Available online at
https://journal.rescollacomm.com/index.php/ijqrm/index

International Journal of Quantitative Research and Modeling

e-ISSN 2721-477X
p-ISSN 2722-5046

Vol. 6, No. 1, pp. 48-59, 2025

Investment Portfolio Optimization Using Mean-Variance Model with Data Envelopment Analysis (DEA) Approach on IDX30 Stocks

Veronica Clasrissa Putrie[1*], Dwi Susanti[2], Sukono[3]

[1,2]Undergrad Program in Mathematics, Faculty of Mathematics and Sciences, Universitas Padjadjaran, Sumedang, Jawa Barat, Indonesia
[2,3]Department of Mathematics, Faculty of Mathematics and Sciences, Universitas Padjadjaran, Sumedang, Jawa Barat, Indonesia

**Corresponding author email: veronica21003@mail.unpad.ac.id*

Abstract

Globalization and technological advancements are driving the importance of careful financial management, including in investments. Stocks have become a popular investment option as they offer potential profits from dividends and capital gains. However, the large selection of stocks in the Indonesian capital market, especially in the IDX30 index, makes investors face challenges in selecting efficient stocks and compiling optimal portfolios. Therefore, this research combines Data Envelopment Analysis (DEA) and Mean-Variance Model to screen efficient stocks and form an optimal investment portfolio. In this study, DEA is used to assess the efficiency of stocks based on company performance, while the Mean-Variance Model is used to determine the optimal weight in the portfolio by balancing risk and return. Of the 13 stocks analyzed, 9 efficient stocks were identified, namely ADRO, ASII, BBCA, BBNI, BBRI, INDF, KLBF, TLKM, and UNTR. The optimal portfolio is obtained with a risk tolerance value (τ) of 0.015, which results in an expected return of 0.00027711 and a variance of 0.00004396.

Keywords: Stock, return, risk, portfolio, DEA model, Mean-Variance model

1. Introduction

In the modern era, globalization and technological advances encourage the importance of careful financial management. With proper management, individuals and companies can make wiser financial decisions despite economic challenges (Aprilia et al., 2024). One of the steps that many choose to improve financial well-being is investment.

Stocks have become one of the most popular investment options because they offer potential profits from dividends and capital gains. One of the preferred indices is the IDX30, which includes 30 large-cap stocks with strong fundamentals (Negara et al., 2020).

The number of Indonesian capital market investors continues to increase significantly, reaching more than 13.6 million as of August 2024 (KSEI, 2024). However, the large number of stock options makes investors face challenges in selecting efficient stocks and compiling optimal portfolios. Diversification is important in minimizing investment risk (Shi, 2022).

The Mean-Variance model introduced by Markowitz (1952) is often used to form an optimal portfolio by balancing return and risk. However, not all stocks should be included in the portfolio. Therefore, Data Envelopment Analysis (DEA) can help filter efficient stocks based on company performance (Suryoaji & Cahyono, 2019).

There are several relevant previous studies, such as those conducted by (Farida & Azhari, 2018) using the DEA model to assess the efficiency of LQ45 stocks and find two efficient stocks even though the level of efficiency does not significantly affect returns. Nugroho & Rusydiana (2020) combined the DEA model with the Single Index Model to select ISSI stocks, resulting in 16 efficient stocks and 6 stocks forming the optimal portfolio. Gusliana & Salih (2022) with the Mean-Variance model produced optimal weights of several stocks, with a return of 0.00209 and a variance of 0.00015.

This study combines DEA and Mean-Variance models to select efficient stocks from IDX30 and form an optimal portfolio. This approach is expected to help investors make the right decision in compiling a portfolio that is not only efficient, but also optimal in minimizing risk and maximizing return.

2. Literature Review

2.1. Capital Market

The capital market is a trading venue for long-term financial instruments that brings together investors and issuers. In Indonesia, the capital market is managed by the Indonesia Stock Exchange (IDX) with the Jakarta Composite Index (JCI) as an indicator of market performance.

1) Market Return

$$R_{Mt} = \frac{JCI_t - JCI_{t-1}}{JCI_{t-1}} \tag{1}$$

R_{Mt} : market return in period t
JCI_t : Composite Stock Price Index in period t
JCI_{t-1} : Composite Stock Price Index in period $t-1$

2) Market Expected Return

$$E(R_M) = \frac{\sum_{t=1}^{n} R_{Mt}}{n} \tag{2}$$

$E(R_M)$: market expected
n : the amount of time

3) Market Variance

$$\sigma_M^2 = \frac{\sum_{t=1}^{n}(R_{Mt} - E(R_M))^2}{n-1} \tag{3}$$

σ_M^2 : market variance

2.2. Stocks

Stocks are securities that show the capital ownership of a person or an entity (business entity) in a company or limited liability company.

1) Return of Stocks

$$R_{it} = \frac{P_{it} - P_{i(t-1)}}{P_{i(t-1)}} \tag{4}$$

R_{it} : i-th stock return in period t
P_{it} : i-th stock price in period t
$P_{i(t-1)}$: i-th stock price in period $t-1$
t : t-th period

2) Expected Return of Stocks

$$E(R_i) = \frac{\sum_{t=1}^{n} R_{it}}{n} \tag{5}$$

$E(R_i)$: expected return of the i-th stock

3) Risk of Stocks

$$\sigma_i = \sqrt{\frac{\sum_{t=1}^{n}(R_{it} - E(R_i))^2}{n-1}} \tag{6}$$

σ_i : standard deviation of the i-th stock

According to (Anugrahayu & Azmi, 2023), in investing, risk is divided into two categories, namely systematic risk and unsystematic risk.

a. Systematic Risk
Systematic Risk is the risk that affects the entire market and is measured by the beta coefficient (β), which indicates how much a stock's return is sensitive to market movements.

$$\beta_i = \frac{Cov(R_i, R_M)}{\sigma_M^2} \tag{7}$$

β_i : beta coefficient
$Cov(R_i, R_M)$: covariance of the i-th stock return with the market return.

b. Unsystematic risk
Unsystematic risk is risk that comes from events within a company or industry, and can be reduced by diversification.

