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International Journal of Research in Community Service

e-ISSN: 2746-3281
p-ISSN: 2746-3273

Vol. 5, No. 3, pp. 138-145, 2024

Analysis of Multistability of Financial Risk Chaos Systems and Its Application to Voice Cryptography

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Abstract

In the chaos literature, the application of modeling and control of dynamic systems in chaos theory arising in several fields is investigated. In this article we analyze complex financial chaos systems with countries as interest rates, investment demand, and price indices. The proposed chaotic flow's dynamic behavior is examined using phase portraits, eigenvalues, bifurcation diagrams, and Lyapunov exponent spectra. A significant quantity of research on secure communication systems has been published in recent years as a result of the major advancements in communications equipment and encryption techniques. A new voice encryption algorithm design is given using a financial chaos model. An application for voice encryption is conducted using the suggested algorithm, and the outcomes are described.

Keywords: Chaos, voice encryption, finance risk, dynamic system.

1. Introduction

Applications for chaotic dynamical systems exist in numerous scientific domains. Chaos theory has been applied in fields like digital signatures in the past few decades such as Cryptography and secure communication (Blahut, 2014), secure communication (Zaher, 2011), biology and medicine (Skinner, 1992), genetic algorithms (Li-Jiang, 2002), image encryption (El Assad), multiphase reactors (van Den Bleek, 2002), systems engineering (Curry, 2012) have been introduced

As new technologies emerge, voice communications become progressively more commonplace and dangerous. As a result, ensuring a high level of security in data communications has become vital (Gharaibeh, 2017). In this sense, the intriguing characteristics of chaos-based cryptography ssuch as its broadband power spectrum, great sensitivity to initial conditions, random-like behavior, and tolerance to sufficiently high noise levels have drawn a lot of attention (Pisarchik, 2010). The high unpredictability of the numbers used in encryption has a significant effect on the encryption's quality (Massoudi, 2008). Discrete chaotic maps have been the primary basis for several encryption systems that have been introduced in recent years, particularly for sound signals. The construction of a sound encryption technique based on a novel chaotic system with boomerang-shaped equilibrium was proposed by Mobayen et al. in 2018. Thus, in this work, voice encryption based on chaotic systems has been used infrequently.

In this work, we propose a new finance chaotic model with state variables as the interest rate, investment demand, and price index. The construction of the new finance chaotic system is carried out by combining two known chaotic models of finance systems. In Section 2, we explain the dynamic equations and properties of Yuningsih's financial model. In Part 3, we build the electronic circuit of a new financial model with the help of MultiSIM. In Section 4, the new voice encryption application is carried out, and the analysis results are given. Section 5 contains conclusions.

2. Yuningsih finance chaotic model

In this paper, we sample the Yuningsih equation to apply it in voice encryption. The mathematical model of Yuningsih's financial system is described by the system of differential equations below:

$$\begin{cases} \dot{x} = z + (y - a)x \\ \dot{y} = 1 - by - |x| - x^2 \\ \dot{z} = -x - z \end{cases} \quad (1)$$

The state variables x and y in Yuningsih's financial dynamics model stand for interest rates and investment demand, respectively, and z stands for the price exponent. In addition, financial parameters a represent household savings and b represent investment costs. Both financial parameters should be positive.

In (Yuningsih, 2022) demonstrated that the chaotic nonlinear finance system (1) becomes chaotic when $(a, b) = (1, 0.04)$, The Lyapunov characteristic exponents of the Yuningsih finance model (2) are obtained in Python for parameter values $(a, b) = 1, 0.04)$, initial state $X(0) = (0.5, 3, -0.4)$ and $LE_1 = 0.18626, LE_2 = 0.00180, LE_3 = -0.53304$.

The Kaplan-Yorke dimension of the Yuningsih finance model is derived as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.353 \quad (2)$$

The two-dimensional phase graphs of the novel finance chaotic system (5) are shown in Figures 1-3. The Lyapnov characteristic exponents for the parameter d of the new financial chaotic system (5) are shown in Figure 4. In these numerical simulations, we have assumed that the new financial chaotic system (5) has an initial state of $X(0) = (0.5, 3, -0.4)$ and parameter values of $(a, b) = (1, 0.04,)$.

