



Dynamical Analysis and Circuit Design for Malasoma System

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Abstract

In this paper, the Malasoma system based cubic function is presented. This system contains operational amplifiers, resistors, capacitors, multipliers, and voltage sources. The first stage, we analyze the Malasoma model and execute its stability. The phase portraits and bifurcation diagram are used to analyze the dynamic behaviors of the Malasoma model. The proposed circuit was modelled by utilizing NI's MultiSim software environment. The electronic circuit is realized by using off-the-shelf components. MATLAB and MultiSim simulation results show a good agreement.

Keywords: chaos, dynamical system, Malasoma system, circuit design

1. Introduction

Chaos is a study that is very intensively researched by scientists in various disciplines such as robotic (Vaidyanathan et al., 2017), video encryption (Preishuber et al., 2020), image encryption (Sambas et al., 2020), sound encryption (Mobayen et al., 2019), touchless fingerprint encryption (Lai et al., 2020), network encryption card (Ding et al., 2007), Permanent magnet synchronous motor (Yin et al., 2020), homopolar dynamo system (Chen et al., 2020), Rotor-disk-Bearing system (Haghani and Safarpour, 2018), electronic circuit (Sambas et al., 2019), neuron network system (Zhou et al., 2008), tumor-immune model (Kumar et al., 2020), epidemiological model (Ghezzi and Piccardi, 1997), financial risk (Sukono et al., 2020) and regular longitudinal waves (Hu et al., 2010).

Sambas et al. (2015) studied behavior of Malasoma system and its application in voice encryption. They show that the Malasoma system is very well used in voice encryption with AWGN techniques. Varan et al. (2016) presented Malasoma system with P-C synchronization and

its application in secure communication system. They have successfully synchronized and implemented a communication security system on Malasoma model using MATLAB-Simulink. However, the researchers found it very difficult to apply the Malasoma model to electronic circuits. Therefore, we will solve this problem with Kirchhoff's legal assistance.

In this study, the Malasoma model is introduced and its dynamical properties are investigated. In Sec. 2, the existence of chaotic behavior in this system is demonstrated. In Section 3, In Section 3, we determine complex behavior, and introduce the MultiSim platform, together with the electronic circuit schema of the system.

2. Mathematical Model and Dynamical Analysis

Malasoma chaotic system, which was proposed by Malasoma (2000) is described by the following chaotic system with only one cubic nonlinear term

$$\begin{aligned}\dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= -az + xy^2 - x\end{aligned}\quad (1)$$

Where x, y, z are a variable and when $a = 2.05$, the Malasoma system in equation (1) shows strange attractors, we have chosen initial conditions for the Malasoma system $(x_0, y_0, z_0) = (0, 0.96, 0.1)$. The Lyapunov exponents are $(0.0541, 0, -2.1041)$, and the Kaplan-Yorke dimension is $D_{KY} = 2.0257$ (Sprott and Linz, 2000).

For numerical simulations of a Malasoma system defined by a set of differential equations, such as the Malasoma system in equation (1), different integration techniques can be used. In Figures 1 (a) - (c), respectively, show the projections of the orbital space of the x - y plane phase, the x - z plane and the y - z plane. For the selected parameter set and initial conditions, the Malasoma system in equation (1) present a dense strange attractor. By using bifurcation diagram (See Figure 2), the range of a over which chaos occurs is very narrow, $2.02 < a < 2.08$. Also, small window of chaos for $0.075351 < a < 0.075362$.