4) Variance

$$\sigma_i^2 = \frac{\sum_{t=1}^{n}(R_{it} - E(R_i))^2}{n-1} \tag{8}$$

σ_i^2 : variance of the i-th stock

5) Covariance

$$\sigma_{ij} = \frac{\sum_{t=1}^{n}((R_{it}-E(R_i))(R_{jt}-E(R_j)))}{n-1} \tag{9}$$

σ_{ij} : covariance of i-th stock return and j-th stock return

2.3. Risk-Free Aset Return

Risk-free asset returns are returns with certainty without the risk of loss (Rahmadi et al., 2024). Time deposit interest rates are the returns that banks give on customer deposits within a certain period.

$$dailyR_f = \left(1 + annualR_f\right)^{\frac{1}{c}} - 1 \tag{10}$$

R_f : risk-free asset return

c : number of trading days in one year

2.4. Portfolio

A portfolio is a collection of financial assets held by investors. According to Gafur (2024), a portfolio is defined as a form of investment in many financial instruments traded on the Stock Exchange and Money Market with the aim of diversifying sources of return and minimizing potential risks. Investment risk can be minimized with a portfolio through diversification to maximize returns for risk-tolerant investors and manage risk for risk-averse investors (Subathra, 2022).

According to Shi (2022), an efficient portfolio has the highest expected return for the same risk or the lowest risk for the same expected return. The optimal portfolio is the best choice from the set of efficient portfolios.

1) Expected Return of Portfolio

$$\mu_p = \sum_{i=1}^{k} w_i E(R_i) \tag{11}$$

μ_p : expected return of portfolio

Σw_i : total weight

2) Risk of Portfolio

$$\sigma_p^2 = w_i^2\sigma_i^2 + w_j^2\sigma_j^2 + 2w_iw_j\sigma_{ij} = \sum_{i=1}^{k}\sum_{j=1}^{k} w_iw_j\sigma_{ij}, \tag{12}$$

σ_p^2 : risk of portfolio

w_j : j-th stock weight

σ_j^2 : j-th stock variance

2.5. Mean-Variance Model

The Mean-Variance model (Markowitz, 1952) maximizes the return and minimizes the risk of a portfolio by using the average return as the expected return and the variance as a measure of risk (Yu, 2023). According to Gusliana & Salih (2022), an efficient portfolio is obtained by maximizing $2\tau\mu_p - \sigma_p^2$, with the condition that $\sum_{i=1}^{N} w_i = 1$ and $\tau \geq 0$. The value of τ indicates the risk tolerance, the larger τ is, the higher the risk of the portfolio chosen for a larger return.

$$\mu_p = E(R_p) = \boldsymbol{\mu}^T\mathbf{w} = \mathbf{w}^T\boldsymbol{\mu} \tag{13}$$

$$\sigma_p^2 = Var(R_p) = \mathbf{w}^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{w} \tag{14}$$

$\boldsymbol{\mu}^T$: transpose stock return expectation vector

$\mathbf{w}$: portfolio weight vector

$\mathbf{w}^T$: transpose portfolio weight vector

$\boldsymbol{\mu}$: stock expected return vector

$\boldsymbol{\Sigma}$: covariance matrix of return variance between stocks

In Mean-Variance portfolio optimization, an efficient portfolio (p^*) is said to be efficient if there exists a portfolio p with $\mu_p \geq \mu_{p^*}$ and $\sigma_p^2 < \sigma_{p^*}^2$. Objective function for portfolio optimization:

$$\max\{2\tau\boldsymbol{\mu}^T\mathbf{w} - \mathbf{w}^T\Sigma\mathbf{w}\} \tag{15}$$
$$s.t\ \mathbf{e}^T\mathbf{w} = 1$$

$\mathbf{e}^T$: transpose vector with entry 1

If the value of ≥ 0, then the portfolio can be expressed as a weight vector.

$$\mathbf{w} = \frac{1}{\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\mathbf{e}}\boldsymbol{\Sigma}^{-1}\mathbf{e} + \tau\left(\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} - \frac{\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\mathbf{e}}\boldsymbol{\Sigma}^{-1}\mathbf{e}\right) \tag{16}$$

2.6. Data Envelopment Analysis (DEA)

Data Envelopment Analysis (DEA) is a mathematical programming technique to evaluate the performance efficiency of decision-making units (DMUs) in various sectors in organizations or other entities.

