



Optimizing Stock Portfolio with Markowitz Method as a Reference for Investment Community Decisions

Viona Prisyella Balqis^{1,*}, Subiyanto², Sudradjat Supian³

¹*Master of Mathematics Student, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Indonesia*

²*Department of Marine Science, Faculty of Fishery and Marine Science, Universitas Padjadjaran, Indonesia*

³*Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Indonesia*

*Corresponding author e-mail address: viona20004@mail.unpad.ac.id

Abstract

An investor who wants to invest by avoiding risk makes investors tend to choose investments with the same expected return and the smallest or lowest possible risk. Therefore, investors expect to be able to maximize profits and minimize risk at the same time in investing. In a stock portfolio, it can be done by investing the funds owned by investors into several stocks so that it can reduce the risk of losses that will occur simultaneously. In choosing the right company to invest in with consideration of expected return and risk, a multi-objective optimization with multivariate objects can be used so that it can meet the expectations of investors. The portfolio concept introduced by Markowitz is a portfolio optimization intended for standard investors because it only refers to one explanation of portfolio returns. The Markowitz method can produce an optimal stock portfolio by considering the expected return and risk simultaneously so that the maximum profit can be obtained without eliminating the existing risk.

Keywords: Optimization, Expected Return, Risk, Multi-objective Model, Stock Portfolio

1. Introduction

Investment is an investment in the form of money or goods which is expected to provide benefits in the future (Lindblad, 2015). Investment is also an agreement in the fund market of one or more assets to be acquired for the future period (Algarvio et al., 2017). The party that invests is called the investor. Investors are generally classified into two groups, namely individual investors and institutional investors (Dian, 2020). An investor buys several shares intending to profit from future increases in the share price in return for the time and risk associated with the investment (Ivanova and Dospatliev, 2017).

Share capital is a unit of ownership in a company in the form of a piece of paper that explains that the owner of the paper is the owner of the company that issued the securities (Lee et al., 2016). The portion of ownership is determined by how much investment is invested in the company. In making an investment in company shares, it will not be separated from the existence of a return and risk. Stock return is the excess of the selling price of the stock over the purchase price. The higher the selling price of the stock is above the purchase price, the higher the return that investors will get. If an investor wants a high return, then he must be willing to bear higher risk, and vice versa if he wants a low return then the risk to be borne is also low.

If investors want to get a minimum risk in investing, then investors must diversify. Diversification is an activity to form a portfolio wherein the portfolio there are various types of investments so that the risk of the portfolio can be reduced (Konno and Yamazaki, 1991). Risk diversification is very important for investors because it can minimize risk without reducing the return received. Investors can diversify in several ways, namely by forming a portfolio that contains many assets, forming a portfolio randomly, or diversifying using the Markowitz method

Markowitz method is one method that can be used to calculate the optimal stock portfolio with multi-objective optimization. The portfolio concept coined by Markowitz is a portfolio optimization aimed at standard investors because it only refers to one explanation of portfolio returns. In multi-objective optimization, it can offer several investment alternatives that can be chosen by investors to adjust the needs of each investor (Farkhati et al., 2014).

2. Literature Review

Several studies discuss the application of the Markowitz Method to obtain an optimal portfolio, Namely Septyanto (2019), the author takes this problem because it is expected to provide information to investors to form a portfolio so

that they can determine the best decision in investing. The author uses the Multi-Objective method because it can minimize risk and maximize expected return simultaneously. This journal uses documentation techniques by recording and collecting closing price stock data on the Jakarta Islamic Index which is listed on the Indonesia Stock Exchange (IDX). In this study, the highest expected return was found when the weight of $k = 10$ was 0.22262% with the investment capital distribution for each stock of 18.61% in ASII shares, 29.60% in CPIN stocks, 0.30% in CTRA shares, 0.44% in EXCL shares, 28.70% in ICBP shares, 6.91% in PTBA shares, 7.92% in SMGR shares, and 7.52% in TLKM shares. So that the highest investment is obtained in CPIN shares and the lowest investment in CTRA shares.

Based on Negara et al. (2021), the author chooses to discuss this problem to determine the optimal portfolio by determining the weight proportion of each stock so that it can find out the best stocks included in the portfolio. This study, using secondary data on closing prices of shares listed on the two periods of the Indonesia Stock Exchange (IDX) as many as 28 shares. The results obtained from this study are the 18 best stocks selected from the 28 stocks that can be included in the portfolio. Two portfolios were formed with the results in portfolio 1 (avoiding risk as much as possible or low risk), the highest weight proportion was on BBCA shares of 0.4654%, while for portfolio 2 (taking into account high profits with existing risks), the proportion of weights the highest is BBCA shares of 0.5663%.

