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Investment Portfolio Optimization Using Genetic Algorithm on Infrastructure Sector Stocks Based on the Single Index Model

Ayyinah Nur Bayyinah[1], Riaman[2], Sukono[3]

[1]*Mathematics Undergraduate Study Program, Faculty of Mathematics and Natural Science, Universitas Padjadjaran, Sumedang, Indonesia*
[2,3]*Department of Mathematics, Faculty of Mathematics and Natural Science, Universitas Padjadjaran, Sumedang, Indonesia*

**Corresponding author email: ayyinah21001@mail.unpad.ac.id*

Abstract

Investment is a strategic step in managing assets to gain profits in the future by allocating some funds in the present. However, behind the promising potential returns, investment also contains risks that cannot be ignored. One way to reduce the level of risk in investing is to implement a portfolio diversification strategy, which is to form an optimal portfolio by allocating investments to various stocks. This study aims to identify the stocks that form the optimal portfolio, determine the optimal weight of each stock, and calculate the expected return and risk of the portfolio. The portfolio optimization process is carried out using Genetic Algorithm, with the calculation of expected return and risk using the Single Index Model (SIM) approach. The data used includes data on stocks in the infrastructure sector for the period July 1, 2023 to June 30, 2024. The results showed that there were six stocks selected in forming the optimal portfolio with the weight of each stock: PGEO 15.0023%, ISAT 32.1522%, GMFI 4.7822%, EXCL 15.3236%, JSMR 29.7379, and OASA 3.0018%. This optimal portfolio provides an expected return of 0.1167% with a portfolio risk of 0.0152%.

Keywords: investment, genetic algorithm, single index model, portfolio optimization.

1. Introduction

The Investment Coordinating Board (BKPM) stated that investment realization is currently showing a positive trend. This is reflected in the 15.24% increase in investment realization during the third quarter of 2024 compared to the same period in the previous year. This increase shows high investor interest in allocating their funds, one of which is through the capital market. In Indonesia, the development of the capital market can be reflected in the index growth indicator, namely the Jakarta Composite Index (JCI). Based on the Jakarta Composite Index and Sectoral indices Movement statistical report on the website www.idx.co.id, the stock price of the infrastructure sector index in Indonesia has strengthened throughout mid-2023 to mid-2024 as shown in Figure 1. According to ipotnews (2023), Muhammad Nafan Aji Gusta as Senior Investment Information from PT Mirae Asset Sekuritas Indonesia said the strengthening of the infrastructure stock index occurred due to investors' optimism about the sustainability of infrastructure development projects.

Despite its promising prospects, investing in the capital market still carries risks. Kein et al. (2021) state that this can be overcome by diversifying stock ownership. Diversification is done by combining investments in various stocks to reduce risk. Through good diversification, it allows investors to get a combination of stocks that provide optimal returns with controlled risk.

In supporting the diversification process, a method is needed to measure the return and risk of each stock. One approach that can be used is the Single Index Model (SIM). The Single Index Model was developed by Sharpe in 1963. This model is considered simpler and more efficient than the complex Markowitz model because it involves the calculation of covariance between stocks. The calculation of covariance in the Markowitz model can be very complicated and inefficient, especially when applied to data with a large number of stocks. In addition, this single index model also considers market aspects and aspects of company uniqueness (Shah, 2014).

