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Investment Portfolio Optimization on Technology Sector Stocks Using Mean-Variance Model with Asset-Liability Based on ARIMA-GARCH Approach

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Abstract

In this era of rapid technological advancement, various sectors are experiencing changes, one of which is investment. Investors are starting to turn their attention to technology sector stocks as new investment targets. However, investments are inherently linked to return and risk levels and stock prices can be highly volatile. Therefore, forming an optimal investment portfolio is very important to achieve a balance between return and risk. In addition, coping with volatile stocks is also very important. The ARIMA-GARCH time series model is a method that can be used to deal with such volatility. A popular strategy for portfolio optimization is to use the Mean-Variance model, also known as the Markowitz model. This study aims to form an optimal portfolio consisting of five technology sector stocks in Indonesia with the codes AXIO, DIVA, EDGE, MCAS, and CASH using the Mean-Variance model with assets-liabilities equipped with the ARIMA-GARCH approach. Based on the results of the study, the optimal portfolio is obtained with the composition of each weight is 23.16% of the capital allocated to AXIO; 2.95% for DIVA; 56.48% for EDGE; 6.36% for MCAS; and 11.05% for CASH. The weight allocation composition can generate a portfolio return of 0.0066 and a variance (risk) return of 0.0082.

Keywords: Portfolio optimization; Mean-Variance; ARIMA; GARCH.

1. Introduction

The rapid development of technology and information in this era of globalization has increased competitiveness in various commercial sectors, as well as having an impact on other fields such as social, educational, political culture, and economic fields (Oktavia & Nirawati, 2022). This can be an opportunity for investors to make big profits by investing in technology stocks because when investing capital, investors will definitely choose a business that is believed to be profitable (Wahdania et al., 2023).

One method for forming a portfolio is Mean-Variance, which is a method that gathers several assets with the aim of maximizing profits and reducing risk (Majidah et al., 2024). The Mean-Variance model solves the portfolio optimization problem by forming the set of optimal portfolios that provide the highest rate of return for a given level of risk or the lowest risk for a given level of return (Kim, 2021).

The most widely used statistical technique for analyzing data that is based on past values of the time series along with the previous error term for estimation is the Autoregressive Integrated Moving Average (ARIMA) model. Meanwhile, to overcome volatility, one model that can be used is the Generalized Autoregressive Correlation Heteroscedasticity (GARCH) model (Kaur & Singla, 2022). Forecasting returns with homoskedastic variance can be described using the ARIMA model, while returns with heteroskedastic variance can be described using the ARIMA-GARCH model (Talumewo et al., 2023).

There are several previous studies in recent years that are relevant to this research. Soeryana et al., (2017) conducted research on investment portfolio optimization by applying mean-variance optimization techniques using the ARMA model for non-constant mean and GARCH for non-constant volatility, as well as the Lagrangian multiplier method to optimize the portfolio. The study used five stocks and obtained the result that the optimal portfolio composition provides a balanced risk and return ratio, with certain weights allocated to each stock. In addition, research by Lesman et al., (2017) discusses stock portfolio optimization using the mean-variance model by considering asset liabilities and asset returns estimated using the ARMA-GARCH model, the results obtained offer the best balance between maximizing returns and minimizing risk, thus providing valuable insight for investment decisions.

Based on previous research, this research discusses the topic of optimizing the technology sector stock portfolio using the Mean-Variance model with assets-liabilities based on the ARIMA-GARCH approach. The difference between this research and previous research lies in the object of research. This research aims to enable investors to better manage stock portfolios in the technology sector and produce optimal portfolios.

2. Literature Review

2.1 Investment

Investment is the investment of capital or money that is planted to be used as capital with the aim of obtaining benefits in the future. Investments are made generally to get profit or income from a business. Investing can also help in planning for future dependents, such as education and health (Heradhyaksa, 2022). Choosing to invest basically means managing money in a certain period of time, namely long or short term. Investors are expected to make thorough calculations that can optimize profits at certain risks before making an investment (Bangun et al., 2012).

2.2 Asset Return

Return is the financial gain derived from an investment made by a person, organization, or business. The amount of asset return can be determined by equation (1) (Tsay, 2005).

$$r_t = \ln P_t - \ln P_{t-1}, \tag{1}$$

with,
r_t: asset-liability return at time t $(t = 1, \dots, n)$,
P_t: the price or value of an asset-liability at time t $(t = 1, \dots, n)$,
n: number of observed data.

2.3 Stationary

Stationary observational data is data that has fixed statistics and does not change over time (Cryer & Chan, 2008). It indicates that the mean and variance are constant over time (E. P. Box et al., 2015). The ADF test is one of the statistical tests to test the stationarity of data against the mean. The ADF test hypotheses are:
$H_0: \delta = 0$ (data is non-stationary),
$H_1: \delta < 0$ (data is stationary).
The test statistic used is:

$$t = \frac{\hat{\delta}}{SE(\hat{\delta})}, \tag{2}$$

with,
$\hat{\delta}$: least squares estimate of δ,
$SE(\hat{\delta})$: standard error *of* $\hat{\delta}$.
The test criteria is to reject H_0 if $|t_{hitung}| > t_{(\alpha,n)}$ or p-value $< \alpha$.

Box-Cox transformation is a transformation method to achieve constant variance with the parameter λ. A rounded value (λ) of 1 indicates that the data is stationary with respect to variance. Otherwise, Box-Cox transformation needs to be performed. The Box-Cox transformation is defined by the equation (3) (Wei, 2006).

$$T(Z_t) = \begin{cases} \dfrac{Z_t^{\lambda} - 1}{\lambda}, & \lambda \neq 0 \\ \ln Z_t\,, & \lambda = 0 \end{cases} \tag{3}$$

with,
$T(Z_t)$: Box-Cox transformation
Z_t : observation value at time t,
λ : transformation parameter.