1) DEA-CCR Model (CRS Model)
The DEA-CCR model is the first model in DEA with the assumption of Constant Return to Scale (CRS), where the ratio of increasing input and output remains the same (Farida & Azhari, 2018). This model assumes that each DMU operates optimally.

$$\min z^{(k)} = \theta^{(k)} \tag{17}$$

$$s.t. \sum_{j=1}^{n} y_j^{(r)} \mu_j \geq y^{(r,k)}\,; r = 1,2,3, \dots, s \tag{18}$$

$$\theta^{(k)} x^{(i,k)} - \sum_{j=1}^{n} x_j^{(i)} \mu_j \geq 0,\ i = 1,2,3, \dots, m \tag{19}$$

$$\mu_j \geq 0; j = 1,2,3, \dots, n \tag{20}$$

$z^{(k)}$: scalar/reduction rate of the k-th DMU's total input
i : i-th input
r : r-th output
j : j-th DMU
k : DMU under study
$\theta^{(k)}$: input reduction rate of the kth DMU
$x_j^{(i)}$: i-th input value for the j-th unit
$y_j^{(r)}$: r-th output value for the j-th unit
μ_j : j-th DMU weight for the calculated DMU

2) DEA-BCC Model (VRS Model)
The DEA-BCC model extends DEA-CCR with the assumption of Variable Return to Scale (VRS), where an increase in input is not always directly proportional to output (Farida & Azhari, 2018). This model is more flexible because not all DMUs operate optimally and adds convexity constraints, namely $\sum_{j=1}^{n} \mu_j = 1$.

$$\min z^{(k)} = \theta^{(k)} \tag{21}$$

$$s.t. \sum_{j=1}^{n} y_j^{(r)} \mu_j \geq y^{(r,k)}\,; r = 1,2,3, \dots, s \tag{22}$$

$$\theta^{(k)} x^{(i,k)} - \sum_{j=1}^{n} x_j^{(i)} \mu_j \geq 0,\ i = 1,2,3, \dots, m \tag{23}$$

$$\mu_j \geq 0; j = 1,2,3, \dots, n \tag{24}$$

$$\sum_{j=1}^{n} \mu_j = 1 \tag{25}$$

3) Input-Output Variables
The input variables used in this study are standard deviation, beta risk coefficient, DER ratio, and PER ratio. Meanwhile, the output variables consist of return, EPS ratio, BV ratio, PBV ratio, ROE ratio, ROA ratio, and NPM ratio.

$$DER = \frac{Total\ liabilities/debt}{Total\ equity} \tag{26}$$

$$PER = \frac{Stock\ price}{EPS} \tag{27}$$

$$EPS = \frac{Profit\ after\ tax}{Number\ of\ stock\ outstanding} \tag{28}$$

$$BV = \frac{Total\ equity}{Number\ of\ stock\ outstanding} \tag{29}$$

$$PBV = \frac{Total\ equity}{Book\ value} \tag{30}$$

$$ROE = \frac{Net\ profit\ after\ tax}{Total\ equity} \times 100\% \tag{31}$$

$$ROA = \frac{Net\ profit\ after\ tax}{Total\ assets} \times 100\% \tag{32}$$

$$NPM = \frac{et\ profit\ after\ tax}{Total\ sales} \times 100\% \tag{33}$$

4) Efficiency Scale
The efficiency scale is calculated from the comparison of the technical efficiency of CRS and VRS, which is obtained through the DEA-CCR and DEA-BCC models. According to (Kraidi et al., 2024), the efficiency scale shows how effectively a DMU operates compared to other units assuming constant returns to scale. If $ES = 1$, the DMU is considered optimal and efficient.

$$SE = \frac{Technical\ Efficiency\ CRS}{Technical\ Efficiency\ VRS} = \frac{TE_{CRS}}{TE_{VRS}} \tag{34}$$

5) Undiserable Variable

In the DEA model, there is a condition that must be met, namely that the input and output values must be greater than or equal to zero. According to Nugroho & Rusydiana (2020), variables such as beta and stock returns are often negative, so they are called undesirable variables. To overcome this, variables can be increased by the lowest negative value plus one so that all data becomes positive.

$$X = X + a \tag{35}$$

$$a = Min\{X\} + 1 \tag{36}$$

X: variable that has a negative value
a : adjustment constant

3. Materials and Methods

3.1. Materials

This research discusses stock portfolio optimization with the Data Envelopment Analysis (DEA) approach and the Mean-Variance model. The data used is the daily closing price of stock from companies included in the IDX30 during the period January 1, 2023 to December 31, 2023, as well as JCI data and annual financial reports obtained from www.investing.com.

3.2. Methods

This research method uses a quantitative approach with concrete data in the form of closing stock prices and financial reports. The research steps are as follow:

1) Calculate Return and Expected Return
   Calculate individual stock returns, JCI returns, and risk-free asset returns.
2) Calculate Stock Risk
3) Calculate Beta Risk Coefficient
4) Calculate Financial Ratios
5) Determine Input-Output Variables
6) Convert negative values in input-output variables to qualify the DEA model
7) Building the DEA Model
   Calculate the value of technical efficiency CRS (TE CRS) and technical efficiency VRS (TE VRS) using the DEA-BCC and DEA-CCR models, and choose the model that is closer to the efficiency value of one.
8) Calculate Efficiency Scale
   Compare TE CRS and TE VRS to calculate the efficiency scale and select efficient stocks as portfolio candidates.
9) Mean-Variance Model Calculation
   Calculate the expected return, variance, covariance, and weight of the efficient portfolio, and determine the optimal portfolio based on the ratio of expected return to variance.

4. Results and Discussion

4.1. Stock Data Graphs

Graphs of the two stocks are given to visualize the movement of stock prices during the study period. The data processing and visualization was done using Microsoft Excel with the resultant graphs shown in Figure 1 and Figure 2.