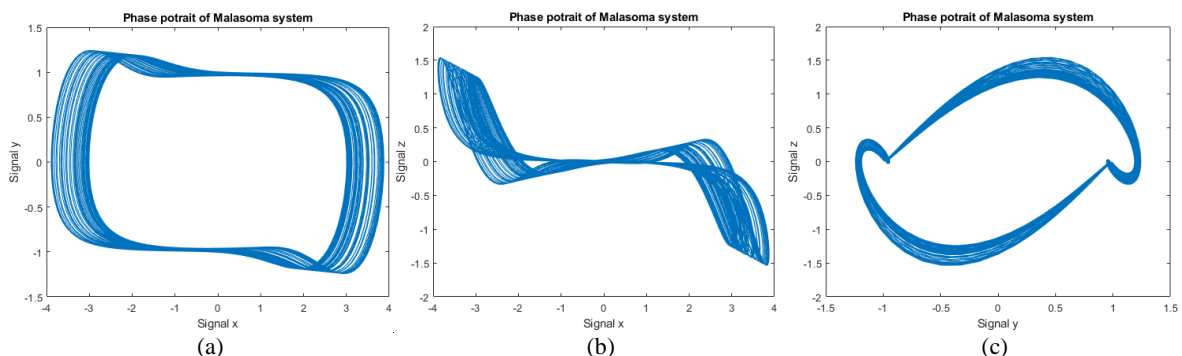


Figure 1. Numerical simulation results using MATLAB, with $a = 2.05$ in the (a) x - y plane, (b) x - z plane, and (c) y - z plane

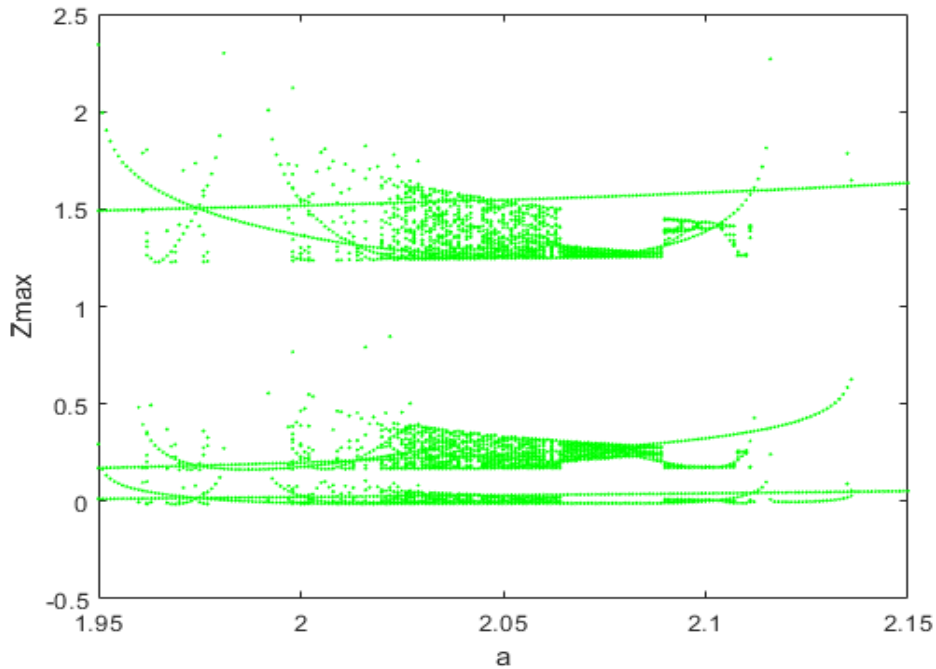


Figure 2. Bifurcation Diagram (between 1.95-2.15).

The equilibria of the Malasoma system (1) are found by setting $\dot{x} = \dot{y} = \dot{z} = 0$, i. e.,

$$\begin{aligned} 0 &= y \\ 0 &= z \\ 0 &= -az + xy^2 - x \end{aligned} \tag{2}$$

The Malasoma system has one equilibrium point $E_0(0, 0, 0)$. For the equilibrium point $E_0(0, 0, 0)$, the Jacobi matrix becomes:

$$J(0,0,0) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & -2.05 \end{bmatrix} \tag{3}$$

To obtain its eigenvalues, let $\det |J_1 - \lambda| = 0$. Then, the characteristic equation has the following form:

$$\lambda^3 + 2.05\lambda^2 + 1 \tag{4}$$

Solving the above characteristic equation, the eigenvalues are found as

$$\lambda_1 = -2.2479, \lambda_{2,3} = -0.0989 \pm 0.6595i$$

Here λ_1 is a negative real number, λ_2 and λ_3 are conjugate pair of complex eigenvalues having positive real parts. That means the equilibrium $E_0(0, 0, 0)$ is a saddle point. So, this equilibrium point is unstable

3. Electronic Circuit

The analog circuit of the Malasoma system was designed in MultiSim. The analog circuit of the Malasoma system is seen in Figure 2. The circuit includes simple electronic elements such as resistors, operational amplifiers.