2.4 ARIMA Model

The Autoregressive Integrated Moving Average (ARIMA) model is a commonly used model in time series analysis introduced by Box and Jenkins in 1970. This model combines elements from the Autoregressive (AR) model, Integrated (I) which represents differencing, and the Moving Average (MA) model (Wang, 2024). The ARIMA model has white noise assumptions, meaning that there is no autocorrelation and is normally distributed with zero mean and constant variance, which can be written as $\{e_t\} \overset{iid}{\sim} N(0, \sigma^2)$ (Tsay, 2005).
The Autoregressive Integrated Moving Average (ARIMA) model is used to analyze and forecast data with non-stationary patterns, so it is necessary to do differencing first and then combine it with AR and MA models. The general form of the ARIMA(p, d, q) model is written in equation (4) (Wei, 2006).

$$\phi_p(B)(1-B)^d Z_t = \mu + \theta_q(B)e_t, \tag{4}$$

with,
$\phi_p(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$,
$\theta_q(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q$,
Z_t : observation at time t,
μ : intercept,
$\phi_p(B)$: autoregressive (AR) operator,
$\theta_q(B)$: moving average (MA) operator,
p : autoregressive (AR) order,
d : orde differencing orde (I),
q : moving average (MA) order,
e_t : residual at time t,
B : back shift operator.

2.5 ARIMA Model Identification

ARIMA model identification can be done by looking at the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) correlograms. ACF is a function used to measure the autocorrelation of each observation with the previous value at various lags. ACF is defined in equation (5) (Wei, 2006).

$$\hat{\rho}_k = \frac{\hat{\gamma}_k}{\hat{\gamma}_0} = \frac{\sum_{t=1}^{n-k}(Z_t - \bar{Z})\,(Z_{t+k} - \bar{Z})}{\sum_{t=1}^{n}(Z_t - \bar{Z})}, k = 0,1,2, \ldots \tag{5}$$

with,
$\hat{\rho}_k$: the estimated ACF at lag k,
$\hat{\gamma}_k$: the estimated autocovariance at lag k,
$\bar{Z}$: mean observation score.

PACF is a function used to identify the autocorrelation between two observation values at different lags after removing the influence of previous lags. PACF is defined in equation (6).

$$\hat{\phi}_{k+1,k+1} = \frac{\hat{\rho}_{k+1} - \sum_{j=1}^{k}\hat{\phi}_{k,j}\hat{\rho}_{k+1-j}}{1 - \sum_{j=1}^{k}\hat{\phi}_{k,j}\hat{\rho}_j}, \tag{6}$$

with,
$\hat{\phi}_{k+1,k+1}$: the estimated PACF at lag $(k+1)$,
$\hat{\phi}_{k,j}$: the estimated PACF at lag k and $lag\ j$.

2.6 Parameter Estimation of ARIMA Model

Maximum Likelihood Estimation (MLE) is a method that can be used to estimate model parameters with the principle of maximizing the likelihood function. The likelihood function of the ARMA(p,q) model can be expressed in equation (7) (Wei, 2006).

$$L\left(e_t \middle| \mu, \phi_p, \theta_q, \sigma_e^2\right) = (2\pi\sigma_e^2)^{-\frac{n}{2}} \exp\left[-\frac{1}{2\sigma_e^2}\sum_{t=1}^{n} e_t^2\right]. \tag{7}$$

2.7 Parameter Significance Test

To ensure that the model used is reliable, parameter testing is required. The parameter significance test aims to determine whether the model parameters significantly contribute to the model formed (8) (Soeryana et al., 2017). The test hypotheses used are:
$H_0: \phi_p = 0$ or $\theta_q = 0$ (model parameters are not significant),
$H_1: \phi_p \neq 0$ or $\theta_q \neq 0$ (model parameters are significant),
The test statistics used are:

$$t_{hitung} = \frac{\hat{\phi}_p}{SE(\hat{\phi}_p)} \quad \text{or} \quad t_{hitung} = \frac{\hat{\theta}_q}{SE(\hat{\theta}_q)}, \tag{8}$$

with,
$\hat{\phi}_p$: estimated AR model parameters of order p,
$\hat{\theta}_q$: estimated MA mode parameters of order q,
$SE(\hat{\phi}_p)$: standard error of $\hat{\phi}_p$,

$SE(\hat{\theta}_q)$: standard error of $\hat{\theta}_q$.
The test criterion is to reject H_0 when $|t_{hitung}| > t_{(\frac{\alpha}{2}, n-n_p)}$ or p-value $< \alpha$.

2.8 Diagnostic Test

Diagnostic tests are used to determine the adequacy of the model by checking whether the model assumptions are met. The assumption is that the residuals are white noise tested by the Ljung-Box test and normally distributed as seen from the visualization of the Quantile-Quantile plot (Q-Q plot) results. According to (Wei, 2006), the test hypotheses used are:
$H_0: \rho_1 = \rho_2 = \cdots = \rho_K = 0$ (residuals meet white noise criteria),
$H_1: \exists \rho_k \neq 0,\ k = 1,2, \dots, K$ (residuals do not meet white noise criteria).
The test statistics used are:

$$Q = n(n+2) \sum_{k=1}^{K} (n-k)^{-1} \hat{\rho}_k^2, \tag{9}$$

with,
n : number of observations,
k : lag k, $k = 1, \dots, K$,
K : number of lags used,
$\hat{\rho}_k^2$: estimated ACF squared at lag k.
The test criterion is to reject H_0, when $Q \geq \chi^2_{(\alpha, K-p-q)}$ or p-value $< \alpha$.