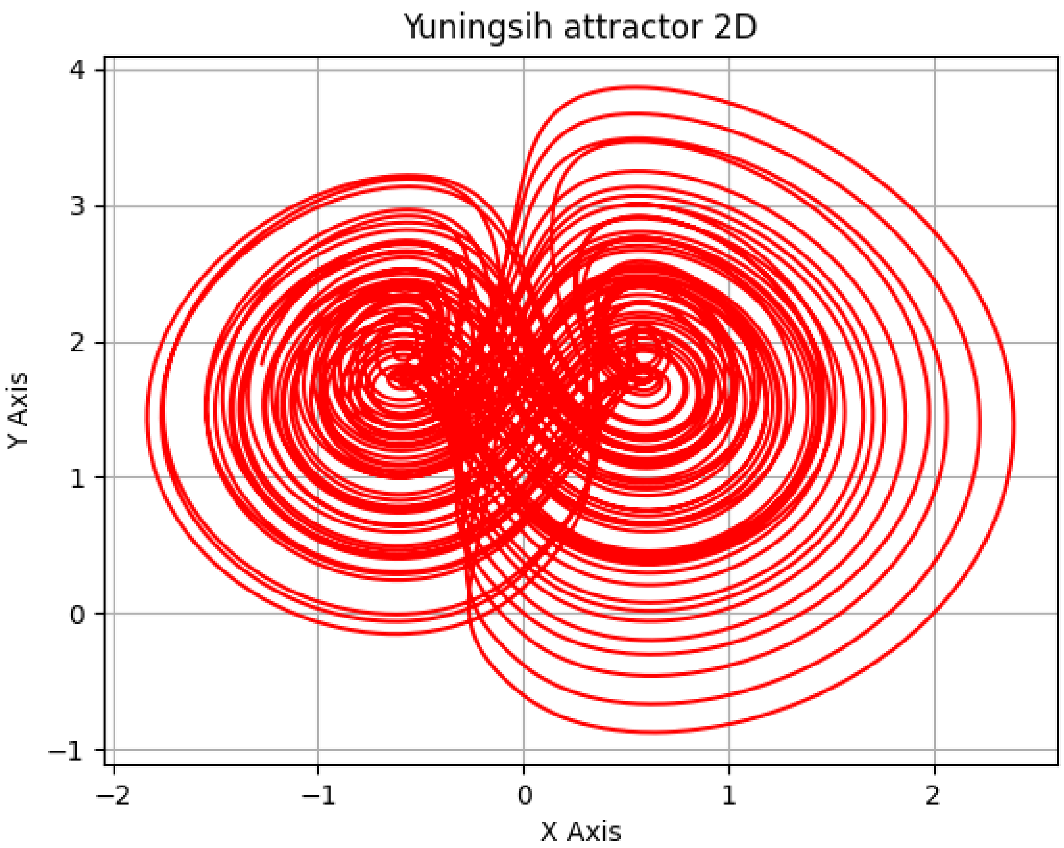

Figure 1: Numerical simulations of the 2-D phase plot in (x, y)plane of Yuningsih finance chaotic system for $X(0) = (0.5, 3, -0.4)$ and $(a, b) = (1, 0.04)$

