Bifurcation Analysis and Electronic Circuit for Sprott Jerk System

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Abstract

In this paper, the Sprott jerk system based quadratic function is presented. The dynamics of this system is revealed through equilibrium analysis, phase portrait, bifurcation diagram and Lyapunov exponents. The Sprott system can exhibit a chaotic attractor, which has complex dynamic behavior. Finally, the circuit implementation is carried out to verify the Sprott Jerk system. The comparison between the MATLAB and MultiSIM simulation results demonstrate the effectiveness of the Sprott system.

Keywords: chaos, dynamical system, Sprott system, circuit design

1. Introduction

Chaos theory has been developed in various disciplines such as robotic (Oliveira et al., 2017; Vaidyanathan et al., 2017), biology (Stollenwerk et al., 2017; Vaidyanathan et al., 2018), financial risk model (Xiao-Dan et al., 2013; Sukono et al., 2020), medical (Anter and Ali, 2020; Belazi et al., 2019), field-programmable gate array (Sambas et al., 2021a; Vaidyanathan et al., 2021), electronic circuit (Johansyah, 2021; Rusyn and Purwandari, 2020; Sambas et al., 2020; Sambas et al., 2019; Mobayen et al., 2021), FitzHugh-Nagumo chaotic neuron models (Baladron et al., 2012; Luo et al., 2010; Zhang and Liao, 2017), radar antenna (Kumar and Sahu, 2016; Zhang et al., 2014), magnetic vortex oscillations (Petit-Watelot et al., 2012; Devolder et al., 2019; Moon et al., 2014) and voice encryption (Mobayen et al., 2019).

Wei et al. (2015) proposed chaotic Jerk system with single non-hyperbolic equilibrium and show that the periodic orbit bifurcation. Li et al. (2016) proposed hypogenetic chaotic jerk flows with complete feedback and amplitude control. They show that the system is symmetric coexisting

attractors from an asymmetric structure. Kengne et al. (2017) presented novel autonomous jerk circuit with single semiconductor diode. They show that the system has antimonotonicity behavior and period doubling bifurcation. Rajagobal et al. (2018) analyzed chaotic jerk system with non-hyperbolic equilibrium and investigate the time-delay effects on the proposed system. Mboupda Pone et al. (2019) presented chaotic Toda jerk oscillator with with an exponential nonlinear term. They show that the system has generate antimonotonicity, periodic oscillations, chaotic oscillations and bubbles. Sambas et al. (2021b) contructed a new chaotic Jerk system with two saddle-focus equilibrium points and proposed backstepping technique for controller in chaotic signal. However, some of the literature above needs to be developed again to be able to discuss it further.

In this study, the Sprott model is introduced and its dynamical properties are investigated. In Sec. 2, the existence of chaotic behavior in this system is demonstrated. In Section 3, we determine complex behavior, and introduce the MultiSim platform, together with the electronic circuit schema of the system.

2. Mathematical Model and Dynamical Analysis

A famous dissipative quadratic jerk chaotic system (Sprott, 1997) is given by the differential equation

$$\dddot{x} + a\ddot{x} - x\dot{x} + x = 0 \tag{1}$$

Sprott showed that the differential equation (1) displays chaotic behaviour when $a = 2.017$.
In system form, Sprott's quadratic jerk system (1) can be expressed as

$$\dot{x} = y \tag{2}$$
$$\dot{y} = z$$
$$\dot{z} = -az - xy - x$$

The system (2) is a five-term Sprott Jerk system with one-quadratic nonlinearity. For numerical simulations, we take the initial values of the Sprott Jerk system (2) as $x(0) = 0$, $y(0) = 0$ and $z(0) = 0.1$. By using Wolf (1985) algorithm, the Lyapunov exponents are (0.44999, 0.45448, -2.0125), and the Kaplan-Yorke dimension is $D_{KY} = 2.449$. So, this system is chaotic behavior.