Q-Q plot is used to check whether the residuals are normally distributed with a graphical analysis approach. According to (Gio & Irawan, 2016) the assumption of normality is fulfilled if the distribution of points is very close to the diagonal line. Meanwhile, the assumption of normality is not fulfilled if the distribution of the dots spreads far (spreads winding on the diagonal line like a snake).

2.9 ARCH-LM Test

The Autoregressive Conditional Heteroscedasticity-Lagrangian Multiplier (ARCH-LM) effect test is conducted to detect the presence or absence of heteroscedasticity or non-constant variance in the residuals. The test hypothesis used is (Tsay, 2005):
$H_0: \alpha_1 = \alpha_2 = \cdots = \alpha_K = 0$ (no heteroscedasticity),
$H_1: \exists \alpha_k \neq 0, k = 1,2, \dots, K$ (heteroscedasticity).
The test statistics used are:

$$LM = nR^2, \tag{10}$$

with,
R^2 : coefficient of determination,
$\hat{Z}_t$: forecasting data at time t,
The test criterion is reject H_0 when $LM > \chi^2_{(\alpha,K)}$ or $p\text{-}value < \alpha$.

2.10 GARCH Model

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) time series model is used to model the volatility of data that fluctuates over time, which has the assumption that the residual variance is not only influenced by the previous period's squared residual, but also the previous period's residual variance. This model is a development of the Autoregressive Conditional Heteroskedasticity (ARCH) model developed by Bollerslev in 1986. The GARCH(m, s) model can be written as equation (11) (Tsay, 2002).

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \cdots + \alpha_m e_{t-m}^2 + \beta_1 \sigma_1^2 + \cdots + \beta_s \sigma_{t-s}^2 \tag{11}$$

with,
σ_t^2 : residual variance at time t,
e_{t-m}^2 : squared residuals at time $(t-m)$,
m : ARCH order,
s : GARCH order,
α_0 : constant,
α_m : ARCH parameter at order m,
β_s : GARCH parameter at order s,

2.11 Parameter Estimation GARCH Model

Parameter estimation of the GARCH model is done by the Maximum Likelihood Estimation (MLE) method. The GARCH(m, s) likelihood function is expressed by the equation (12).

$$L(e_t|\alpha_0, \alpha_m, \beta_s) = \prod_{t=1}^{n} (2\pi\sigma^2)^{-\frac{1}{2}} \exp\left(-\frac{e_t^2}{2\sigma^2}\right) \tag{12}$$

2.12 Best Model Selection Criteria

For a given data set, when there are several adequate models, the selection criterion is usually based on the summary statistics of the residuals. One of the criteria that can be used is Akaike's Information Criterion (AIC). According to (Wei, 2006), the most optimal model is the one with the minimum AIC value. The AIC value is calculated with the following equation.

$$AIC = 2M - 2\ln L, \tag{13}$$

with,

M : number of parameters in the model,
L : value of the likelihood function on the estimated parameters.

2.13 Asset-Liability Model

The asset-liability surplus return modeling at time $t = 0$ is described by the following equation (Alex Keel & Muller, 1995).

$$S_0 = A_0 - L_0, \tag{14}$$

with,

S_0 : surplus at time u $t = 0$,
A_0 : asset asset at time $t = 0$,
L_0 : liability at time $t = 0$.

The surplus obtained after one period is written as follows.

$$S_1 = A_1 - L_1 = A_0[1 + r_A] - L_0[1 + r_L] \tag{15}$$

Suppose the surplus return is expressed as:

$$r_S = \frac{S_1 - S_0}{A_0} = \frac{A_0 r_A}{A_0} - \frac{L_0 r_L}{A_0} = r_A - \frac{1}{f_0} r_L, \tag{16}$$

with,

$f_0 = \frac{L_0}{A_0}$.

Based on equation (16), the mean surplus return value can be calculated as follows.

$$\mu_S = E[r_S] = \mu_A - \frac{1}{f_0}\mu_L, \tag{17}$$

with,

μ_S : mean return surplus,
μ_A : mean assets,
μ_L : mean liability.

Also obtained the variance of surplus which can be determined by this equation.

$$\sigma_S^2 = \sigma_A^2 - \frac{2}{f_0}\sigma_{AL} + \frac{1}{f_0^2}\sigma_L^2, \tag{18}$$

with,

σ_S^2 : variance of *return* surplus,
σ_A^2 : varians of asset,
σ_L^2 : varians of liability,
σ_{AL} : covariance between assets and liabilities

2.14 Mean-Variance Model Portfolio Optimization

The Mean-Variance model is a tool used to optimize the weight of assets in an investment portfolio in a certain period. The main objective is to maximize the average return and minimize the variance of the return (Majidah et al., 2024). The surplus return and portfolio variance can be determined by the following equations respectively. (Bakry et al., 2021).

$$\hat{\mu}_{S_p} = \boldsymbol{\mu}_S^T \mathbf{w}, \tag{19}$$

$$\hat{\sigma}_S^2 = \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w}, \tag{20}$$

$\boldsymbol{\mu}_S$: mean vector of surplus returns,

N : number of observations,
$\mathbf{w}$: weight vector,
$\mathbf{\Sigma}$: variance-covariance matrix of surplus returns between stocks.

In Lesman et al., (2017), referring to Panjer *et al*. (1998) and Bjork *et al*., (2005) surplus return optimization refers to equation (2.55), where τ is risk tolerance. The greater the risk tolerance value, the greater the investor's courage in facing risk.

$$Max\{2\tau\boldsymbol{\mu}^T\mathbf{w} + 2\boldsymbol{\gamma}^T\mathbf{w} - \mathbf{w}^T\mathbf{\Sigma}\mathbf{w}\},\quad s.t.\, \mathbf{e}^T\mathbf{w} = 1 \tag{21}$$

with,
$\boldsymbol{\gamma}$: covariance vector between asset return and liability return,
$\mathbf{e}$: unit vector.

