods

3.1. Materials

The primary source of data for this study is Yahoo! Finance, a widely recognized financial platform known for providing accurate and up-to-date information on various financial instruments. For the purpose of this research, historical data for the S&P 500, NASDAQ, and Bitcoin were obtained. These data sets include monthly opening and closing prices spanning the most recent six-month period, which serve as the foundation for the compound interest model analysis.

In addition to stock and cryptocurrency price data, the study incorporates historical interest rates and relevant market data from Yahoo! Finance to enhance the accuracy of the analysis. These supplementary data points aid in identifying factors that influence the performance of each investment portfolio.

The selection of the S&P 500 and NASDAQ represents traditional stock indices, offering insights into the performance of diversified equity portfolios. Meanwhile, Bitcoin is included as a cryptocurrency asset to explore the dynamics of high-risk, high-return investments in a rapidly evolving financial landscape. By utilizing these data sources, this study aims to provide a thorough and comparative analysis of investment performance across different asset classes.

3.2. Methods

3.2.1. Structure

This research examines the application of the compound interest model to analyze the long-term growth of three distinct investment portfolios: S&P 500, NASDAQ, and Bitcoin. The study utilizes historical price data for a comparative analysis to highlight the growth trajectories and risk factors of these assets under the compound interest framework. The research process is structured as follows:

a. Identification and collection of historical data for the S&P 500, NASDAQ, and Bitcoin.
b. Determination of relevant variables influencing portfolio performance, including annualized returns and volatility.
c. Application of the compound interest model to assess the future value of investments, focusing on the hypothetical scenario of a $100,000 initial investment.

This approach aims to provide a comprehensive understanding of how compound interest impacts portfolio growth, emphasizing the differences between traditional stock indices and cryptocurrencies in long-term investment strategies.

The compound interest model, widely regarded as one of the most powerful tools for understanding long-term financial growth, is utilized in this study. The model assumes a consistent interest rate, periodic reinvestment of returns, a fixed compounding frequency, and a stable principal amount over the investment horizon.

3.2.2. Formula / Equation

The compound interest model, a foundational concept in finance, is utilized to calculate the growth of an investment by accounting for the effect of reinvested interest on the principal. This model assumes consistent interest rates, periodic reinvestment, a fixed compounding frequency, and a stable principal amount over the investment period. Mathematically, the compound interest formula is expressed as:

$$FV = PV \cdot (1 + \frac{r}{n})^{n \cdot t} \tag{1}$$

where:

FV: Future value of the investment
PV: Present value or initial value of the investment
r: Annual interest rate (in decimal form)

n: Number of times interest is compounded per year
t: Number of years the money is invested

Variables that Determine the Future Value of Investment

a. PV: Principal amount, representing the initial investment.
b. r: Annual interest rate, derived from the monthly returns. The monthly return can be calculated as:

$$RM = \frac{Closing\ price - Opening\ price}{Opening\ price} \tag{2}$$

The annual interest rate (r) is then computed as:

$$r = (1 + RM)^{12} - 1 \tag{3}$$

c. r: The number of times interest is compounded per year. For monthly data, $n = 12$
d. t: The total time period, measured in years, can be calculated from the number of monthly observations:

$$t = \frac{Total\ observations}{12} \tag{4}$$

Assumptions

- Compounding is performed annually for traditional indices (S&P 500, NASDAQ) and monthly for Bitcoin due to its higher volatility.
- The analysis assumes consistent reinvestment of returns without withdrawal over the investment horizon.
- Transaction fees and tax implications are excluded for simplicity.

This structured methodology allows for a detailed comparison of the three asset classes, emphasizing their respective growth potentials and risk profiles under the compound interest model.

4. Results and Discussion

Table 1 below provides the historical data of three different assets—S&P 500 Index, NASDAQ Composite Index, and Bitcoin—collected on the 1st of January for the past 10 years. This data includes the closing prices of each asset, forming the basis for analyzing long-term investment growth using the compound interest model.

Table 1: Historical Closing Prices on January 1

Year	S&P 500 Index	NASDAQ Composite Index	Bitcoin (BTC-USD)
2014	$1,831.98	$4,143.07	$767.74
2015	$2,058.90	$4,726.81	$314.25
2016	$2,043.94	$5,007.41	$434.33
2017	$2,257.83	$5,429.08	$998.33
2018	$2,695.81	$7,065.53	$14,156.40
2019	$2,510.03	$6,635.28	$3,843.52
2020	$3,257.85	$9,092.19	$7,194.89
2021	$3,700.65	$12,888.28	$29,374.15
2022	$4,796.56	$15,832.80	$46,311.74
2023	$3,839.50	$10,466.48	$16,540.69

Table 2 illustrates the results of calculating investment growth for each asset with an initial investment (PV) of $100,000. The calculations are performed using varying annual interest rates (r) derived from historical data and assume compounding occurs annually ($n = 1$) over a 10-year period ($t = 10$).

The analysis presented in Table 2 demonstrates how the value of an investment evolves over time under the influence of different rates of return, emphasizing the significance of asset selection in achieving optimal growth.