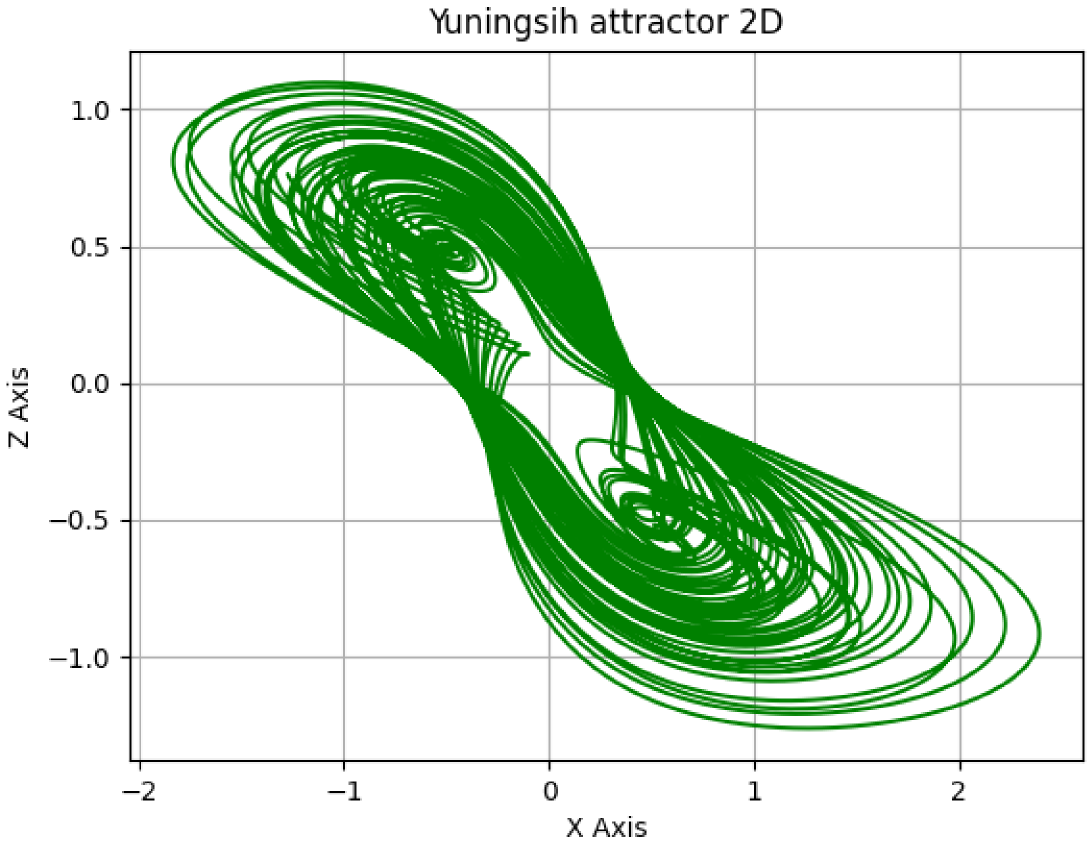

Figure 2: Numerical simulations of the 2-D phase plot in (x,z)plane of Yuningsih finance chaotic system for $X(0) = (0.5, 3, -0.4)$ and $(a,b) = (1, 0.04)$