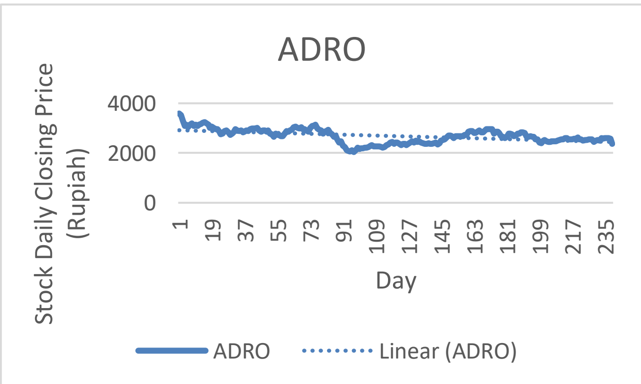

Figure 1: ADRO Stock Daily Closing Price Data Chart

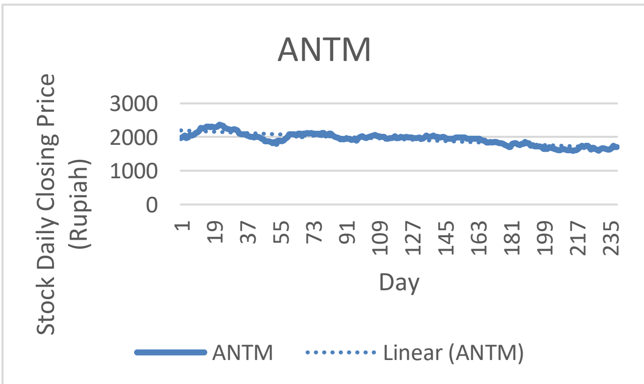

Figure 2: ANTM Stock Daily Closing Price Data Chart

Figures 1 and Figure 2 show the daily closing prices of ADRO and ANTM stock which fluctuate with a downward trend, characterized by a negative trendline. This indicates the possibility of ADRO and ANTM stock returns being negative during the study period.

4.2. Return and Expected Return of Individual Stock, JCI, and Risk-Free Asset Return

Return is the profit earned by investors in investing. The expected return value is used to help investors make decisions by considering the potential profits and risks in investing.

Table 1: Expected return values of 13 IDX30 stocks and JCI

No	Stocks	Expected Return
1	ADRO	-0.00143702
2	ANTM	-0.00048982
3	ASII	0.00007328
4	BBCA	0.00045858
5	BBNI	0.00072937
6	BBRI	0.00076878
7	BMRI	0.00096690
8	CPIN	-0.00030518
9	INDF	-0.00013066
10	KLBF	-0.00083910
11	SMGR	0.00001682
12	TLKM	0.00024279
13	UNTR	-0.00033092
IHSG		0.00026983

Based on Table 1, there are 7 companies with positive and 6 negative expected returns. A positive expected return indicates a potential gain, while a negative return indicates a loss. The JCI expected return value is positive, indicating that the market as a whole has the potential to provide profits. BMRI has the highest expected return (0.09669%) and ADRO the lowest (-0.143702%). Risk-free assets using a 12-month time deposit interest rate of 5.54% per annum produces a risk-free asset return of 0.00021380.

4.3. Stock Risk

Stock risk is calculated through standard deviation, which describes the deviation between expected return and actual return. Variance is calculated first before calculating standard deviation. The results of the calculation of variance and standard deviation of other stocks are presented in Table 2.

Table 2: Value variance and stock risk of 13 IDX30 stocks

No	Stocks	Variance	Risk
1	ADRO	0.00057247	0.02392634
2	ANTM	0.00027761	0.01666164
3	ASII	0.00022145	0.01488120
4	BBCA	0.00012108	0.01100354
5	BBNI	0.00017359	0.01317541
6	BBRI	0.00017798	0.01334095
7	BMRI	0.00022614	0.01503797
8	CPIN	0.00041289	0.02031958
9	INDF	0.00015132	0.01230119
10	KLBF	0.00039346	0.01983585
11	SMGR	0.00032600	0.01805543
12	TLKM	0.00016021	0.01265737
13	UNTR	0.00044161	0.02101454

4.4. Beta Risk Coefficient

The beta risk coefficient measures the sensitivity of stock returns to movements in market returns, calculated by comparing the covariance of stock and market returns with the variance of market returns. The value of the market return variance is 0.00003744, and the value of the stock return covariance with the market is presented in Table 3.

Table 3: Value of beta risk coefficient (β) of the stock

No	Stocks	Beta Risk Coefficient (β)
1	ADRO	1.60124491
2	ANTM	0.95919672
3	ASII	0.91357396
4	BBCA	0.88018834
5	BBNI	0.83420788
6	BBRI	1.20223133
7	BMRI	1.43509815
8	CPIN	0.75589344
9	INDF	0.18539855
10	KLBF	0.68987503
11	SMGR	0.98502405
12	TLKM	0.63641490
13	UNTR	1.27773274

4.5. Financial Ratios

The calculated financial ratios are presented in Table 4.