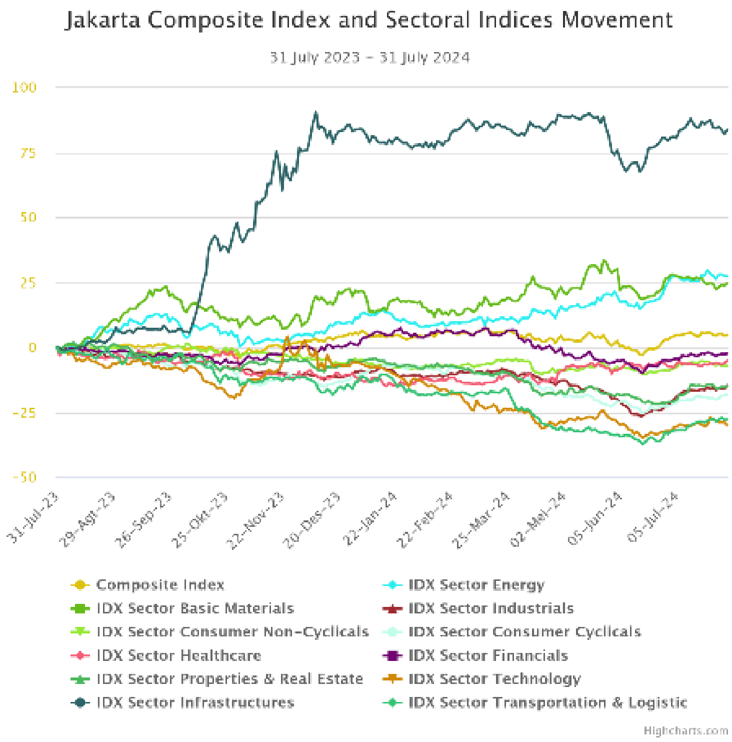

Figure 1: *Jakarta Composite Index and Sectoral indices Movement*

Along with the increasing need for diversification processes in forming optimal portfolios, mathematical and computational approaches are being used, one of which is Genetic Algorithm (GA). The Genetic Algorithm created by John Holland in the 1960s is a technique that adopts the process of natural evolution, namely Darwin's theory of evolution (Farid & Rosadi, 2022).

Genetic Algorithm (GA) has been widely applied in research for portfolio optimization and shows superior results compared to conventional methods. Dubinskas and Urbšienė (2017) proved that GA can produce portfolios with better risk-return ratios than deterministic and stochastic approaches. Wahyono *et al.* (2018) also implemented GA with the Single Index Model (SIM) approach on LQ45 stocks, and obtained a portfolio with higher returns and lower risk. In addition, Farid and Rosadi (2022) combined the Self Organizing Maps (SOM) clustering method with GA, and the results showed that the GA-based portfolio had a higher Sharpe ratio value compared to Markowitz theory.

Following up on these findings, this study aims to apply Genetic Algorithm in portfolio optimization using the Single Index Model approach. In this study, stocks that have a positive expected return are selected as candidates for optimal portfolio formation. Furthermore, these stocks are further analyzed using Genetic Algorithm to determine the allocation weight of each stock in the portfolio.

2. Literature Review

2.1. Investment

In Rudianto (2023), Halim states that investment is an activity of allocating a number of funds in the present with the hope of generating profits in the future. This investment activity has the aim of obtaining a return or the level of profit obtained by investors from an investment (Jayati et al., 2017). Investment capital can be allocated into 2 types of assets, namely real assets (real investment) which includes investment in real company projects, gold, derivative bonds, deposits, etc., and financial assets (financial investment) which can be done through the capital market, banking, money market and so on (Jayati et al., 2017). The capital market is a means to accommodate all tradable long-term financial instruments, such as debt or equity. These instruments can be documents issued by the government, private companies or public institutions (Suriyanti & Hamzah, 2024).

2.2. Stocks

Ibnas et al. (2017) states that stock are a type of security (securities) as proof of capital ownership in a company. There are 2 types of stock traded on the Stock Exchange, including common stock and preferred stock. Common stock is a type of stock that benefits after preferred stock profits are paid. Meanwhile, preferred stock is a type of stock that has the right to get profits paid in the year of profit and not compared to the year of loss.

2.3. Return

Return is the result of an investor's courage to take risks on his investment. Stock returns are divided into 2, namely realized return and expected return (Pratiwi et al., 2014). Ramadhan et al., (2019) state that stock realization returns are returns that have occurred and are calculated based on historical data. Stock realization return $(R_{i,t})$ can be calculated using the following equation (Pratiwi et al., 2014):

$$R_{i,t} = \frac{P_{i,t} - P_{i,(t-1)}}{P_{i,(t-1)}}, \tag{1}$$

Where,

$P_{i,t}$: Price of stock i at time t,
$P_{i,(t-1)}$: Price of stock i at time t.

While the realization of market return or actual market return can be calculated using the equation (2):

$$R_{m,t} = \frac{JCI_t - JCI_{t-1}}{JCI_{t-1}}. \tag{2}$$

with,

JCI_t : Jakarta stock exchange composite index period t
$JCI_{(t-1)}$: Jakarta stock exchange composite index period $t-1$

The expected return is the rate of return expected by investors in the future. In calculating the expected return of stocks using the Single Index Model requires the value of the market expected return $E(R_m)$. Market expected return can be calculated using the following equation (Setiawan & Dewi, 2021):

$$E(R_m) = \frac{\sum_{t=1}^{n} R_{m,t}}{n}. \tag{3}$$

2.4. Risk

In investment, risk can be interpreted as uncertainty that results in investment objectives not being achieved. Investment risk can also be interpreted as the amount of deviation between the expected return level and the actual return level (Suriyanti and Hamzah, 2024). In Jayati et al. (2017), Halim states that risk consists of 2 types, namely systematic risk and unsystematic risk. Systematic risk is a risk that cannot be reduced through diversification, because changes in this risk are influenced by macro factors that can affect the market as a whole. Meanwhile, unsystematic risk is a risk that can be reduced through diversification, because changes in this risk are influenced by micro factors that can affect local or regional markets.

