



Some Forms of Investor Risk Tolerance in Investing: Review Theory

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Abstract

Investors in investing are always accompanied by a sense of tolerance for the risk of funds invested in an asset. Each investor has a different form of risk tolerance, depending on the function of the utility. This paper aims to conduct a theoretical study of the forms of investor risk tolerance for several utility functions. This study is carried out by reviewing several utility functions which include: square root utility, cubic fraction utility, quadratic utility, exponential negative utility, and logarithmic utility. Based on the results of the study for each of these utility functions, successively obtained risk tolerance in the form of linear, linear, linear, constant, and linear. Linear risk tolerance illustrates that an investor changes the value of his investment in line with changes in the level of risk faced.

Keywords: Investment, investors, risk, utility function, risk tolerance.

1. Introduction

In investing, investors can choose to invest their funds in various assets, both risk assets and risk-free assets, or a combination of the two assets. The choice of investors for these assets will depend on the extent of investor preference for risk. The more reluctant an investor is to risk (risk-averse) or the less tolerant of risk, the investment choice will tend to be more on risk-free assets. An important concept in choice theory is what is referred to as a utility concept (Bjork et al., 2011). Utility functions can be interpreted as a mathematical function that shows the value of a choice that exists. The higher the value of a choice, the higher the alternative utility. In the context of portfolio management, the utility function shows an investor's preference for various investment choices with each risk and the expected rate of

return. The utility function for each investor can be different, and this will form a different level of risk tolerance for each investor (Ardehali, 2004).

Referring to Baber and Odean (2011), risk tolerance can be analyzed from the utility function owned by each investor. Investment analysis usually uses the assumption that investors are risk-averse or do not like risk. This means that investors expect compensation for a greater level of profits if the investor must bear the greater risk. According to Brandt (2010), the expected utility model states that investors will choose an investment opportunity that gives utilities the highest expectations. The highest expected utility is not always the same as the highest expected profit rate. Referring to Husnan (2001), this hopeful utility model uses the assumption of investor attitudes towards risk. These attitudes are grouped into three, namely attitudes that are risk-averse (do not like risk), risk-neutral (neutral against risk), and risk seekers (like risk). Janecek (2004), for investors whose risk-averse shape of the utility function curve is curved, with an increasing decrease. For those who are risk-neutral, the shape of the curve will be a straight line, and for investors who are risk seekers the shape of the curve will be curved with an increasing increase.

How is the relationship between utility functions, risk tolerance, and portfolio optimization models, in this section discussed about risk tolerance models and several utility function models. The following is discussed first about risk tolerance in the Markowitz model.

2. Risk Tolerance in the Markowitz Model

Refer to Panjer et al. (1998) and Kirby & Ostdiek (2012), Markowitz's in 1952 popularized the efficient portfolio selection method. Suppose a p portfolio is given with a weight vector \mathbf{w} , the investor has two (objective) objectives,

- (i) Maximizing the expected value μ_p of portfolio returns, and
- (ii) Minimizing portfolio risk, which is measured σ_p^2 or σ_p .

Based on individual preferences, an investor puts weight on these two different goals and maximizes them

$$2\tau\mu_p - \sigma_p^2 \text{ with } \tau \geq 0 \quad (1)$$

The τ parameter is called risk tolerance (Panjer et al., 1998). The 2τ factor is obtained as follows. For example, given initial capital W_0 , under a portfolio with a weight vector \mathbf{w} , at the end of the period, the capital becomes $W_0(1 + R_p)$ (Panjer et al., 1998). Where is the random variable portfolio return R_p with the weight vector \mathbf{w} . The utility equation of $W_0(1 + R_p)$ is $u[W_0(1 + R_p)]$. If the utility equation is translated using the second-order Taylor approximation, the utility equation is obtained as

$$u[W_0(1 + R_p)] \approx u(W_0) + W_0 \cdot u'(W_0) \cdot R_p + \frac{1}{2} W_0^2 \cdot u''(W_0) \cdot R_p^2 \quad (2)$$