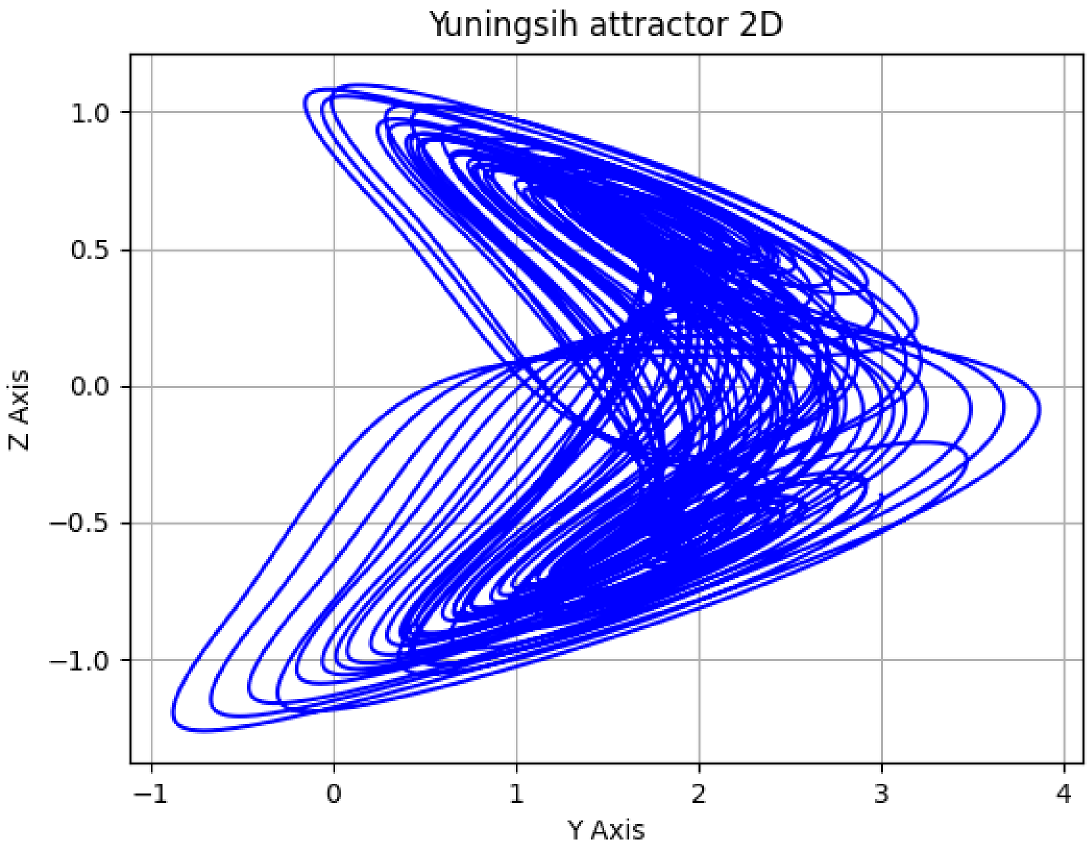

Figure 3: Numerical simulations of the 2-D phase plot in (y, z)plane of Yuningsih finance chaotic system for $X(0) = (0.5, 3, -0.4)$ and $(a, b) = (1, 0.04)$

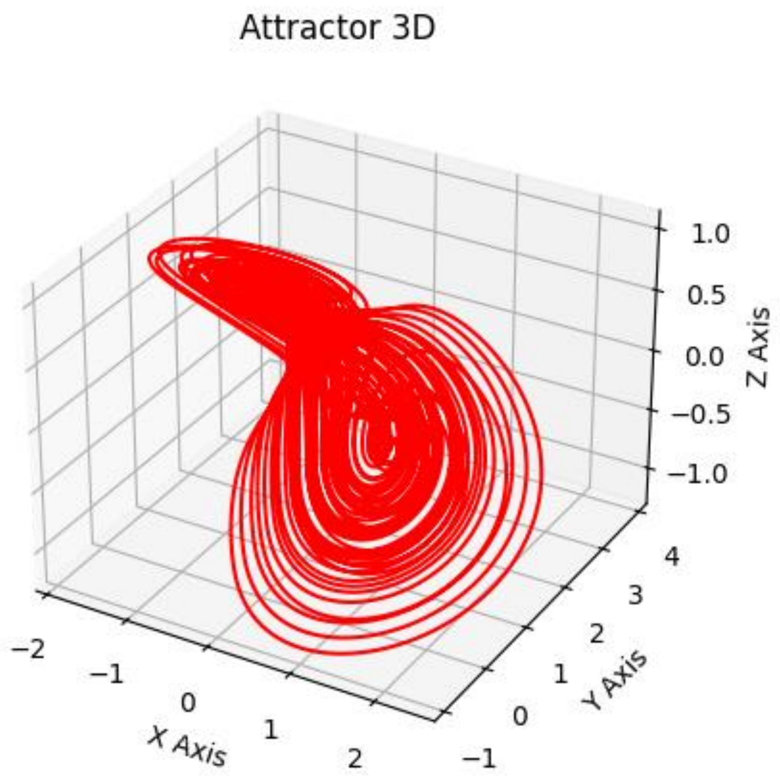

Figure 4: Numerical simulations of the 3-D phase plot in $x, (y, z)$ plane of Yuningsih finance chaotic system for $X(0) = (0.5, 3, -0.4)$ and $(a, b) = (1, 0.04)$

The following system of equations must be solved in order to determine the equilibrium points of the chaotic system of Yuningsih finance (5).

$$x_3 + (x_2 - 1)x_1 = 0 \quad (3a)$$
$$1 - 0.04x_2 - |x_1| - x_1^2 = 0 \quad (3b)$$
$$-x_1 - x_3 = 0 \quad (3c)$$

From Eq. (3c), it follows that

$$x_3 = -x_1 \quad (4)$$

By using (4), equations (3a) and (3b) can be balanced as equations (5) and (6):

$$x_1(x_2 - 1) - 2 = 0 \quad (5)$$
$$1 - 0{,}04x_2 - |x_1| - x_1^2 = 0 \quad (6)$$

In this case there are two cases that must be considered (A): $x_1 = 0$ and (B): $x_1 \neq 0$.