2.5. Single Index Model (SIM)

The Single Index Model (SIM) is a simple model of the mean-variance model (Markowitz) developed by Sharpe in 1963. This model assumes that securities stock prices fluctuate in the same direction as the market price index, so that when the stock index price increases, securities stock prices will also increase (Setiawan and Dewi, 2021). Based on this assumption, the relationship between individual stock returns and market stock returns in the Single Index Model (SIM) is represented by the regression function equation in equation (4) (Lestari et al., 2019):

$$R_i = \alpha_i + \beta_i R_m + e_i. \tag{4}$$

According to Hartono (2014) as cited in Pratiwi et al. (2014), the expected return of individual stocks $(E(R_i))$ can be calculated using equation (5).

$$E(R_i) = \alpha_i + \beta_i . E(R_m). \tag{5}$$

While the Single Index Model (SIM) stock risk $\left(\sigma_i^2\right)$ can be calculated using equation (6) (Collins & Barry, 1988)

$$\sigma_i^2 = (\beta_i \sigma_m)^2 + \sigma_{ei}^2. \tag{6}$$

After obtaining the estimated return and risk of each stock, the next step is to form these stocks into a portfolio to obtain a balance between return and risk. The portfolio is then analyzed by calculating the estimated return and risk of the portfolio as a whole. The portfolio return can be calculated using the formula as in equation (7) below (Collins and Barry, 1988):

$$R_p = \sum_{i=1}^{l} w_i R_i, \tag{7}$$

Expected return portfolio $(E(R_p))$ or the rate of return that investors expect from this portfolio, can be calculated using the following equation (8):

$$E(R_p) = \sum_{i=1}^{l} w_i \alpha_i + \left(\sum\nolimits_{i=1}^{l} w_i \beta_i\right) E(R_m) \tag{8}$$

While the portfolio risk (σ_p^2) is obtained using the following equation (9):

$$\sigma_p^2 = \left(\sum_{i=1}^{l} w_i \beta_i\right)^{-} \sigma_m^2 + \sum_{i=1}^{l} w_i^2 \sigma_{ei}^2. \tag{9}$$

2.6. Genetic Algorithm (GA)

Genetic algorithm is one of the heuristic methods used to solve optimization problems. This method was first introduced by John Holland in the 1960s who was inspired by Darwin's theory of evolution. This theory states that each individual in the population has different characteristics. Surviving individuals will undergo changes or transformations based on the principles of genetics that make it possible to form new individuals (Farid and Rosadi, 2022).

2.6.1 Encoding

In genetic algorithms, chromosome encoding can be done using binary strings (0 and 1) or real-values. In the context of portfolio optimization, chromosomes are represented by real-values that represent the weight of each stock. The chromosome is randomly generated in the interval [0,1]. The length of the chromosome is the number of stocks in the portfolio. In this case, the total genes must be equal to 1 (Wahyono et al., 2018).

2.6.2 Fitness Function

Fitness function is the objective function of a problem. Genetic algorithms are able to optimize chromosome functions to find a solution. Each chromosome has a fitness value. The fitness value of each chromosome determines the quality of the solution. The higher the fitness function value, the better the resulting solution (Farid and Rosadi, 2022). The fitness function in optimizing the stock portfolio is done by maximizing the excess return to risk ratio. The fitness function for the s-th chromosome (f_s) is written with the formula as in equation (10):

$$f_s = \frac{R_{s,p} - R_f}{\sigma_{s,p}^2}, (s = 1,2,3, \dots, N). \tag{10}$$

with R_f is risk-free return.

2.6.3 Selection

In 1989, Goldberg introduced the roulette wheel selection method in genetic algorithms that uses the principle of fitness proportional selection, where the chance of each chromosome being selected is proportional to its fitness value. This method is similar to playing a roulette wheel where each chromosome is placed in a slot on the wheel. The size of the slot is proportional to the ratio of the fitness value of a chromosome to the total fitness value of all chromosomes (Farid and Rosadi, 2022).

In using the roulette wheel selection method, the probability of each individual (p_s) to be selected is required by normalizing the fitness value using the formula as in equation (11) (Chiamenti, 2015):

$$p_s = \frac{f_s}{\sum_{s=1}^{N} f_s}, \qquad (s = 1,2,3, \dots, N). \tag{11}$$

The steps in the selection stage of the genetic algorithm using roulette wheel selection (Han, 2013):

1. Calculate the cumulative probability c_s for each chromosome $(s = 1,2,3, \dots N)$.
2. Generate a random number r in the interval [0,1].