Table 4: Value of financial ratios

No	Stocks	DER	PER	EPS	BV	PBV	ROE	ROA	NPM
1	ADRO	0.41355978	44,828.60076	0.05309111	0.23963072	475,651,738,461.54	22.15538384	17.7115362	28.45971711
2	ANTM	0.37495292	13.31291979	128.0710789	1,296.907111	24,030,765,000	9.87511579	7.18215297	7.49773684
3	ASII	0.20374027	6.75949644	835.8610809	2,076.32645	40,484,000,000	40.2567275	9.9849892	14.05746055
4	BBCA	4.77317982	23.82414448	394.5577146	1,967.450778	123,275,050,000	20.05426103	3.45556798	48.68468999
5	BBNI	6.02285458	9.57725057	561.2257882	4,153.135186	37,256,798,316	13.51330412	1.94229571	33.63681522
6	BBRI	5.20909954	14.37264002	398.3262638	2,097.495063	150,880,994,955	18.99056979	3.07505505	32.96685387
7	BMRI	5.77555448	17.50210078	589.9291822	3,080.303164	93,333,333,332	19.15165978	2.76199673	43.34858949
8	CPIN	0.51582252	35.53891082	141.3943164	1,648.296012	16,398,000,000	8.5782114	5.6579027	3.76216185
9	INDF	0.8572454	6.95146911	927.8614199	11,441.9147	8,780,426,500	8.1093195	6.15995436	10.289491
10	KLBF	0.17030893	26.91664785	59.81428331	499.8314001	46,255,641,410	11.96689189	10.2684942	9.12474165
11	SMGR	0.66462143	19.90781677	321.4817614	7,080.010689	6,751,540,089	4.54069599	2.80565407	5.93925026
12	TLKM	0.83340785	15.9322376	247.9249995	1,580.441114	99,062,216,600	15.68707605	11.2206576	21.58481664
13	UNTR	0.83289363	3.98654064	5,675.346639	23,138.7617	3,631,809,000	24.52744322	14.36755679	17.21071259

4.6. Input-Output Variables

Based on Table 3, beta is already positive. Therefore, only the expected return variable must be converted so that the value becomes positive and is presented in Table 5 and Table 6.

Table 5: Input variable value of each DMU

No	DMU	SD (x_1)	BETA (x_2)	DER (x_3)	PER (x_4)
1	ADRO	0.02392634	1.60124491	0.41355978	44,828.60075653
2	ANTM	0.01666164	0.95919672	0.37495292	13.31291979
3	ASII	0.01488120	0.91357396	0.20374027	6.75949644
4	BBCA	0.01100354	0.88018834	4.77317982	23.82414448
5	BBNI	0.01317541	0.83420788	6.02285458	9.57725057

6	BBRI	0.01334095	1.20223133	5.20909954	14.37264002
7	BMRI	0.01503797	1.43509815	5.77555448	17.50210078
8	CPIN	0.02031958	0.75589344	0.51582252	35.53891082
9	INDF	0.01230119	0.18539855	0.85724540	6.95146911
10	KLBF	0.01983585	0.68987503	0.17030893	26.91664785
11	SMGR	0.01805543	0.98502405	0.66462143	19.90781677
12	TLKM	0.01265737	0.63641490	0.83340785	15.93223760
13	UNTR	0.02101454	1.27773274	0.83289363	3.98654064

Table 6: Output variable value of each DMU

No	DMU	RETURN (y_1)	EPS (y_2)	BV (y_3)	PBV (y_4)	ROE (y_5)	ROA (y_6)	NPM (y_7)
1	ADRO	0.9969016	0.05309111	0.23963072	475,651,738,461.54	22.15538384	17.7115362	28.45971711
2	ANTM	0.9978488	128.07107890	1,296.907111	24,030,765,000	9.87511579	7.18215297	7.49773684
3	ASII	0.9984119	835.86108090	2,076.32645	40,484,000,000	40.2567275	9.9849892	14.05746055
4	BBCA	0.9987972	394.55771460	1,967.450778	123,275,050,000	20.05426103	3.45556798	48.68468999
5	BBNI	0.999068	561.22578820	4,153.135186	37,256,798,316	13.51330412	1.94229571	33.63681522
6	BBRI	0.9991074	398.32626380	2,097.495063	150,880,994,955	18.99056979	3.07505505	32.96685387
7	BMRI	0.9993056	589.92918220	3,080.30316400	93,333,333,332	19.15165978	2.76199673	43.34858949
8	CPIN	0.9980335	141.39431640	1,648.29601200	16,398,000,000	8.5782114	5.6579027	3.76216185
9	INDF	0.998208	927.86141990	11,441.91470000	8,780,426,500	8.1093195	6.15995436	10.289491
10	KLBF	0.9974996	59.81428331	499.83140010	46,255,641,410	11.96689189	10.2684942	9.12474165
11	SMGR	0.9983555	321.48176140	7,080.01068900	6,751,540,089	4.54069599	2.80565407	5.93925026
12	TLKM	0.9985815	247.92499950	1,580.44111400	99,062,216,600	15.68707605	11.2206576	21.58481664
13	UNTR	0.9980077	5,675.34663900	23,138.76170000	3,631,809,000	24.52744322	14.36755679	17.21071259

4.7. Conversion of Negative Values in Input-Output Variables

Based on Table 3, beta is already positive. Therefore, only the expected return variable must be converted so that the value becomes positive and is presented in Table 7.