In case (A), we suppose that $x_1 = 0$. Since $x_3 = -x_1$, its is immediate that $x_3 = 0$. From Equation (6), we get $1 - 0.04x_2 = 0$ or $x_2 = 25$. Thus, $E_0 = (0, 25, 0)$ is a balance point of the Yuningsih finance system.

In case (B), we suppose that $x_1 \neq 0$. From Equation (5), we get $x_2 = 2$.

Thus Equation (6) can be simplified as follows:

$$x_1^2 + |x_1| = 0 \quad (7)$$

To solve equation (7) we obtain 2 real roots $x_1 = \pm 0.5816653820$.

Cause $x_3 = -x_1$ it can be said that $x_3 = \mp 0{,}5816653820$.

So that in case (B), the chaotic system of the financial risk model (1) has two equilibrium points, $E_1 = (0.5816, 2, -0.5816)$ and $E_2 = (-0.5816, 2, 0.5816)$.

The Jacobian matrix J_0 of the Yuningsih chaotic system (1) at E_0 has the spectral values

$$\lambda_1 = -0.4505, \lambda_2 = 0.7869, \lambda_3 = 0.7869 - 0.8194i$$

This show that the equilibrium point E_0 is a saddle-point and unstable.

The Jacobian matrix J_1 of the Yuningsih chaotic system (1) at E_1 has the spectral values

$$\lambda_1 = -0.4505, \lambda_2 = 0.7896 + 0.8194i, \lambda_3 = 0.7869 - 0.8194i$$

This show that the equilibrium point E_1 is a saddle-point and unstable.
The Jacobian matrix J_2 of the Yuningsih chaotic system (1) at E_2 has the spectral values

$$\lambda_1 = -0.4505, \lambda_2 = 0.7869, \lambda_3 = 0.7869 - 0.8194i$$

This show that the equilibrium point E_2 is a saddle-point and unstable.

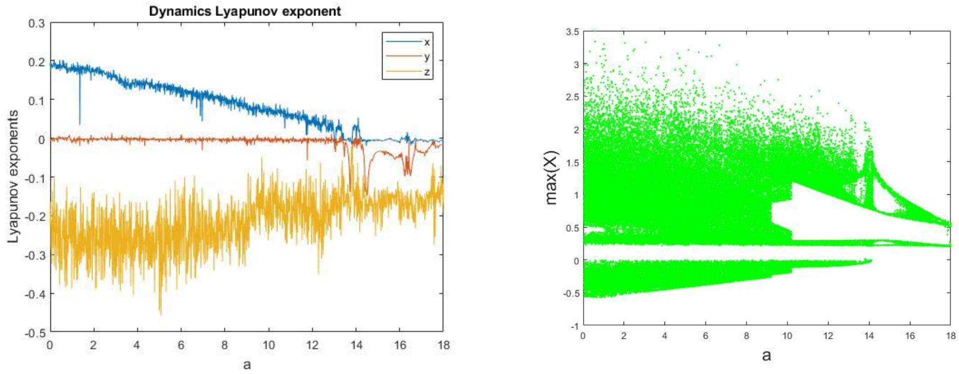

Figure 5: Diagram of the Lyapunov characteristic exponents versus a and the bifurcation diagram of X3max versus a for $b = 0.04$