For circuit implementation, we rescale the state variables of the Malasoma system (1) as follows: $X = 6x$, $Y = 2y$, $Z = 2z$. The rescaled chaotic system is given as follows:

$$\begin{aligned} \dot{X} &= 3Y \\ \dot{Y} &= Z \\ \dot{Z} &= -aZ + \frac{XY^2}{12} - \frac{X}{3} \end{aligned} \quad (5)$$

Applying the Kirchhoff laws, the circuit presented in Figure 3 is described by the following equations:

$$\begin{aligned} \dot{X} &= \frac{1}{C_1 R_1} Y \\ \dot{Y} &= \frac{1}{C_2 R_2} Z \\ \dot{Z} &= -\frac{1}{C_3 R_3} Z + \frac{1}{100 C_3 R_4} XY^2 - \frac{1}{C_3 R_5} X \end{aligned} \quad (6)$$

We get the value of electronic components $R_1 = 200 \text{ k}\Omega$, $R_2 = 600 \text{ k}\Omega$, $R_3 = 292.68 \text{ k}\Omega$, $R_4 = 72 \text{ k}\Omega$, $R_5 = 1.8 \text{ M}\Omega$, $R_6 = R_7 = R_8 = R_9 = R_{10} = R_{11} = 100 \text{ k}\Omega$, and $C_1 = C_2 = C_3 = 1 \text{ nF}$. Figure 4 illustrates phase portraits which are obtained from the designed circuit. As it can be seen from the MultiSim outputs in Figure 4 and MATLAB simulation in Figure 1, the results are similar.