Table 7: Expected return conversion yield

No	Stocks	Expected Return
1	ADRO	0.99690163
2	ANTM	0.99784883
3	ASII	0.99841193
4	BBCA	0.99879724
5	BBNI	0.99906802
6	BBRI	0.99910743
7	BMRI	0.99930556
8	CPIN	0.99803348
9	INDF	0.99820799
10	KLBF	0.99749956
11	SMGR	0.99835548
12	TLKM	0.99858145
13	UNTR	0.99800774

4.8. DEA Model Formation and Calculate Efficiency Scale

The DEA-CCR model is used to find the technical efficiency CRS (TE CRS) value. The DEA-BCC model is almost the same as the DEA-CCR model with the only difference being the addition of one constraint function in the DEA-BCC model. The DEA-BCC model is used to find the value of technical efficiency VRS (TE VRS).The efficiency scale is obtained by comparing the value of technical efficiency CRS and the value of technical efficiency VRS. This technical efficiency value shows the ability of a number of DMUs to produce the maximum possible output from a number of inputs. The calculation uses DEAP 2.1 software and Python to obtain the technical efficiency CRS value from the DEA-CCR model and the technical efficiency VRS value from the DEA-BCC model, as well as the efficiency scale presented in Table 8.

Table 8: Efficiency scores of 13 IDX30 stocks

No	Stocks	DEA-CCR (TE CRS)	DEA-BCC (TE VRS)	Efficiency Scale
1	ADRO	1	1	1
2	ANTM	0.86414701	0.86460211	0.99947363
3	ASII	1	1	1
4	BBCA	1	1	1
5	BBNI	1	1	1
6	BBRI	1	1	1
7	BMRI	0.99021667	1	0.99021667
8	CPIN	0.78312614	0.79026456	0.99096705
9	INDF	1	1	1
10	KLBF	1	1	1
11	SMGR	0.77672590	0.84696696	0.91706753
12	TLKM	1	1	1
13	UNTR	1	1	1

The efficiency scale (ES) is used to determine whether a DMU is operating optimally. If $ES < 1$, the DMU is considered inefficient. Based on Table 8, the DMUs that operate optimally are ADRO, ASII, BBCA, BBNI, BBRI, INDF, KLBF, TLKM, and UNTR, which are selected as optimal portfolio candidates using the Mean-Variance model.

4.9. Optimal Portfolio Formation Using the Mean-Variance Model

4.9.1. Shape of Vectors μ, e, and Matrix Σ

The vector $\boldsymbol{\mu}$ is a vector with entries of expected return values. The vector $\mathbf{e}$ is a vector with entries of value one with as many rows as the selected efficient stocks. $\boldsymbol{\Sigma}$ matrix is a matrix with entries from the calculation of variance and covariance values among the selected efficient stocks in Table 9.

$$\boldsymbol{\mu} = \begin{bmatrix} -0.00143702 \\ 0.00007328 \\ 0{,}00045858 \\ 0.00072937 \\ 0.00076878 \\ -0.00013066 \\ -0.00083910 \\ 0.00024279 \\ -0.00033092 \end{bmatrix}$$

$$\mathbf{e} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Table 9: Variance-covariance of 9 efficient stocks

Stocks	ADRO	ASII	BBCA	BBNI	BBRI	INDF	KLBF	TLKM	UNTR	ADRO
ADRO	0.00057247	0.00005874	0.00001120	-0.00000848	0.00001291	-0.00001728	-0.00001865	0.00002000	0.00024069	0.00057247
ASII	0.00005874	0.00022145	0.00001999	0.00001707	0.00002883	0.00000370	-0.00000945	0.00002098	0.00010452	0.00005874
BBCA	0.00001120	0.00001999	0.00012108	0.00005644	0.00006509	0.00001556	0.00001388	0.00002054	0.00002998	0.00001120
BBNI	-0.00000848	0.00001707	0.00005644	0.00017359	0.00005233	0.00001985	0.00000838	0.00003068	-0.00000747	-0.00000848
BBRI	0.00001291	0.00002883	0.00006509	0.00005233	0.00017798	-0.00000120	0.00003432	0.00002651	0.00001135	0.00001291
INDF	-0.00001728	0.00000370	0.00001556	0.00001985	-0.00000120	0.00015132	0.00003168	-0.00000570	-0.00001006	-0.00001728
KLBF	-0.00001865	-0.00000945	0.00001388	0.00000838	0.00003432	0.00003168	0.00039346	0.00004420	-0.00001705	-0.00001865
TLKM	0.00002000	0.00002098	0.00002054	0.00003068	0.00002651	-0.00000570	0.00004420	0.00016021	0.00001571	0.00002000
UNTR	0.00024069	0.00010452	0.00002998	-0.00000747	0.00001135	-0.00001006	-0.00001705	0.00001571	0.00044161	0.00024069

4.9.2. Investment Portfolio Optimization

After forming the $\boldsymbol{\mu}$ vector, $\mathbf{e}$, and $\boldsymbol{\Sigma}$ matrix, the next step is to calculate the efficient stock weight vector with risk tolerance (τ) values starting from 0. The ratio of expected return to portfolio variance is calculated to determine the optimal portfolio. The calculation is done using Microsoft Excel, and the results are presented in Table 10.