Table 2: Calculation Results with Compound Interest Model

Asset	Present Value (PV)	Annual Interest Rate (r)	Compound Per Year (n)	Period (t)	Future Value (FV)
S&P 500 Index	$100,000	8.73%	1	10	$230,659.48
NASDAQ Composite Index	$100,000	10.68%	1	10	$276,365.63
Bitcoin (BTC-USD)	$100,000	34.53%	1	10	$1,860,841.25

This Table 2 provides an overview of how the value of an investment can evolve over a 10-year period, incorporating variations in annual interest rates (r) and different investment assets. Using this data, we can analyze long-term investment growth trends, evaluate the impact of interest rates and compounding frequency on returns, and gain deeper insights into financial decision-making and the principles of Financial Mathematics.

The calculation results presented in Table 2 offer significant insights into the dynamics of investment growth using the compound interest model. The analysis is structured around several key variables:

a). Initial Investment ($\boldsymbol{PV}$)
Each scenario begins with an initial investment of $100,000, which serves as the foundation for calculating growth over the 10-year investment period. This standardized starting value allows for a consistent comparison across assets.

b). Annual Interest Rate ($\boldsymbol{r}$)
The annual interest rates for each asset vary, reflecting historical performance over the past decade. These rates are critical in determining how rapidly the investment grows, with higher rates typically resulting in more substantial returns.

c). Interest Rate and Time ($\boldsymbol{t}$)
The interaction between the interest rate and time is particularly important. The compound interest model demonstrates how a higher interest rate, when sustained over a long period, can exponentially increase the future value of an investment. Conversely, lower interest rates yield more modest growth over the same period.

d). Compound Frequency per Year ($\boldsymbol{n}$)
In all cases, the compound frequency per year (n) is set at 1, meaning interest is compounded annually. This assumption simplifies the calculations while still demonstrating the power of compounding over time.

e). Investment Period ($\boldsymbol{t}$)
The investment period is fixed at 10 years for each asset. This duration highlights the effects of long-term compounding and provides a realistic perspective on how investments grow in a decade.

f). Future Value (FV)
The future value of each investment, calculated using the formula $\boldsymbol{FV} = \boldsymbol{PV} \cdot (\mathbf{1} + \frac{\boldsymbol{r}}{\boldsymbol{n}})^{\boldsymbol{n \cdot t}}$ reflects the total value of the investment after 10 years. This value captures the impact of the initial investment, interest rate, and compounding over the specified period. The results reveal substantial growth across all assets, with Bitcoin showing the most dramatic increase due to its higher historical returns. This detailed analysis emphasizes the importance of asset selection, time horizon, and understanding compounding effects when planning long-term investments. By applying the compound interest model, investors can gain a clearer perspective on potential returns and make more informed financial decisions.

5. Conclusion

Considering the calculation results in the above tables, which reflect the application of the compound interest model in analyzing long-term investments, the following conclusions can be drawn:

a. Variation in Initial Investment
Different initial investments in each period provide valuable insights into the impact of fund allocation and highlight the importance of early decisions when designing investment portfolios. This variation helps understand how different starting points can affect the growth trajectory of an investment.

b. Impact of Annual Interest Rate ($\boldsymbol{r}$)
The varying annual interest rates directly affect investment growth, demonstrating how different interest rates influence the final returns. Higher interest rates generally result in greater profit potential, while lower rates may limit growth over time, making it essential to consider interest rate expectations when choosing investments.

c. Compound Frequency per Year ($\boldsymbol{n}$)
   By setting the compound frequency per year to 1, the interest is calculated annually. This assumption simplifies the analysis, but in real-world scenarios, compounding more frequently (e.g., quarterly or monthly) could result in slightly higher returns. Customizing the compounding frequency according to the financial strategy and investment goals can optimize investment performance.
d. Investment Period ($\boldsymbol{t}$)
   The analysis conducted over a 10-year period reveals how investments develop over time, reinforcing the benefits of long-term investing. Longer investment periods allow for compounding to fully take effect, demonstrating the importance of patience and long-term planning when aiming for higher returns.
e. Future Value (FV)
   The Future Value at the end of each period reflects the investment's returns after applying the compound interest model. The resulting range of values (from $230,659.48 to $1,860,841.25) illustrates the significant variation in potential final returns based on the chosen asset, interest rate, and other factors.

References

Bierwag, G. O. (1977). Immunization, duration, and the term structure of interest rates. *Journal of financial and quantitative analysis*, *12*(5), 725-742.

Ho, L. C., Cadle, J., & Theobald, M. (2022). Portfolio insurance strategies. In *Encyclopedia of finance* (pp. 1437-1465). Cham: Springer International Publishing.

Hue, B., Jinks, A., Spain, J., Bora, M., & Siew, S. (2019). Investment risk for long-term investors: risk measurement approaches: Considerations for pension funds and insurers. *British Actuarial Journal*, *24*, e16.

Kremer, M., Rao, G., & Schilbach, F. (2019). Behavioral development economics. In *Handbook of behavioral economics: applications and foundations 1* (Vol. 2, pp. 345-458). North-Holland.

Lewin, C. G. (2019). The emergence of compound interest. British Actuarial Journal, 24, e10, 1-23.

Lewin, C. G. (2019). The emergence of compound interest. *British Actuarial Journal*, *24*, e34.

Morgan Stanley. (2024). Compound Interest. Morgan Stanley Investment Management, 15(3), 12-25.

Ramsinghani, M. (2021). *The Business of Venture Capital: The Art of Raising a Fund, Structuring Investments, Portfolio Management, and Exits*. John Wiley & Sons.

Saini, S. (2021). Power of Compounding in Mutual Funds. International Journal of Science and Research, 10(11), 469-470.