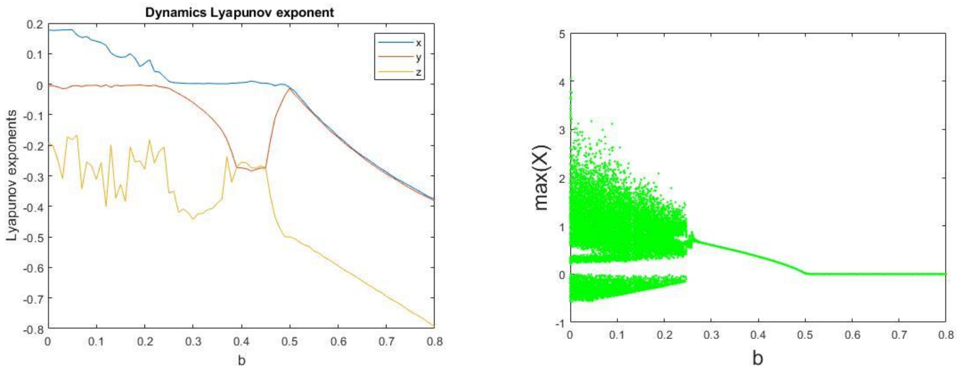

Figure 6: Diagram of the Lyapunov characteristic exponents versus b and the bifurcation diagram of X3max versus b for $a = 1$

Our bifurcation analysis is described as follows.
Case (A) Effect of varying a

Here, we fix $b = 0.04$ and cary a. When $0 < a < 15$ the chaotic system (1) displays chaotic behavior. When $a > 15$ the chaotic system (1) displays periodic behavior. The bifurcation analysis for the case of varying a is describe by Figure 5.

Case (B) Effect of varying b

Here, we fix $a = 1$ and cary b. When $0 < a < 0.25$ the chaotic system (1) displays chaotic behavior. When $a > 0.25$ the chaotic system (1) displays periodic behavior. The bifurcation analysis for the case of varying a is describe by Figure 6.

3. Circuit Realization of The Yuningsih Chaotic System

4. Voice encryption application and its analysis

The implementation of voice encryption follows the design of the encryption algorithm. The original voice files utilized in the encryption process, both encrypted and decrypted, are displayed in Figure 7-11. It looks that a totally different voice file is obtained from the original file in Figure 7 when we examine the encrypted voice file's graph. The voice file has been successfully decrypted, and the original voice file has been recovered, as can be observed by looking at Figures 7 and 8. The frequency spectrum analysis findings of the original, encrypted, and decrypted voice recordings are displayed in Figure 10 and 11. The spectrum graph of the encrypted voice file has far larger frequency values than the original voice file of the spectrum analysis results, and it has nearly equal spectrum values for each sample value, as can be seen when looking at Figures 10 and 11. The proposed encryption technique has been shown to successfully conduct voice encryption and decryption, based on the findings of frequency spectrum analysis.