Table 10: Portfolio optimization results

τ	Bobot Saham ($\mathbf{w}^{\mathrm{T}}$)									μ_p	σ_p^2	$\frac{\mu_p}{\sigma_p^2}$
	ADRO	ASII	BBCA	BBNI	BBRI	INDF	KLBF	TLKM	UNTR			
0.000	0.04665514	0.107517094	0.150679565	0.092651925	0.091440398	0.245773107	0.056431269	0.175673601	0.033177901	0.00010002	0.00004131	2.42120788
0.001	0.04355769	0.107080134	0.151199232	0.095122126	0.094951742	0.244228295	0.053557442	0.176571455	0.033731885	0.00011182	0.00004132	2.70624826
0.002	0.040460239	0.106643175	0.151718899	0.097592327	0.098463086	0.242683483	0.050683615	0.177469308	0.034285869	0.00012363	0.00004136	2.98941793
0.003	0.037362789	0.106206215	0.152238566	0.100062529	0.101974429	0.241138671	0.047809787	0.178367162	0.034839852	0.00013544	0.00004141	3.27023735
0.004	0.034265339	0.105769256	0.152758232	0.10253273	0.105485773	0.239593859	0.04493596	0.179265015	0.035393836	0.00014724	0.00004150	3.54823716
0.005	0.031167888	0.105332296	0.153277899	0.105002932	0.108997117	0.238049047	0.042062132	0.180162869	0.03594782	0.00015905	0.00004160	3.82296080
0.006	0.028070438	0.104895337	0.153797566	0.107473133	0.112508461	0.236504234	0.039188305	0.181060722	0.036501804	0.00017085	0.00004173	4.09396699
0.007	0.024972987	0.104458378	0.154317233	0.109943334	0.116019805	0.234959422	0.036314477	0.181958576	0.037055787	0.00018266	0.00004189	4.36083198
0.008	0.021875537	0.104021418	0.1548369	0.112413536	0.119531149	0.23341461	0.033440650	0.182856429	0.037609771	0.00019447	0.00004206	4.62315167
0.009	0.018778087	0.103584459	0.155356567	0.114883737	0.123042493	0.231869798	0.030566823	0.183754283	0.038163755	0.00020627	0.00004226	4.88054354
0.010	0.015680636	0.103147499	0.155876234	0.117353939	0.126553837	0.230324986	0.027692995	0.184652136	0.038717739	0.00021808	0.00004249	5.13264828
0.011	0.012583186	0.10271054	0.15639559	0.11982414	0.130065181	0.228780174	0.024819168	0.18554999	0.039271722	0.00022989	0.00004274	5.37913128

0.012	0.009485735	0.102273581	0.156915567	0.122294341	0.133576525	0.227235361	0.02194534	0.186447843	0.039825706	0.00024169	0.00004301	5.61968378
0.013	0.006388285	0.101836621	0.157435234	0.124764543	0.137087869	0.225690549	0.019071513	0.187345697	0.04037969	0.00025350	0.00004330	5.85402383
0.014	0.003290834	0.101399662	0.157954901	0.127234744	0.140599213	0.224145737	0.016197686	0.18824355	0.040933674	0.00026531	0.00004362	6.08189700
0.015	0.000193384	0.100962702	0.158474568	0.129704945	0.144110556	0.222600925	0.013323858	0.189141404	0.041487657	0.00027711	0.00004396	6.30307678
0.016	-0.002904066	0.100525743	0.158994235	0.132175147	0.1476219	0.221056113	0.010450031	0.190039257	0.042041641	0.00028892	0.00004433	6.51736483

The results show that an increase in τ increases the ratio of expected return and portfolio variance, with the risk tolerance value taken in the interval $0 \leq \tau \leq 0.015$ because when the tolerance value $\tau > 0.015$ results in the value of $w_i < 0$.

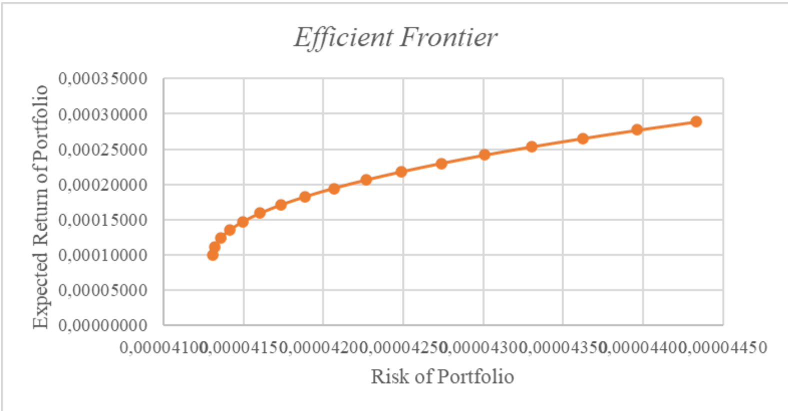

Figure 3: Efficient Frontier Graph

Based on Table 10, the efficient portfolio is along the line with a risk tolerance value in the interval 0≤τ≤0.016 with the highest expected return value of 0.00027711 and the lowest expected return value of 0.00010002 as shown by the efficient frontier graph in Figure 3.

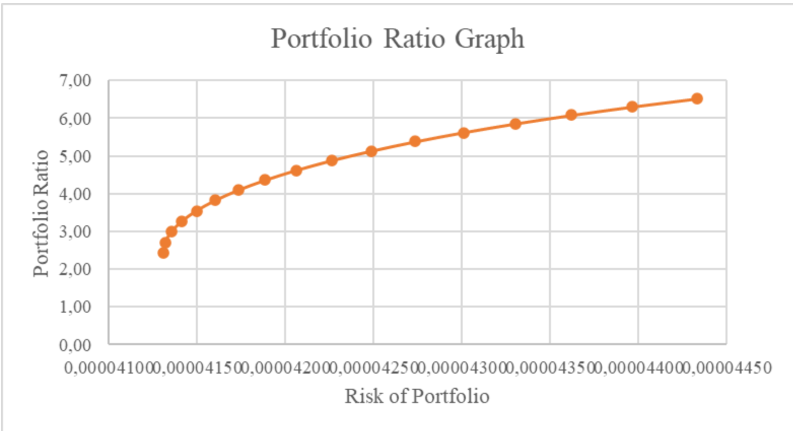

Figure 4: Portfolio Ratio Chart

Based on Figure 4.4, the ratio of expected return to portfolio variance increases in the interval $0 \leq \tau \leq 0.016$, with the largest ratio reached at $\tau = 0.015$. At this point, the expected return of the portfolio is 0.00027711 and the variance is 0.00004396, with a profit of about 0.028%. So, the optimal portfolio using the Mean-Variance model is obtained at $\tau = 0.015$.