Based on Hanif et al. (2021), the author discusses this problem because of the pandemic conditions that occur in the world, causing concern for investors in global financial markets and also having a major impact on stock investment. By paying attention to the pandemic conditions, the author hopes that investors can gain insight on how to choose stocks to be formed as portfolios so that they get the expected profit and are balanced with the chosen risk. This study uses a purposive sampling technique with inclusion criteria for the selection of LQ-45 stock samples, namely stocks that have not experienced auto-reject more than 1 (one) time in 2019-2020. The results obtained from this study are that the two BBCA and BRPT stocks have a proportion of 78.09% and 21.91%, respectively, and the optimal portfolio using the Markowitz method can be used for investors to avoid high risks and obtain optimal returns.

3. Methods

In this study, a systematic literature search was carried out or commonly referred to as a systematic literature review. The literature review is an important feature of academic research conducted with keyword search as the initial relevance is determined by title (Xiao and Watson, 2019). The literature review is carried out with several objectives, including summarizing the development of practice or technology, identifying a gap that can be used for further research, can help position new research activities in the future, and to test hypotheses with existing empirical evidence.

In this study, literature searches were carried out on national journals and international journals obtained through the Google Scholar and Science Direct internet sites. The search criteria used in national and international journals consist of information on optimizing stock portfolio models using the Markowitz Method by considering expected returns and risks. Journals related to the search criteria will be selected, then read and analyzed to determine the methods used and the results obtained. Comparisons for the results obtained were not carried out because most of the journals used different stock data.

4. Results and Discussion

In 1952, Markowitz has introduced and popularized the theory of portfolio optimization which has since become the most efficient portfolio preparation technique (Siregar and Pangruruk, 2021; Hali and Yuliati, 2020). Markowitz explained that the formation of portfolio diversification can be done to minimize risk but still get a large enough return (Verdiyanto, 2020). In his approach, Markowitz uses several statistical measures including the expected return, the standard deviation of securities and portfolios, and correlation between returns. According to Markowitz, the optimal portfolio theory involves elements of risk and return in investment by minimizing risk through diversification and combining various investment instruments in a portfolio (Muis and Adhitama, 2021).

Based on the results of the literature study, several calculations must be done as a step in determining the optimal stock portfolio using the Markowitz Method. These include the calculation of stock returns, expected returns, variance, and covariance between stocks, stock weights, and optimal portfolio calculations based on the capital owned by investors.

4.1. Portfolio

A portfolio is an investment consisting of various stocks so that an efficient combination can be made so that investors can obtain the desired return with minimum risk. There are several steps in the portfolio management process, first, the construction of a policy statement, namely determining the type of risk and the objectives of the investment made. Second, conducting a study of portfolios with current financial and economic conditions and conducting a study of forecasted trends in the future. Third, the allocation process sets as an estimate of the financial

market that is used as an investment strategy. Fourth, overseeing policy statements, capital market conditions and investment strategies to verify any changes. These steps are a continuous procedure in a portfolio (Kamali, 2014).

4.1.1. Return

In a portfolio, Markowitz refers to return. A portfolio return is an investment return in various parts of finance within a certain period. Stock returns can be written as follows (Farkhati et al., 2014; Verdiyanto, 2020).

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \quad (1)$$

4.1.2. Portfolio Return

If there are n observations of stock returns, then the calculation of stock returns is formulated as follows:

$$R_t = w_1R_1 + w_2R_2 + \dots + w_NR_N \quad (2)$$

with:

R_t : The rate of return of the portfolio during the current period

R_i : The rate of return on assets i , $i = 1, 2, \dots, N$

w_i : Weight of asset i in portfolio i , $i = 1, 2, \dots, N$

From equation (2) it can be summarized into the following form:

$$R_t = \sum_{i=1}^n w_i R_i, \sum_{i=1}^n w_i = 1$$

The matrix notation of equation (1) can be written as follows:

$$R_t = w_1R_1 + w_2R_2 + \dots + w_NR_N = [w_1 \quad \dots \quad w_N] \begin{bmatrix} R_1 \\ \vdots \\ R_N \end{bmatrix} = \mathbf{w}^T \mathbf{R}$$

with:

\mathbf{w}^T : Vector transpose of \mathbf{w}

\mathbf{R} : Column vector consisting of a single asset return

4.1.3. Expected Return

The expected return is expected from an investment can be calculated based on recent performance. Expected return (μ_i) on stock i where $i = 1, \dots, n$ is determined using equation (3) where is stock return i is between periods t and $t - 1$ and m reflects the number of periods.

$$\mu_i = E(R_t) = \frac{\sum_{t=1}^m R_t^i}{m} \quad (3)$$

4.1.4. Expected Return Portfolio

The expected return value can be formulated as follows (Farkhati et al., 2014):

$$E(R_t) = \sum_{i=1}^n w_i \mu_i \quad (4)$$

and in matrix notation, equation (4) becomes:

$$E(R_t) = w_1\mu_1 + w_2\mu_2 + \dots + w_N\mu_N = [w_1 \quad \dots \quad w_N] \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_N \end{bmatrix} = \mathbf{w}^T \boldsymbol{\mu}$$

with:

\mathbf{w}^T : Vector transpose of \mathbf{w}

$\boldsymbol{\mu}$: Column vector consisting of the expected return of a single asset

4.1.5. Portfolio Variance

The variance of a mathematical calculation of a data set those spreads between numbers. The calculation of variance is the deviation from the average of each number in the set and every other number in the set (Verdiyanto, 2020). The variance of a portfolio with N assets, gives the portfolio variance as follows (Farkhati et al., 2014):

$$\sigma_p^2 = Var\left(\sum_{i=1}^N w_i R_i\right) = \sum_{i=1}^N \sum_{j=1}^N w_i w_j Cov(R_i, R_j) \quad (5)$$

in the form of matrix notation as follows:

with:

\mathbf{w}^T : Vector transpose of w

Σ : Variance-covariance matrix of assets

4.1.6. Portfolio Risk

Portfolio risk is the investment risk of a group of financial instruments in the Portfolio. Portfolio risk can be obtained using the following equation (Farkhati et al., 2014):

$$\sigma_p = \sqrt{\mathbf{w}^T \Sigma \mathbf{w}} = \left[[w_1 \quad \dots \quad w_N] \begin{bmatrix} \sigma_{11} & \dots & \sigma_{1N} \\ \vdots & \ddots & \vdots \\ \sigma_{N1} & \dots & \sigma_{NN} \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix} \right]^{\frac{1}{2}} \quad (6)$$

with:

\mathbf{w}^T : Vector transpose of w

Σ : Variance-covariance matrix of assets

4.2. Markowitz Method (Mean-Variance)

Markowitz's Modern Portfolio Theory is becoming an important part of the field of assignment for portfolio profit and advancement. The Markowitz model is one of the basic theories for current asset allocation which focuses on minimizing portfolio variance with a fixed average return for the entire portfolio (Goel). Markowitz proposed the well-known MV portfolio model, assuming that investors seek to maximize returns and minimize risk. The Mean-Variance portfolio has been widely applied in the scientific community and has been accepted by professionals in the field. The main advantage of the MV approach is that it is easy to use because the theory directly introduces the concepts of profitability and risk and easily formulate optimization problems (Fernández-Navarro et al., 2021). Using the Markowitz mean-variance method, stocks with a correlation less than +1 reduce portfolio risk (Siregar and Pangruruk, 2021).

Markowitz's model has several assumptions about investor behavior:

- 1) Investors consider each investment alternative based on a probability distribution of the Expected return over multiple holding periods (average length of time an investor uses to hold a stock for a specific period);
- 2) The investor maximizes his expected utility over a period and his utility curve shows the diminishing marginal utility of wealth;
- 3) Investors estimate portfolio risk based on expected return variability;
- 4) Investors base their decisions on expected return and risk so that their utility curve is only their function;

For a given level of risk, investors prefer higher returns to lower returns and, similarly, for expected rates of return, investors prefer less risk to more risk.