After obtaining an efficient portfolio, the value of the ratio between return and variance is then calculated, which is shown in the following equation.

$$Ratio = \frac{\hat{\mu}_{S_p}}{\hat{\sigma}_S^2} \tag{22}$$

The ratio is said to be favorable when the ratio value is high, and bad when it is low. Since the ratio indicates that the portfolio can generate the largest return, the ratio with the highest value is chosen when determining the ideal portfolio.

3. Materials and Methods

3.1. Materials

This study discusses the optimization of the technology sector stock portfolio. The object of research is stock price data of five technology companies, namely Tera Data Indonusa Tbk (AXIO), Distribusi Voucher Nusantara T (DIVA), Indointernet Tbk (EDGE), M Cash Integrasi Tbk (MCAS), and Cashlez Worldwide Indonesia Tb (CASH). The stock price data taken is the price at closing time starting from January 1, 2023 to June 30, 2024. This data is obtained from the website https://finance.yahoo.com/. While the liability data used is simulated data.

3.2. Methods

This research uses the Mean-Variance model with assets-liabilities based on the ARIMA-GARCH approach. The stages in conducting this research are as follows:

1. Calculating returns from historical data
2. Test the stationarity of return data in the mean using Augmented Dickey Fuller (ADF) and test the stationarity of return data in variance using Box-Cox transformation.
3. Identify ARIMA models based on ACF and PACF plots.
4. Calculate ARIMA model parameter estimates using the MLE method.
5. Test the significance of parameters and diagnostic tests of ARIMA models
6. Test the ARCH effect on model residuals.
7. If there is a heteroscedasticity effect, identify the GARCH model using ACF and PACF.
8. Calculate parameter estimates of the ARIMA-GARCH model using the MLE method.
9. Test the significance of parameters and diagnostic tests of the ARIMA-GARCH model.
10. Test the ARCH effect on model residuals.
11. Choose the best ARIMA-GARCH model based on the smallest AIC value and forecast stock return data one period ahead using the best ARIMA-GARCH model for each company.
12. Calculate the mean and variance of surplus return then form its vector and matrix.
13. Optimization of return surplus, where the value of τ is simulated gradually from 0, with an increase of 0.01.

4. Results and Discussion

4.1 Company Stock Data

Descriptive statistics are used to provide an overview of the characteristics of research data before further analysis is carried out. Through descriptive statistics, various important aspects of the data can be known, such as distribution, central tendency, and variability. In this study, 349 closing stock price data were analyzed to understand their patterns and characteristics. As an illustration, the following is a plot of AXIO stock data as an example of the dataset used.

Figure 1: Plot of AXIO Closing Price

The data is used to calculate asset returns based on equation (1). The following is the calculation of return from the daily closing stock price for AXIO. The asset return plot for AXIO stock is presented in figure 2.

For $t = 1$, $P_1 = 234$, and assumed $P_0 = 234$,

$$r_1 = \ln(P_1) - \ln(P_0) = \ln(234) - \ln(234) = 0.$$

For $t = 2$, $P_2 = 244$, and $P_1 = 234$,

$$r_2 = \ln(P_2) - \ln(P_1) = \ln(244) - \ln(234) = 0.0418.$$

$$\vdots$$

For $t = 349$, $P_{349} = 179$, and $P_{348} = 176$,

$$r_{349} = \ln(P_{349}) - \ln(P_{348}) = \ln(179) - \ln(176) = 0.0169.$$

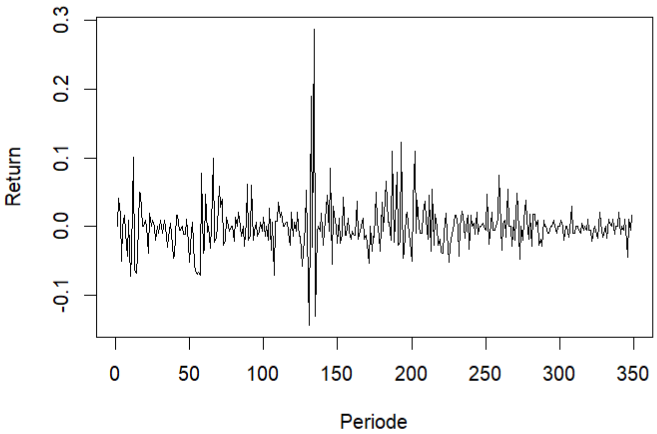

Figure 2: Plot of AXIO Return

In Figure 4.2, the horizontal axis represents the observation time of the data, while the vertical axis represents the percentage change in stock price at the observation time. In AXIO stock, stock returns tend to be stable and consistent throughout the period, but there is a high spike around the 130th period. The characteristics of each company's stock return data can be observed from its descriptive statistics. The following table presents the descriptive statistics for AXIO stocks.

Table 1: Descriptive Statistics

Statistics	Value	Statistics	Value
Minimum	-0.1437	Mean	0.2877
Median	0.0000	Standard Deviation	0.0360
Maximum	-0.0008		

From Table 1, the mean value of stock returns is negative, indicating a loss in investment. While the standard deviation shows the extent to which a return deviates from its mean value or the level of stock fluctuation.

4.2 Stationarity Test

The return data is then tested for stationarity against the mean using the Augmented Dickey-Fuller (ADF) test based on equation (2) with a significant level α used is 5%. While the stationarity of the variance is tested using Box-Cox Transformation. The work was done with the help of RStudio software, the results are presented in Table 2.

Table 2: Stationarity Test Results

Stock	Value of λ	*p-value*
AXIO	$0.9469 \approx 1$	0.01

Based on Table 2, it is obtained that AXIO stock has a p-value $= 0.01 < \alpha = 0.05$ so that H_0 is rejected. It can be said that all stock returns are stationary on mean. It is also obtained that λ is 1, so it can be said that stock returns are stationary with respect to variance. Based on the test results, the same results are obtained for other stocks, so the five stock returns are stationary both on mean and variance.