For the numerical simulations of the novel jerk chaotic system (2), we have taken the parameter values as in the chaotic case (2) and the initial conditions as (2). Figures 1 (a) - (c) show the projections of the orbital space of the x-y plane, the x-z plane and the y-z plane. For the selected parameter set and initial conditions, the Sprott system in equation (2) present a dense strange attractor.

We derived the bifurcation plots with parameter a as it governs the equilibrium points of the proposed system. The parameter a is varied between [2 2.25], and the local maximum of the state variable is plotted as shown in Figures 2(a) and 2(b) which show the corresponding LEs of the system. It can be seen from Figures 2(a) and 2(b), under the change of parameter a, the chaos state and periodic state of the system appear alternately.

The equilibria of the Sprott Jerk system (2) are found by setting $\dot{x} = \dot{y} = \dot{z} = 0$, i. e.,

$$
\begin{aligned}
0 &= y \\
0 &= z \\
0 &= -az - xy - x
\end{aligned}
$$
(3)

The Sprott Jerk system has one equilibrium point E_0 (-2.017, 0, 0). For the equilibrium point E_0 (-2.017, 0, 0), the Jacobi matrix becomes:

$$
J(0,0,0) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}
$$
(4)

To obtain its eigenvalues, let det $|J_1 - \lambda| = 0$. Then, the characteristic equation has the following form:

$$
\lambda^3 + 1
$$
(5)

Solving the above characteristic equation, the eigenvalues are found as

$$
\lambda_1 = -1, \lambda_{2,3} = \frac{1}{2} \pm \frac{1}{2} i\sqrt{3}
$$

Here λ_1 is a negative real number, λ_2 and λ_1 are conjugate pair of complex eigenvalues having positive real parts. That means the equilibrium E_0 (-2.017, 0, 0) is a saddle point. So, this equilibrium point is unstable
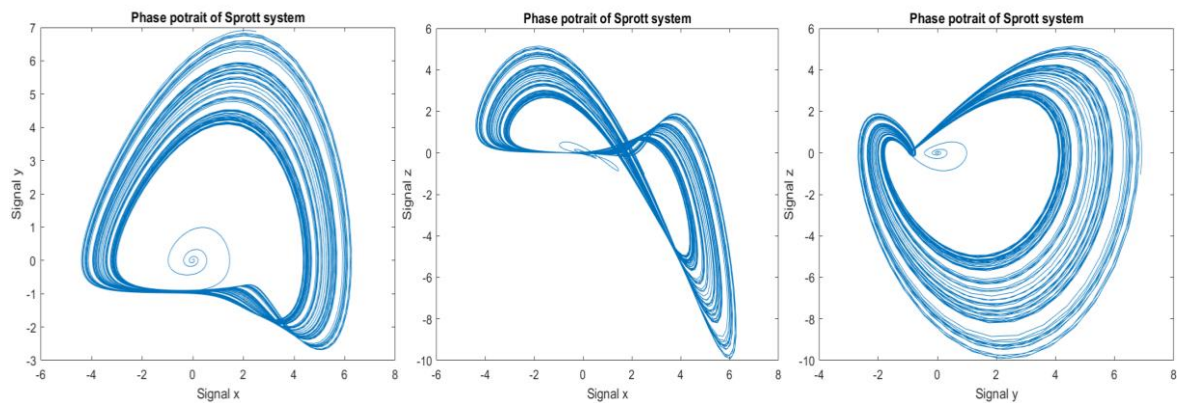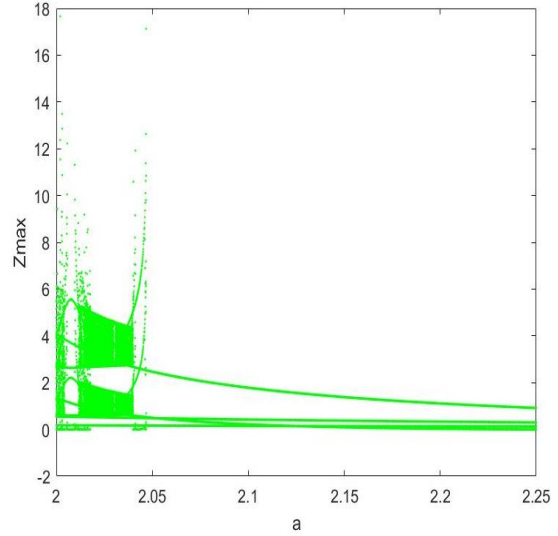
(a)   (b)   (c)