Markowitz's efficient portfolio concept is also known as the Mean-Variance Efficient Portfolio (MVEP). To solve the optimization problem, you can use the Lagrange function as follows:

$$L = \mathbf{w}^T \Sigma \mathbf{w} + \lambda_1 (\mu_p - \mathbf{w}^T \boldsymbol{\mu}) + \lambda_2 (1 - \mathbf{w}^T \mathbf{1}) \quad (7)$$

To obtain the optimal solution to w , then from equation (7) it is partially derived from w and equalized to zero so that the weighting equation is obtained:

$$\mathbf{w} = \frac{1}{2} \Sigma^{-1} (\lambda_1 \boldsymbol{\mu} + \lambda_2 \mathbf{1}) \quad (8)$$

by substituting $\mathbf{1}^T \mathbf{w}$ into equation (7) we get:

$$\lambda_2 = \frac{2 - \mathbf{1}^T \Sigma^{-1} \lambda_1 \boldsymbol{\mu}}{\mathbf{1}^T \Sigma^{-1} \mathbf{1}} \quad (9)$$

by substituting equation (9) into equation (7) we get:

$$\mathbf{w} = \frac{1}{2} \lambda_1 \left(\Sigma^{-1} \boldsymbol{\mu} - \frac{\mathbf{1}^T \Sigma^{-1} \boldsymbol{\mu}}{\mathbf{1}^T \Sigma^{-1} \mathbf{1}} \Sigma^{-1} \mathbf{1} \right) + \frac{\Sigma^{-1} \mathbf{1}}{\mathbf{1}^T \Sigma^{-1} \mathbf{1}}$$

For the case of a portfolio with an efficient variance, there is no limitation on the mean portfolio ($\lambda_1 = 0$) so that the weight vector is:

$$\mathbf{w} = \frac{\Sigma^{-1} \mathbf{1}}{\mathbf{1}^T \Sigma^{-1} \mathbf{1}}$$

The conditions for obtaining the minimum weight are:

$$\frac{\partial^2 L}{\partial \mathbf{w}^T \partial \mathbf{w}} = 2\Sigma > \mathbf{0}$$

Then the optimal weight on the mean-variance of the efficient portfolio with a return $R_t \sim N_N(\boldsymbol{\mu}, \Sigma)$ is:

$$\mathbf{w} = \frac{\Sigma^{-1} \mathbf{1}}{\mathbf{1}^T \Sigma^{-1} \mathbf{1}} \quad (10)$$

4.3. Stock Portfolio Optimization Model

The Mean Variant method uses parameters in the form of return, variance, and covariance of the selected stocks. The Markowitz model approach emphasizes the aspect of investment diversification to optimize investment returns in a way that is called efficient portfolio formation. There are several criteria for the formation of an efficient portfolio, namely:

- 1) A portfolio that offers a higher rate of return at the same rate of return.
- 2) Portfolios that offer less risk with the same rate of return.

An efficient portfolio is a portfolio that generates the highest return with the lowest risk. According to Markowitz, a portfolio can be said to be efficient if it meets the requirements, namely: The formed portfolio can provide high returns but is accompanied by certain risks.

The optimal portfolio model using the Markowitz method can be expressed in the objective function as follows:

$$\min(\sigma^2) = \min \sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij}, \quad i \neq j \quad (11)$$

with constraints:

1. $\sum_{i=1}^n w_i = 1$
2. $w_i \geq 0$ for $i = 1, \dots, n$
3. $\sum_{i=1}^n w_i R_i = R_p$

where $\sigma_i^2 = E[(R_i - E(R_i))^2]$ is the variance of risk on asset i and $\sigma_{ij} = E[(R_i - E(R_i))(R_j - E(R_j))]$ is the covariance between assets i and j .

5. Conclusion

Based on the results of the reviews that have been carried out, there are several studies regarding determining the optimal stock portfolio using the Markowitz Method. There are advantages to each article, firstly, the articles of Septyanto (2019) have the advantage of being able to show the best stock portfolio choices and can provide investors with an overview on how to determine the best stock portfolio by considering the minimum possible risk and get as much profit as possible. Second, the Dian (2020) article has the advantage of providing consideration with other methods to determine an optimal portfolio. Third, the article Negara et al. (2021) has the advantage of, namely the selection of candidates for the optimal portfolio using stocks that have a positive expected return value so that the return calculation can be greater than the original value so that it can provide the possibility of obtaining profits. the greater one. Fourth, articles by Hanif et al. (2021) have the advantage of considering other conditions, namely conditions during Covid-19 that affect stock movements. While the shortcomings of the four articles are the research data which is still incorporated in the same scope of the stock index and from each article only discusses its application using the Markowitz Method.

In its development, it is expected to consider existing methods, using uncertain constraints such as constraints on the stock portfolio optimization model. One method that can be used by considering the unknown value is the Robust Optimization of the stock portfolio optimization model.

Based on this literature review, it is hoped that it can provide a little description of the formation of a stock portfolio using the Markowitz Method and in the future, it is hoped that the Markowitz method can be applied to various other fields.

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