4.3 Identification ARIMA Model

The identification of the ARIMA model is done to determine the order p for the AR model, order d for the differencing amount, and order q for the MA model. Because the stock return data has been stationary, no differencing is required or in other words, order d is 0. Meanwhile, to obtain order p and q is done by looking at the ACF and PACF plots presented in the following Figure.

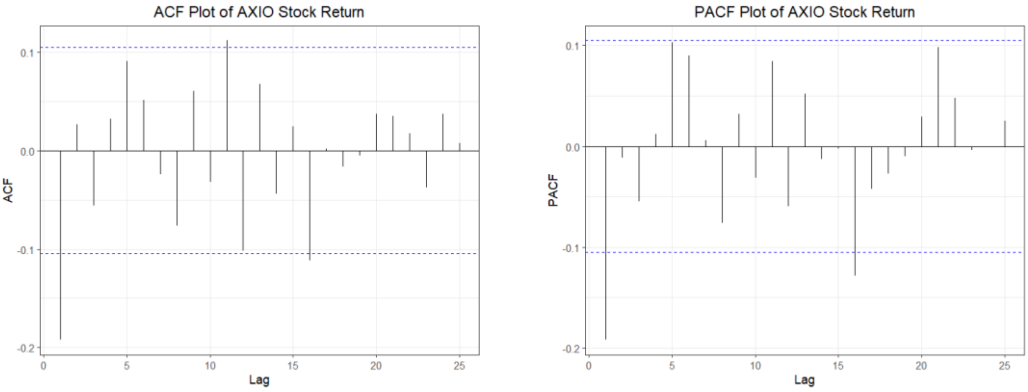

Figure 3: ACF and PACF Plot of AXIO Stock Return

Based on the plots, the ACF and PACF cut-off at lag 1, so the provisional models that can be selected are ARIMA(1,0,0), ARIMA(0,0,1), and ARIMA(1,0,1).

4.4 Significance Test of ARIMA Model Parameters

The temporary model that has been obtained will then be estimated using the Maximum Likelihood Estimation (MLE) method. The following are the results of estimating and testing the significance of model parameters on AXIO shares.

Table 3: Parameter Estimation and Significance Test Results

Model	Parameter	Parameter Estimation	*p-value*	Significant
ARIMA(1,0,0)	$\hat{\mu}$	-0.0008	0.6249	Not Significant
	$\hat{\phi}_1$	-0.1916	0.0003	Significant
ARIMA(0,0,1)	$\hat{\mu}$	-0.0008	0.6085	Not Significant
	$\hat{\theta}_1$	-0.1932	0.0002	Significant
ARIMA(1,0,1)	$\hat{\mu}$	-0.0008	0.6141	Not Significant
	$\hat{\phi}_1$	-0.0838	0.8126	Not Significant
	$\hat{\theta}_1$	-0.1115	0.7536	Not Significant

Based on Table 3, all parameters in the ARIMA(1,0,1) model are not significant, because the p-value is greater than the significance level ($\alpha = 5\%$) so that the model cannot be continued to the next stage. Meanwhile, in the ARIMA(1,0,0) and ARIMA(0,0,1) models, there is one significant parameter, which indicates that the parameter contributes to the model.

4.5 Diagnostic Test of ARIMA Model

To determine whether the assumptions of the ARIMA model have been met, the model is tested using the Ljung-Box test and visualized using the Quantile-Quantile Plot (Q-Q Plot). The AXIO stock test results are presented in Table 4.

Table 4: Ljung-Box Test Results

Model	*p-value*	Note
ARIMA(1,0,0)	0.2811	White Noise
ARIMA(0,0,1)	0.3163	White Noise

Models that meet the white noise criteria can proceed to the visualization stage to check for normality. The Q-Q plot for AXIO stock is presented in Figure 4.

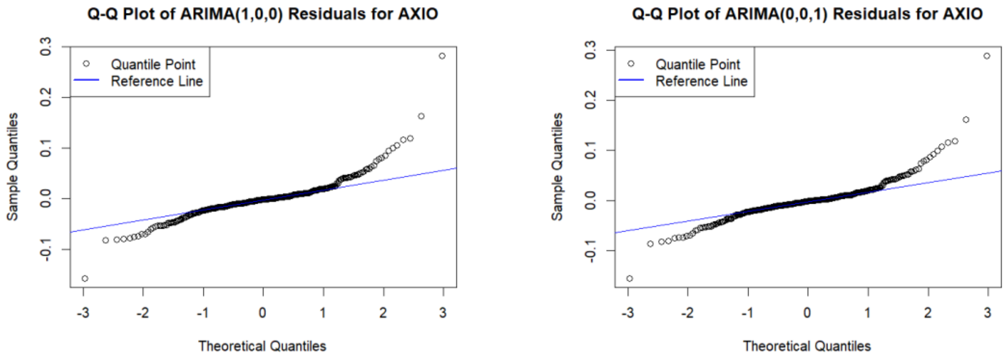

Figure 4: Q-Q Plot of Residuals

Figure 4 shows that the quantile points on the residuals of both models move close to their reference lines. This indicates that the ARIMA model residuals are close to normal distribution.

4.6 Selection of the Best ARIMA Model

The ARIMA model that has met the assumptions is then selected the best for each stock using the AIC method in equation (13). Table 5 presents the calculation results for each model.

Table 5: AIC Value of ARIMA Model

Model	AIC
ARIMA(1,0,0)	-1337.89
ARIMA(0,0,1)	-1337.923

Based on Table 5, the ARIMA model is selected according to the smallest AIC value. So, the best model for AXIO stock is ARIMA(0,0,1).