Taking the expectation of equation (2) the form is obtained

$$\begin{aligned} E[u[W_0(1 + R_p)]] &\approx u(W_0) + W_0 \cdot u'(W_0) \cdot \mu_p + \frac{1}{2} W_0^2 \cdot u''(W_0) \sigma_p^2 \\ &\approx u(W_0) - \frac{W_0^2}{2} u''(W_0) \cdot \left[-\frac{2u'(W_0)}{W_0 \cdot u''(W_0)} \mu_p - \sigma_p^2 \right] \end{aligned}$$

Based on the approximation above, maximizing the utility of expectations is approached, equivalent to maximizing

$$\frac{2}{R_R} \mu_p - \sigma_p^2 \tag{3}$$

Where $R_R = -W_0 u''(W_0) / u'(W_0)$ states the relative size of risk aversion. If $\tau = 1 / R_R$ is taken, then equation (3) is equivalent to (1) (Panjer et al., 1998).

3. Risk Tolerance for Multiple Utility Functions

The basic proportion of modern treatment of utilities is that it is possible to obtain several expressions regarding a person's preferences for investors. Based on these numerical expressions regarding one's preferences, then the utility curve equation can be determined. Each investor generally has different equation curves or utility functions, according to his preference for attitudes towards investment risk. Based on the utility function owned by an investor, the risk aversion function and the risk tolerance function can be determined. The following discusses several forms of utility functions to determine the risk aversion function and the risk tolerance function. For example, W state the assets (funds) owned to invest.

3.1. The square root utility function

The first form of utility functions is to consider an investor with a square root utility function as (Ardia and Boudt, 2013):

$$U(W) = \sqrt{k + W} = (k + W)^{0.5}; \text{ with } W > k \text{ and } k \geq 0$$

The first and second derivatives of this utility function are:

$$U'(W) = 0.5(k + W)^{-0.5} > 0 \text{ and } U''(W) = -0.25(k + W)^{-1.5} < 0$$

So the risk avoidance function R_R is:

$$R_R(W) = -\frac{U''(W)}{U'(W)} = -\frac{-0.25(k + W)^{-1.5}}{0.5(k + W)^{-0.5}} = 0.5(k + W)^{-1}$$

and thus the risk tolerance function τ is as follows:

$$\tau(W) = \frac{1}{R_R(W)} = 2(k + W). \tag{4}$$

3.2. Cubic fractional utility function

The second form of the utility function, suppose that an investor has a cubic fraction utility function as (Gorter and Bikker, 2011):

$$U(W) = \frac{-k}{(c + W)^3}; \text{ with constants } W \neq -c \text{ and } k > 0$$

The first and second derivatives of this utility function are as follows:

$$U'(W) = 3k(c + W)^{-4} > 0 \text{ and } U''(W) = -12k(c + W)^{-5} < 0$$

Therefore, the risk aversion function R_R takes the form of:

$$R_R(W) = -\frac{U''(W)}{U'(W)} = -\frac{-12k(c+W)^{-5}}{3k(c+W)^{-4}} = 4(c+W)^{-1} .$$

So the form of the risk tolerance function τ can be determined as follows:

$$\tau(W) = \frac{1}{R_R(W)} = 0.25(c+W) \quad (5)$$

3.3. Quadratic utility function

The third form of the utility function, suppose that an investor has a quadratic utility function as follows (Gurrib and Alshahrani, 2012):

$$U(W) = W - bW^2 ; \text{ with parameter coefficients } b > 0 .$$

The first and second derivatives of this utility function are given as follows:

$$U'(W) = 1 - 2bW > 0 \text{ for } W < 1/2b , \text{ and } U''(W) = -2b < 0 .$$

The risk aversion function R_R for an investor can be determined as:

$$R_R(W) = -\frac{U''(W)}{U'(W)} = -\frac{-2b}{1-2bW} = \frac{2b}{1-2bW}$$

Therefore, the risk tolerance function τ can be obtained as:

$$\tau(W) = \frac{1}{R_R(W)} = \frac{1-2bW}{2b} \quad (6)$$

3.4. Exponential negative utility function

The fourth form of the utility function is that an investor has an exponential utility function as follows (Ardehali, 2004):

$$U(W) = -e^{kW} ; \text{ with constants } k > 0 .$$

The first and second derivatives of this utility function are as follows:

$$U'(W) = ke^{-kW} > 0 \text{ and } U''(W) = -k^2e^{-kW} < 0$$

Thus the risk avoidance function can be determined as follows:

$$R_R(W) = -\frac{U''(W)}{U'(W)} = -\frac{-k^2e^{-kW}}{ke^{-kW}} = k ,$$

and the risk tolerance function is a constant function shaped as:

$$\tau(W) = \frac{1}{R_R(W)} = \frac{1}{k} = c \quad (7)$$

3.5. Logarithmic utility function

The fifth form of the utility function is that an investor is assumed to have a utility function that is shaped as (Ardehali, 2004):

$$U(W) = \log_e(k+W) ; \text{ on the condition } W > k \text{ and } k \geq 0 .$$

The first and second derivatives of the utility function are as follows:

$$U'(W) = \frac{1}{k+W} = (k+W)^{-1} > 0 \text{ and } U''(W) = -\frac{1}{(k+W)^2} < 0$$

So the risk avoidance function R_R can be obtained as:

$$R_R(W) = -\frac{U''(W)}{U'(W)} = -\frac{-1/(k+W)^2}{1/(k+W)} = \frac{1}{k+W}$$

Therefore, a risk tolerance function can be determined as follows:

$$\tau(W) = \frac{1}{R_R(W)} = \frac{1}{1/(k+W)} = k+W \quad (8)$$

Some of the above risk tolerance functions are then used to formulate the optimum portfolio theorem under risk tolerance, which is discussed below.

4. Conclusion

In this paper, a review of risk tolerance for utility functions has been carried out, including square root utility, cubic fraction utility, quadratic utility, exponential negative utility, and logarithmic utility. From these utility functions, successively generated risk tolerances are linear, linear, linear, constant, and linear. For an investor who has a linear risk tolerance will make changes in the value of the invested capital in line with changes in the level of risk faced. For an investor who has a constant form of risk tolerance, tends not to change the value of the investment even though the risks faced are relatively changing.

References

- Ardehali, P.H. (2004). *Assessing financial risk tolerance of portfolio investors using data envelopment analysis*. Working Paper. Faculty of Applied Science and Engineering, University of Toronto, Canada.
- Ardia, D. & Boudt, K. (2013). *Implied expected returns and the choice of a mean-variance efficient portfolio proxy*. Working Paper. A Departement de finance, assurance et immobilier, Universite Laval, Quebec City (Quebec), Canada.
- Baber, B.M. & Odean, T. (2011). *The behavior of individual investors*. Working Paper. Electronic copy available at: <http://ssrn.com/abstract=1872211>.
- Bjork, T., Murgoci, A. & Zhou, X.Y. (2011). *Mean-variance portfolio optimization with state-dependent risk aversion*. Working Paper. Department of Finance, Stockholm School of Economics, Box 6501, SE-113 83 Stockholm, SWEDEN. E-mail: tomas.bjork@hhs.se.
- Brandt, M.W. (2010). *Portfolio choice problems*. Working Paper. Fuqua School of Business, Duke University, Durham, NC.
- Gorter, J. & Bikker, J.A. (2011). *Investment risk-taking by institutional investors*. DNB Working Paper, No. 294/ May 2011.
- Gurrib, I. & Alshahrani, S. (2012). Diversification in Portfolio Risk Management: The Case of the UAE Financial Market. *International Journal of Trade, Economics, and Finance*, 3(6).

- Husnan, S. (2001). *Dasar-dasar Teori Portofolio dan Analisis Sekuritas, Edisi Ketiga*, Yogyakarta, UPP AMP YKPN.
- Janecek, K. (2004). *What is a realistic aversion to risk for real-world individual investors?*. Working Paper. Department of Mathematics, Carnegie Mellon University, Pittsburgh, PA 15213, USA.
- Kirby, C. & Ostdiek, B. (2012). *Optimizing the performance of sample mean-variance efficient portfolios*. Working Paper. Belk College of Business, the University of North Carolina at Charlotte.
- Panjer, H.H., Boyle, D.D., Cox, S.H., Dufresne, D., Gerber, H.U., Mueller, H.H., Pedersen, H.W., & Pliska, S.R. (1998). *Financial economics. With applications to investments, insurance, and pensions*. Schaumburg, Illinois: the Actuarial Foundation.