5. Conclussion

Based on the research results and discussion, the conclusions are as follows:

1) Of the 13 stocks studied, 9 efficient stocks were selected that have an efficient scale value equal to one ($ES = 1$), which indicates that these stocks have operated optimally. The efficient stocks are ADRO, ASII, BBCA, BBNI, BBRI, INDF, KLBF, TLKM, and UNTR. These stocks are candidates for optimal portfolio formation using the Mean-Variance model.

2) The composition of the optimal investment portfolio weight for ADRO is 0.02%, ASII 10.1%, BBCA 15.85%, BBNI 12.97%, BBRI 14.41%, INDF 22.26%, KLBF 1.33%, TLKM 18.91%, and UNTR 4.15%. The optimal portfolio is obtained when the risk tolerance value is $\tau = 0.015$, with an expected return of 0.00027711 and a variance of 0.00004396.

References

Anugrahayu, M., & Azmi, U. (2023). Stock Portfolio Optimization Using Mean-Variance and Mean Absolute Deviation Model Based On K-Medoids Clustering by Dynamic Time Warping. *Jurnal Matematika, Statistika Dan Komputasi*, *20*(1), 164–183. https://doi.org/10.20956/j.v20i1.27755

Aprilia, E., Apriliani, H. J., Ridiansyah, M. H. R., & Djasuli, M. (2024). The Effect of Financial Literacy on Investment Decisions for Millennial Investors. Journal of Management Economics, *28*(5), 89–98. https://jurnalhost.com/index.php/jekma/article/view/1069

Farida, N., & Azhari, M. (2018). Efficiency Measurement Using DEA and Its Effect on Stock Return. SIKAP (Information Systems, Finance, Auditing and Taxation), *2*(2), 112–121. http://jurnal.usbypkp.ac.id/index.php/sikap

Gafur, A. (2024). Formation of Optimal Investment Portfolios Using the Markowitz Model and the Single Index Model on Risk-Free Assets and Risky Assets (LQ45 Stocks on the Indonesia Stock Exchange). *Journal of Management and Innovation Entrepreunership (JMIE)*, *1*(2), 228–245. https://doi.org/10.59407/jmie.v1i2.338

Gusliana, S. A., & Salih, Y. (2022). Mean-Variance Investment Portfolio Optimization Model Without Risk-Free Assets in Jii70 Share. *International Journal of Business, Economics and Social Development*, *3*(4), 168–173. https://doi.org/10.46336/ijbesd.v3i4.352

Kraidi, A. A., Daneshvar, S., & Adesina, K. A. (2024). Weight-restricted approach on constant returns to scale DEA models: Efficiency of internet banking in Turkey. *Heliyon*, *10*(10). https://doi.org/10.1016/j.heliyon.2024.e31008

Negara, I. N. W., Langi, Y., & Manurung, T. (2020). Markowitz Mean-Variance Model Stock Portfolio Analysis Using the Lagrange Method. D'Cartesian: Journal of Mathematics and Applications, *9*(2), 173–180. https://doi.org/10.35799/dc.9.2.2020.29656

Nugroho, T., & Rusydiana, A. S. (2020). The Selection of Efficient Sharia Stocks for the Formation of Optimal Portfolio. *Jurnal Ekonomi Syariah Teori Dan Terapan*, *7*(5), 985–1001. https://doi.org/10.20473/vol7iss20205pp985-1001

Rahmadi, M. D., Yahya, L., & Nuha, A. R. (2024). *Business Index 27 Stock Portfolio Optimization Using the Black Litterman Model Accompanied by Value At Risk Calculation*. *21*(1), 136–146. https://doi.org/10.20956/j.v21i1.36306

Shi, H. (2022). Portfolio Optimization for US. Stock with Mean-variance Model, CAPM, Fama French Three-factor Model. *BCP Business & Management PGMEE*, *35*, 737–744. https://doi.org/10.54691/bcpbm.v35i.3390

Subathra. (2022). Selection Among Two Competing Objectives for an Optimal Portfolio with Respect to the Investor's Attitude. *International Journal of Scientific Research in Science and Technology*, *9*(4), 123–131. https://doi.org/10.32628/IJSRST22949

Suryoaji, O., & Cahyono, E. F. (2019). Comparative Efficiency & Productivity of Conventional and Sharia Life Insurance Companies in Indonesia in 2014 - 2017, with DEA & Malmquist Index Approaches. Journal of Islamic Economics Theory and Applied, *6*(9), 1877–1893. https://doi.org/10.20473/vol6iss20199pp1877-1893

Yu, J. (2023). On Frontier Portfolio in Shanghai Stock Exchange Based on Mean Variance Model. *Advances in Economics, Management and Political Sciences*, *45*(1), 38–46. https://doi.org/10.54254/2754-1169/45/20230252