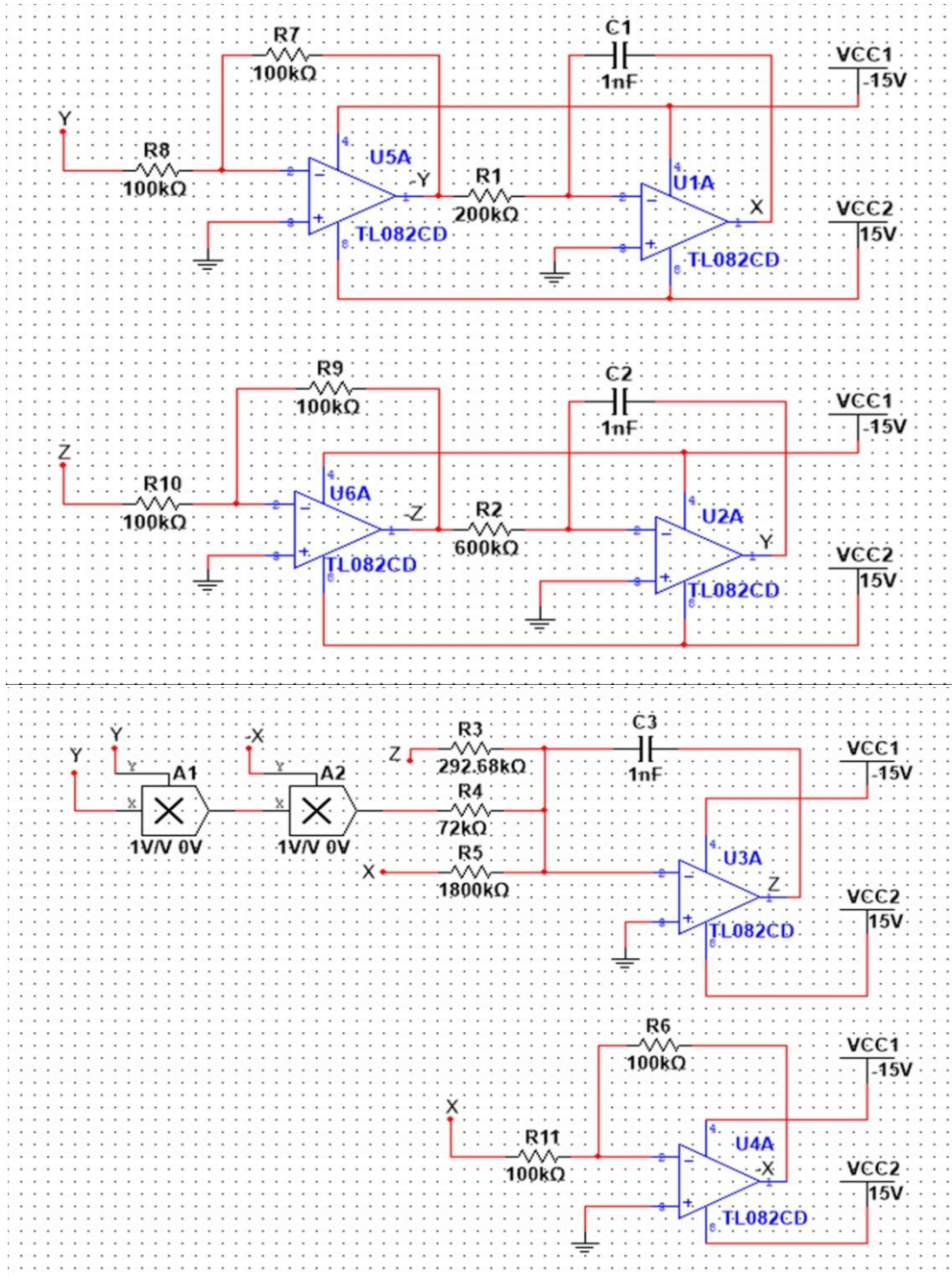


Figure 3. The electronic circuit schematic of the Malasoma system

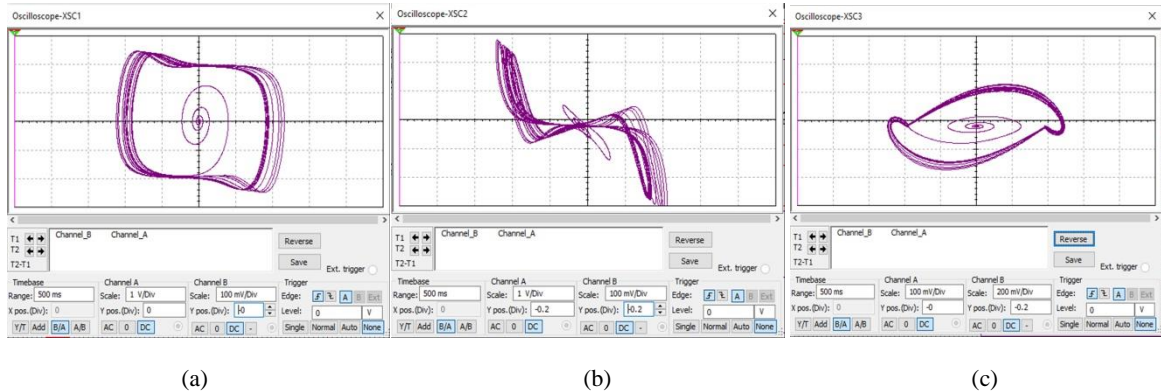


Figure 4. Multisim outputs of the in the Malasoma system (a) X-Y plane, (b) X-Z plane, and (c) Y-Z plane

4. Conclusion

In this study, we have studied a Malasoma model with cubic function which is obtained obtained by Malasoma (2000). Fundamental dynamical properties of the Malasoma model have been discovered through equilibrium point analysis, bifurcation diagram and Lyapunov exponents. Finally, the MATLAB simulations and MultiSim simulation are performed to verify the theoretical model. For hardware electronic circuit can be investigated in our future works.

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