4.7 ARCH-LM Test of ARIMA Model

Before performing GARCH modeling, it is necessary to check first whether the model has heteroscedasticity effect or not. The ARCH-LM test result for ARIMA(0,0,1) on AXIO stock results in a p-value of 2.232×10^{-5} which is smaller than the significance level ($\alpha = 5\%$). This indicates that the test criterion is to reject H_0 or the residuals have a heteroscedasticity effect, so GARCH modeling is required.

4.8 Identification GARCH Model

Identification of the order of the GARCH model is done with the ACF and PACF plots of the squared residuals of the ARIMA model. The ACF and PACF values are calculated using equations (5) and (6) which are then presented in the correlogram in the following figure.

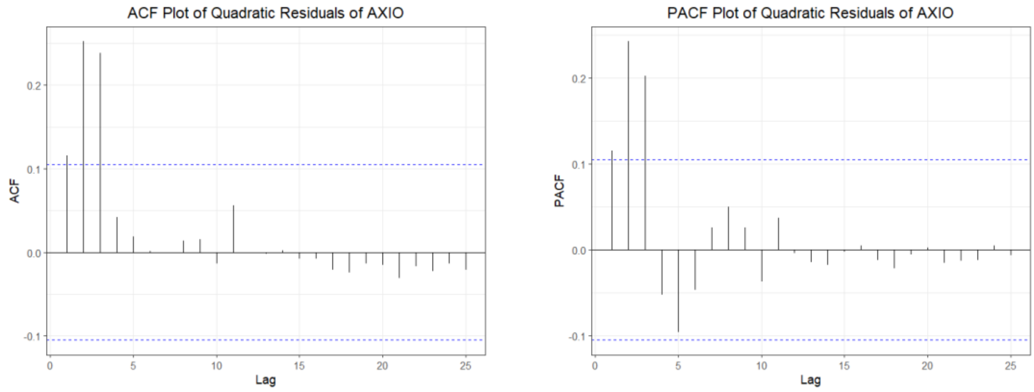

Figure 5: ACF and PACF Plot of Quadratic Residuals

Figure 5 shows the ACF and PACF of the squared residuals of AXIO stock cut-off at lag 1 to lag 3, so the provisional models that can be selected are GARCH(3,0), GARCH(1,3), GARCH(2,3), GARCH(3,1), GARCH(3,2), and GARCH(3,3).

4.9 Estimation and Significance Test of ARIMA-GARCH Model Parameters

The temporary model that has been obtained will then be estimated using the Maximum Likelihood Estimation (MLE) method. The estimation results and significance tests of two sample ARIMA-GARCH models for AXIO stock are presented in Table 6 as an illustration of the overall analysis.

Table 6: Parameter Estimation and Significance Test Results

Model	Parameter	Parameter Estimation	*p-value*	Significance
ARIMA(0,0,1)-GARCH(3,0)	$\hat{\mu}$	0.0005	0.7307	Not Significant
	$\hat{\theta}_1$	0.1180	0.1305	Not Significant
	$\hat{\alpha}_0$	0.0004	0.0000	Significant
	$\hat{\alpha}_1$	0.2132	0.0273	Significant
	$\hat{\alpha}_2$	0.3988	0.0008	Significant
	$\hat{\alpha}_3$	0.1359	0.0340	Significant
ARIMA(0,0,1)-GARCH(1,3)	$\hat{\mu}$	-0.0002	0.8756	Not Significant
	$\hat{\theta}_1$	0.1438	0.0311	Significant
	$\hat{\alpha}_0$	0.00006	0.0091	Significant
	$\hat{\alpha}_1$	0.2742	0.0009	Significant
	$\hat{\beta}_1$	0.7012	0.1142	Not Significant
	$\hat{\beta}_2$	0.0000	1.0000	Not Significant
	$\hat{\beta}_3$	0.0000	1.0000	Not Significant

Table 6 shows that all models have more than one significant parameter. This is indicated by a p-value that is smaller than the significance level. As an illustration, the ARIMA(0,0,1)-GARCH(3,0) mean return model shows that only the $t-1$ residual contributes. Then, the residual variance model at time t shows that only the squared residual in period $t-1$ and the residual variance of period $t-2$ contribute to the model.

4.10 Test and ARCH-LM test ARIMA-GARCH Model

The provisional model that has met the significance test is then tested diagnostically using the Ljung-Box test based on equation (9). The test results for AXIO stock are presented in Table 7.

Table 7: Diagnostic Test Results of ARIMA-GARCH Model

	ADF Test		ARCH-LM Test	
Model	*p-value*	Note	*p-value*	Note
ARIMA(0,0,1)-GARCH(3,0)	0.6364	*White Noise*	0.1082	There is no heteroscedasticity effect
ARIMA(0,0,1)-GARCH(1,3)	0.4752	*White Noise*	0.8661	There is no heteroscedasticity effect
ARIMA(0,0,1)-GARCH(2,3)	0.8018	*White Noise*	0.3586	There is no heteroscedasticity effect
ARIMA(0,0,1)-GARCH(3,1)	0.8129	*White Noise*	0.8715	There is no heteroscedasticity effect
ARIMA(0,0,1)-GARCH(3,2)	0.8486	*White Noise*	0.3851	There is no heteroscedasticity effect
ARIMA(0,0,1)-GARCH(3,3)	0.8489	*White Noise*	0.5496	There is no heteroscedasticity effect

Based on Table 7, it is obtained that all models meet the white noise criteria, with a p-value greater than the significance level ($\alpha = 5\%$). This means that the test criterion is to accept H_0 or there is no autocorrelation in the data. Table 7 also shows that all models have no heteroscedasticity effect, because the p-value is greater than the significance level ($\alpha = 5\%$) so the test criterion is to accept H_0.