Figure 1. Numerical simulation results using MATLAB, with a = 2.017 in the
(a) x-y plane, (b) x-z plane, and (c) y-z plane
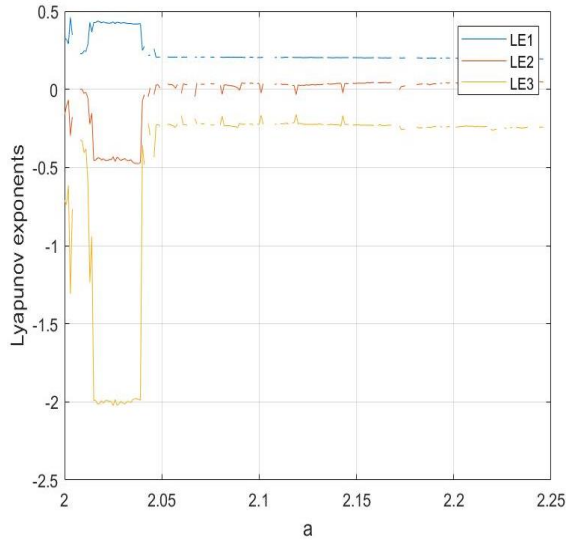
(a)   (b)

Figure 2. Dynamics of the proposed system with the variation of the parameter a:
(a) Lyapunov exponents (LEs); (b) Bifurcation diagram

3. Electronic Circuit

The circuit in Fig. 3 has been designed by using methods in (Sambas et al., 2020; Sambas et al., 2021a), which are based on five operational amplifiers. As can be seen in Fig. 3, X, Y and Z are the voltages at the operational amplifiers U_1, U_2 and U_3.

For circuit implementation, we rescale the state variables of the Sprott Jerk system (2) as follows: X = 1/5 x, Y = 1/5 y, Z = 1/5 z. The rescaled chaotic system is given as follows:

$$\dot{X} = Y$$
$$\dot{Y} = Z \tag{5}$$
$$\dot{Z} = -aZ - 5xy - x$$

Applying the Kirchhoff laws, the circuit presented in Figure 3 is described by the following equations:

$$\dot{X} = \frac{1}{C_1 R_1} Y$$

$$\dot{Y} = \frac{1}{C_2 R_2} Z \tag{6}$$

$$\dot{Z} = -\frac{1}{C_3 R_3} Z - \frac{1}{C_3 R_4} XY - \frac{1}{C_3 R_5} X$$

We get the value of electronic components $R_1 = R_2 = R_5 = 400$ kΩ, $R_3 = 198.3$ kΩ, $R_4 = 80$ kΩ, $R_7 = R_8 = R_9 = R_{10} = 100$ kΩ, and $C_1 = C_2 = C_3 = 1$ nF. Figure 4 illustrates phase portraits which are obtained from the designed circuit. As it can be seen from the MultiSim outputs in Figure 4 and MATLAB simulation in Figure 1, the results are similar.
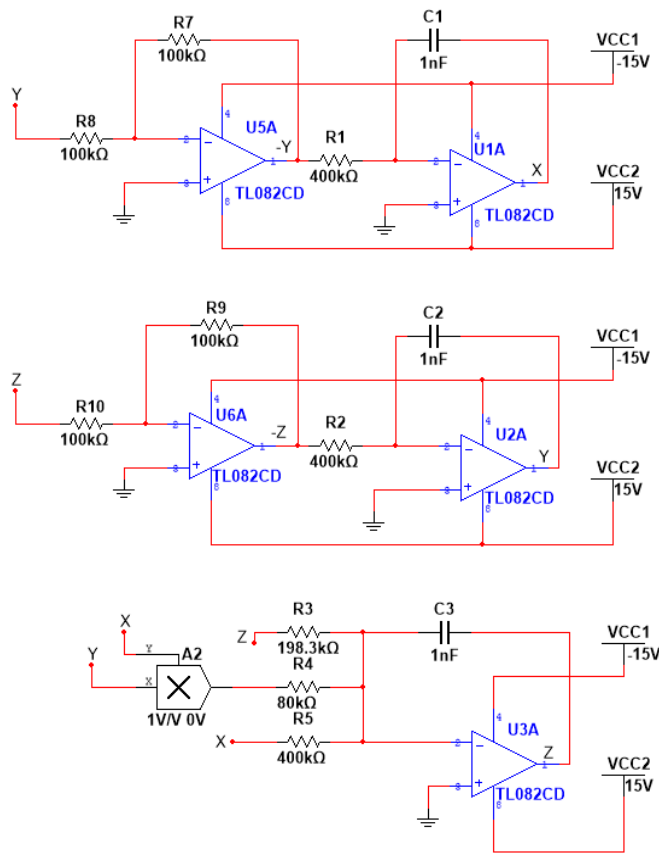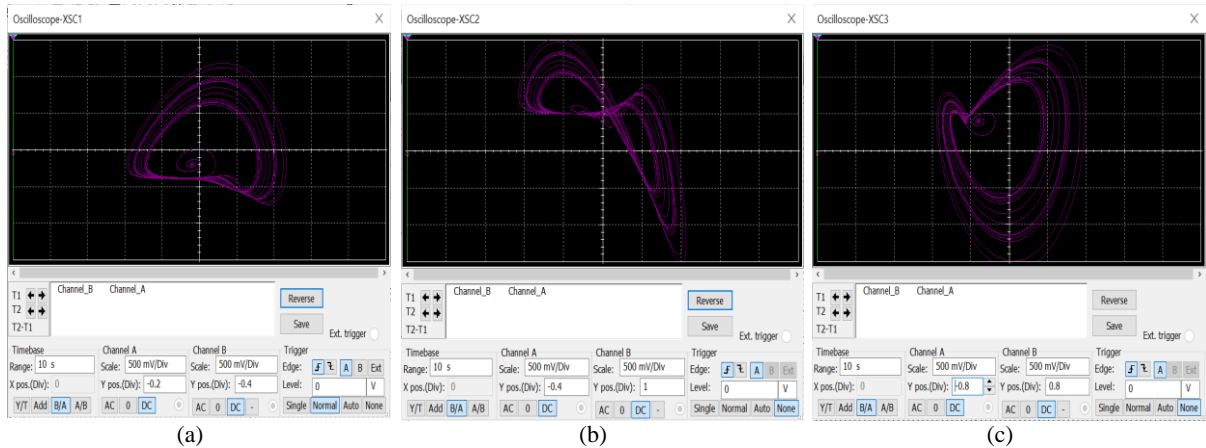
Figure 3. The electronic circuit designed for the Sprott jerk system

(a)   (b)   (c)

Figure 4. Multisim outputs of the in the Malasoma system (a) *X-Y* plane, (b) *X-Z* plane, and (c) *Y-Z* plane

4. Conclusion

In this study, we have studied a Sprott Jerk model with quadratic function which is obtained obtained by Sprott (1997). Dynamical properties of this model were analyzed by the help of Lyapunov exponents' spectrum and bifurcation diagram. Finally, the MATLAB simulations and MultiSim simulation are performed to verify the theoretical model. For hardware electronic circuit can be investigated in our future works.

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