3. If $r \le c_s$ for $s = 1$, then the 1st chromosome is selected. If $c_{s-1} < r \le c_s$ for $s > 1$ then the sth chromosome is selected.
4. Repeat steps 2 and 3 until the N-th chromosome.

2.6.4 Crossover

Crossover is the process of creating a new individual by combining the chromosomes of two individuals. This operation will exchange some elements (genes) in one pair of parent chromosomes to produce two child chromosomes. This crossing over process is not applied to all individuals, but only to randomly selected individuals. If crossing over does not occur, then the value of the parent will be passed on to its offspring (Farid and Rosadi, 2022).

According to Chiamenti (2015), there are several techniques that can be used in crossing chromosomes, one of which is arithmetic crossover. The following is the process of crossing chromosomes using arithmetic crossover:

$$\begin{aligned} ChildA &= \alpha * ParentA + (1-\alpha) * ParentB, \\ ChildB &= (1-\alpha) * ParentA + \alpha * ParentB, \end{aligned} \tag{12}$$

where α is a random number with interval [0,1].

Farid and Rosadi (2022) suggested that the crossover process is carried out with a certain probability, namely P_c. Crossover only occurs if a random number with the interval [0, 1] is smaller than the predetermined P_c value. In general, the value of P_c ranges in the interval [0.6, 1). Determining the right P_c value is highly dependent on the problem at hand.

2.6.5 Mutation

Mutation is the process of creating a new individual through changes in the chromosome structure of one individual. This change produces a mutant, which is a new chromosome that is genetically different from the original chromosome. One of the mutation techniques in genetic algorithms used for real value genes is uniform mutation. This mutation technique changes the selected gene directly with a random value from a certain interval. In the context of a stock portfolio, this random value must be in the interval [0,1] because the value in the gene represents the weight of the individual stock. The mutation probability (P_m) is the ratio between the number of genes in the population and the number of genes expected to undergo mutation. Since mutation is a support operator, the value of P_m used is quite low, ranging from 0.001 to 0.2 (Farid and Rosadi, 2022).

2.6.6 Elitism

In genetic algorithms, the evolutionary process allows for a decrease in the quality of individuals in the population. One way to prevent this is by applying elitism. Elitism is one of the strategies in genetic algorithms that aims to maintain the quality of the best individuals in a population. In this process, the best individual with the highest fitness is copied to the next generation without change, so that the superior traits of the best individual are still passed on to the next generation (Fanggidae & Pandie, 2020). In this study, elitism is done by replacing m worst individuals with m best individuals. Elite members in the elitism process in this study were selected as much as 2% of the total population.

3. Materials and Methods

3.1. Materials

The object of this study is secondary data in the form of daily closing stock prices data from 25 companies in the infrastructure sector and daily closing stock prices data from Jakarta Composite Index for the period July 1, 2023 to June 30, 2024. Data obtained from yahoo finance (https://finance.yahoo.com/).

3.2. Methods

1) Calculate the realized return of 25 infrastructure sector stocks as well as the market return using equations (1) and (2).
2) Conduct a significance test to ensure the basic assumptions of the single index model are met.
3) Calculate the expected return of each company's stock and select stocks that meet the positive expected return criteria to be used as candidates for optimal portfolio formation.

4) Optimizing portfolio composition by applying the Genetic Algorithm method. The individual selection process is carried out using roulette wheel selection, crossover using arithmetic crossover, and mutation is carried out with uniform mutation. In addition, an elitism mechanism is applied, where one best individual from the previous generation replaces one worst individual.
5) Determining the optimal weight of each stock in the portfolio based on the individual with the highest fitness value generated by the Genetic Algorithm process.
6) Calculating the expected return and variance of the optimal portfolio using equations (8) and (9).

4. Results and Discussion

After obtaining the market return and realized return of 25 infrastructure sector companies, a significance test was conducted to assess the suitability of the model. Of the 25 stocks in the infrastructure sector, there are 6 insignificant stocks indicating that the market return does not affect the return of these stocks. Because they do not meet the basic assumptions of the Single Index Model, the six companies are not included in the portfolio formation process. Furthermore, from the remaining 19 stocks, the expected return value is calculated using the single index model approach, as shown in table 1.