4.11 Selection of the Best ARIMA-GARCH Model

Selection of the best ARIMA-GARCH model is done by calculating the AIC value based on equation (13). The results of the AIC value calculation are shown in Table 8.

Table 8: AIC Value of ARIMA-GARCH Model

Model	AIC value
ARIMA(0,0,1)-GARCH(3,0)	-4.2331
ARIMA(0,0,1)-GARCH(1,3)	-4.2577
ARIMA(0,0,1)-GARCH(2,3)	-4.2575
ARIMA(0,0,1)-GARCH(3,1)	-4.2607
ARIMA(0,0,1)-GARCH(3,2)	-4.2576
ARIMA(0,0,1)-GARCH(3,3)	-4.2519

The best model is selected based on the smallest AIC value, obtained the ARIMA (0,0,1)-GARCH (3,1) model for AXIO shares. Based on the calculation results, the ARIMA(2,0,0)-GARCH(1,1) model is obtained for DIVA shares, ARIMA(0,0,1)-GARCH(1,3) for EDGE shares, ARIMA(2,0,2)-GARCH(1,3) for MCAS shares, and ARIMA(1,0,0)-GARCH(1,0).

4.12 Stock Return Forecasting

The model that has been obtained in the previous stage is then forecast one period ahead for each stock. Forecasting includes two aspects, namely the mean return and volatility. The forecasting results are presented in Table 9.

Table 9: Mean Forecasting Results and Volatility of Stock Returns

Stocks	Model	Mean predicted return	Predicted volatility of return
AXIO	ARIMA(0,0,1)-GARCH(3,1)	0.002093	0.023221
DIVA	ARIMA(2,0,0)-GARCH(1,1)	0.0007793	0.09585
EDGE	ARIMA(0,0,1)-GARCH(1,3)	0.01175	0.0199
MCAS	ARIMA(2,0,2)-GARCH(1,3)	-0.004396	0.01584
CASH	ARIMA(1,0,0)-GARCH(1,0)	-0.00008558	0.0354

A positive mean return prediction indicates a potential increase in stock prices, while a negative one indicates a potential decrease in stock prices so investors should be careful.

4.13 Surplus Value Calculation

The calculation of surplus value involving asset return and liability return is carried out. The return on liabilities used is simulated data with the assumption of normal distribution which is generated as much as 349 data using EasyFit, then the mean value and variance are estimated, the results of which are presented in the following table.

Table 10: Mean and Volatility of Stock Liability

Stocks	Mean ($\hat{\mu}_L$)	Variance ($\hat{\sigma}_L^2$)
AXIO	-0.00005	0.00011
DIVA	0.00271	0.00012
EDGE	0.00020	0.00003
MCAS	0.00057	0.00001
CASH	0.00019	0.00007

Furthermore, the covariance value between asset return and liability return of each stock is calculated using Microsoft Excel. Then a gamma vector is formed which is the covariance between asset returns and liability returns as follows.

$$\boldsymbol{\gamma}^T = [-0.000008 \quad -0.000059 \quad 0.000002 \quad -0.000004 \quad 0.0000004]$$

To estimate the mean and variance of surplus return, it is assumed that the ratio between initial assets and liabilities is $f_0 = 1$.

Table 11: Mean and Volatility of Stock Surplus

Stocks	Mean ($\hat{\mu}_S$)	Variance ($\hat{\sigma}_S^2$)
AXIO	0.00214	0.02335
DIVA	-0.00194	0.09609
EDGE	0.01155	0.01993
MCAS	-0.00496	0.01585
CASH	-0.00027	0.03547

4.14 Formation of Mean Vector, Unit Vector, and Variance Covariance Matrix

Based on the mean value of surplus returns in Table 11, the following mean vector is formed:

$$\boldsymbol{\mu}_S^T = [0.00214 \quad -0.00194 \quad 0.01155 \quad -0.00496 \quad -0.00027]$$

Then, a unit vector is formed which has five elements, according to the number of stocks analyzed.

$$\mathbf{e}^T = [1 \quad 1 \quad 1 \quad 1 \quad 1]$$

Furthermore, the variance-covariance matrix between stock surplus returns is formed. The variance value of an asset uses the information in Table 11, while the covariance value between surplus returns obtained is very small, so it is assumed to be equal to zero. The inverse matrix is obtained as follows:

$$\boldsymbol{\Sigma}^{-1} = \begin{bmatrix} 42.8215 & 0 & 0 & 0 & 0 \\ 0 & 10.4069 & 0 & 0 & 0 \\ 0 & 0 & 50.1829 & 0 & 0 \\ 0 & 0 & 0 & 63.0724 & 0 \\ 0 & 0 & 0 & 0 & 28.1898 \end{bmatrix}$$

4.15 Mean-Variance Model Portfolio Optimization

In this section, the optimal portfolio of surplus returns is formed. Based on the values of the mean return surplus vector, the unit vector, and the inverse of the variance-covariance matrix, the optimization process is performed with reference to equation (21). In this process, risk tolerance values are simulated and tested starting with 0; 0.01; 0.02 and so on which are multiples of 0.01. The calculation is done using Microsoft Excel and the results are presented in Table 12.