Table 1: expected return of infrastructure sector stocks

No	Companies	Expected return	No	Companies	Expected return
1	TLKM	-0.0009336	11	ACST	-0.0004362
2	MTEL	-0.0000225	12	PPRE	-0.0025750
3	TOWR	-0.0013021	13	WEGE	-0.0016879
4	PGEO	0.0020182	14	ADHI	-0.0031056
5	WIKA	-0.0067096	15	JSMR	0.0013796
6	ISAT	0.0010039	16	OASA	0.0004342
7	GMFI	0.0002310	17	PTPP	-0.0024068
8	JKON	-0.0003585	18	DGIK	-0.0013761
9	POWR	-0.0005342	19	IPCM	-0.0003850
10	EXCL	0.0006992			

Based on the calculation results shown in table 1, only 6 companies have positive expected return values, while 13 other companies show negative expected return values. Companies with negative expected return values are eliminated from the portfolio because they have the potential to provide losses to investors. Conversely, companies with positive expected return values become optimal portfolio candidates. Companies that are candidates for this optimal portfolio include PGEO, ISAT, GMFI, EXCL, JSMR and OASA.

The stocks selected as optimal portfolio candidates are subsequently analyzed using a Genetic Algorithm to determine the optimal weight allocation for each stock within the portfolio. The algorithm is implemented in Python, with an initial population of 50 individuals and a total of 300 generations. The crossover parameter (α) is set at 0.5. Multiple runs of the Genetic Algorithm are conducted using various combinations of crossover probability (P_c) and mutation probability (P_m). Each combination yields a best-performing individual, and the results of the most optimal Genetic Algorithm run are summarized in Table 2.

Table 2: Best individual of Genetic Algorithm optimization result

No	P_c	P_m	PGEO	ISAT	GMFI	EXCL	JSMR	OASA	Fitness
1		0.01	0.148573	0.317297	0.060278	0.143359	0.299975	0.030517	5.991837
2	0.6	0.05	0.150766	0.326202	0.051607	0.157496	0.288319	0.025610	5.994222
3		0.1	0.147727	0.307191	0.061669	0.163343	0.297204	0.022866	5.985673
4		0.01	0.150023	0.321522	0.047822	0.153236	0.297379	0.030018	5.996917
5	0.7	0.05	0.146688	0.321800	0.053342	0.153363	0.293944	0.030863	5.995733
6		0.1	0.144630	0.330782	0.046601	0.146196	0.306016	0.025775	5.992935
7		0.01	0.139130	0.318938	0.061406	0.154938	0.287976	0.037612	5.977030
8	0.8	0.05	0.148326	0.327996	0.048316	0.150984	0.298248	0.026129	5.996578
9		0.1	0.145499	0.320860	0.054879	0.169083	0.282852	0.026827	5.985192

Based on Table 2, the best chromosome is obtained when the value of P_c is 0.7 and P_m is 0.01. The best chromosome obtained has a fitness value of 5.996917 and forms an investment portfolio with the following stock weight distribution.

1. PGEO: 0.150023
2. ISAT: 0.321522
3. GMFI: 0.047822

4. EXCL: 0.153236
5. JSMR: 0.297379
6. OASA: 0.030018

Then the expected return of the portfolio is calculated using equation (8) and the portfolio risk using equation (9), obtained an expected return portfolio of 0.001167 with a portfolio risk of 0.000152.

The results showed that parameters in Genetic Algorithm such as crossover probability and mutation probability have a significant influence in exploring and maintaining individual diversity. Choosing the right parameter value allows Genetic Algorithm to work more effectively in finding the optimal solution. In addition, the application of Genetic Algorithm combined with Single Index Model (SIM) in portfolio optimization produces an efficient portfolio. This is indicated by the fitness value of 5.996917 which indicates that the return generated is quite high compared to the level of portfolio risk. Therefore, the portfolio with weight allocation is as follows: PGEO 15.0023%, ISAT 32.1522%, GMFI 4.7822%, EXCL 15.3236%, JSMR 29.7379%, OASA 3.0018% and expected portfolio return of 0.1167% and portfolio risk of 0.0152% can be considered to get optimal investment for investors who seek a balance between return and risk.

5. Conclussion

The results of the expected stock return analysis using the Single Index Model (SIM) approach show that of the 25 infrastructure sector stocks that have been analyzed, there are 6 selected stocks. These stocks include PGEO, ISAT, GMFI, EXCL, JSMR, and OASA. The weight of stock in the formation of the optimal portfolio using Genetic Algorithm on infrastructure sector stocks for the period July 1, 2023 to June 31, 2024 consists of PGEO 15.0023%, ISAT 32.1522%, GMFI 4.7822%, EXCL 15.3236%, JSMR 29.7379%, OASA 3.0018%. This optimal portfolio produces an expected return value of 0.1167% with a portfolio risk of 0.0152%.

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