Table 12: Portfolio Optimization Results

τ	$\mathbf{w}^T$ AXIO	DIVA	EDGE	MCAS	CASH	$\hat{\mu}_{S_p}$	$\hat{\sigma}^2_{S_p}$	Rasio
0	0.2199	0.0529	0.2582	0.3241	0.1450	0.0017	0.0051	0.33134
0.01	0.2200	0.0525	0.2631	0.3199	0.1444	0.0018	0.0051	0.34671
0.02	0.2202	0.0521	0.2681	0.3157	0.1439	0.0019	0.0051	0.36197
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0.6	0.2312	0.0302	0.5549	0.0720	0.1117	0.0065	0.0080	0.80764
0.61	0.2314	0.0298	0.5598	0.0678	0.1111	0.0065	0.0081	0.80785
0.62	0.2316	0.0295	0.5648	0.0636	0.1105	0.0066	0.0082	0.80790
0.63	0.2318	0.0291	0.5697	0.0594	0.1100	0.0067	0.0083	0.80780
0.64	0.2320	0.0287	0.5747	0.0552	0.1094	0.0068	0.0084	0.80755
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0.76	0.2343	0.0242	0.6340	0.0048	0.1028	0.0077	0.0097	0.79467
0.77	0.2345	0.0238	0.6390	0.0006	0.1022	0.0078	0.0098	0.79293
0.78	0.2346	0.0234	0.6439	−0.0036	0.1017	0.0079	0.0100	0.79110

From Table 12, for a risk tolerance of 0, the weights of the minimum portfolio for each stock are: AXIO by 0,2199; DIVA by 0,0529; EDGE by 0,2582; MCAS by 0,3241; and CASH by 0,1450. With a mean portfolio surplus return of 0,0017 and a variance of 0,0051, the smallest ratio of 0,33134 is obtained. Furthermore, for a risk tolerance of 0,62, the optimal portfolio is obtained because the ratio value of 0,80790 is the largest. This value has a weight for each of its shares, namely: AXIO by 0,2316; DIVA by 0,0295; EDGE by 0,5648; MCAS by 0,0636; and CASH by 0,1105. Then, for a risk tolerance of $\geq$ 0,78, the portfolio obtained is said to be not feasible because there is a negative portfolio weight.

The efficient frontier graph of the mean value of the surplus portfolio return and its variance with a risk tolerance limit of $0 \leq \tau < 0{,}78$ plotted using RStudio is presented in Figure 6.

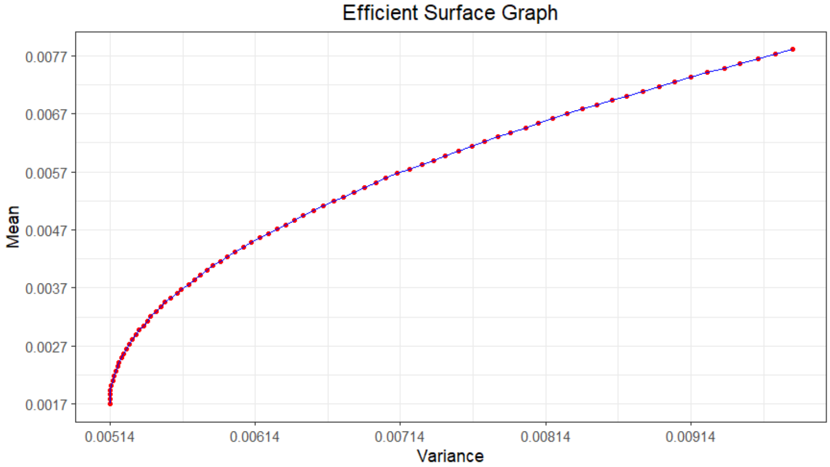

Figure 6: Efficient Surface Graph

In Figure 6, the horizontal axis shows the level of risk as measured by the variance, while the vertical axis shows the expected return. The shape of the curve illustrates that portfolios with higher variance provide higher mean returns. However, increased risk does not provide significant gains after a certain point, as can be seen in Figure 7.

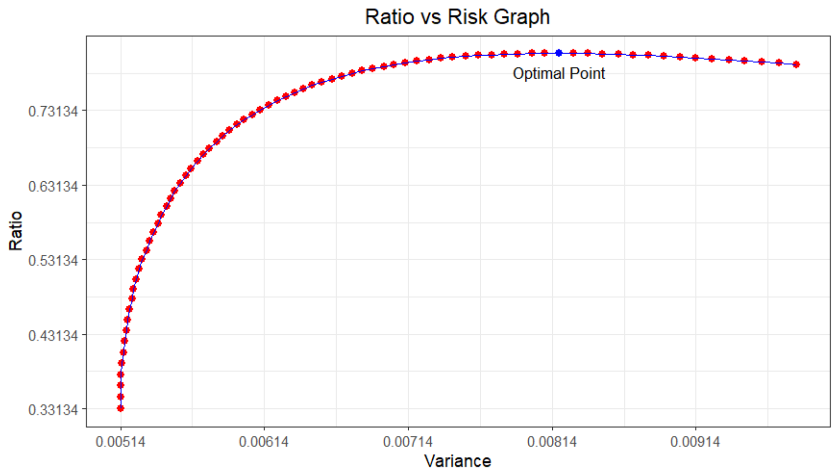

Figure 7: Ratio vs Risk Graph

Figure 7 shows that as the variance increases, the ratio increases significantly. But after reaching the highest point indicated by the optimal point, the ratio starts to decline even as the variance increases. At this point, the portfolio yields the highest ratio, which indicates the most efficient combination of risk and return.

5. Conclussion

The application of the ARIMA-GARCH model to the technology sector stock return data produces the best model based on the smallest AIC value, namely ARIMA(0,0,1)-GARCH(3,1) for AXIO stock, ARIMA(2,0,0)-GARCH(1,1) for DIVA, ARIMA(0,0,1)-GARCH(1,3) for EDGE, ARIMA(2,0,2)-GARCH(1,3) for MCAS, and ARIMA(1,0,0)-GARCH(1,0) for CASH. In the formation of the optimal portfolio, the weight allocation of each stock is obtained: AXIO by 23.16%; DIVA by 2.95%; EDGE by 56.48%; MCAS by 6.36%; and CASH by 11.05%. The composition of the weight allocation can produce a portfolio return of 0.0066 and a variance (risk) return of 0.0082.

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