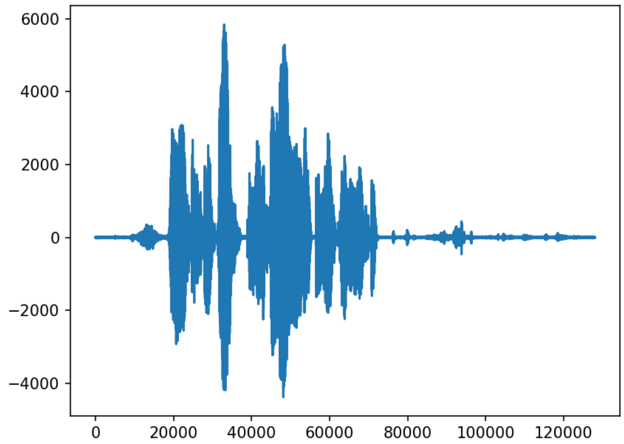

Figure 7: The original voice file

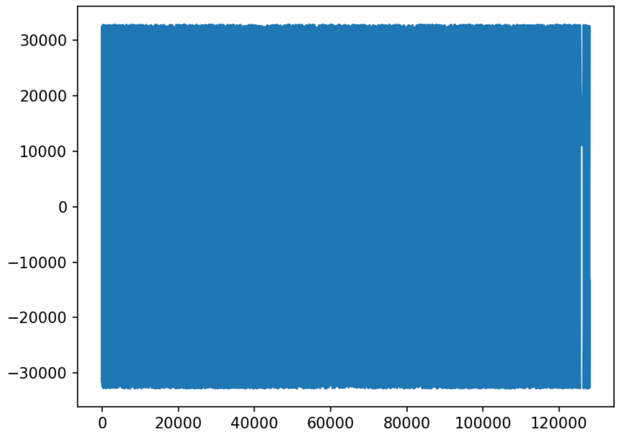

Figure 8: The encrypted files

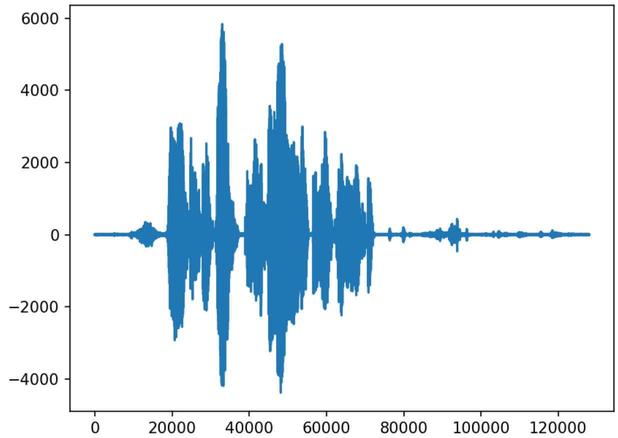

Figure 9: The decrypted files

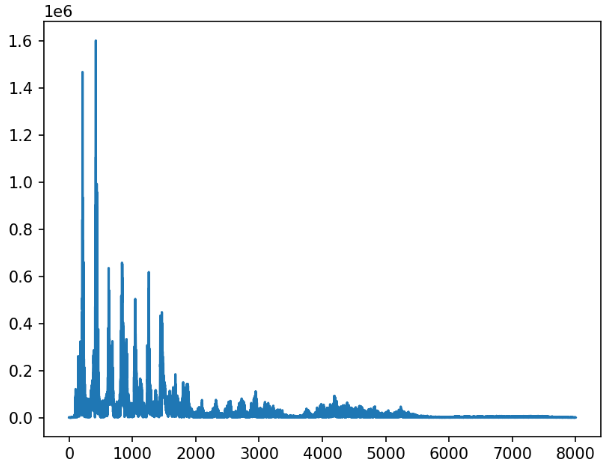

Figure 10: The spectrum analysis outcomes of original

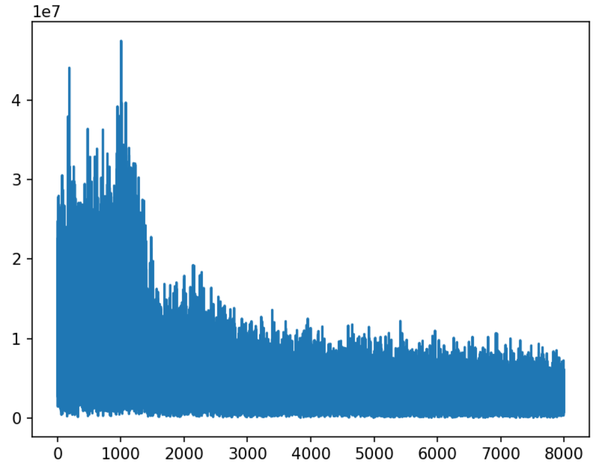

Figure 11: The spectrum analysis outcomes of encrypted voice files.

5. Conclusion

This study presents Yuningsih's intricate model of financial chaos, which includes three state variables: interest rate, investment demand, and price index. A thorough analysis is conducted of the new financial chaos model's dynamic characteristics. The suggested system's dynamic behaviour was investigated, and its bifurcation diagram and spectrum of Lyapunov exponents were shown. Multisim is used to construct and analyse the new financial model's electrical chaotic circuit. All the principal findings reported in this article are explained using numerical simulations. The frequency spectrum analysis of the encryption process is used to carry out the voice encryption application. The analysis's findings indicate that the suggested encryption method completes the encryption process satisfactorily.

Acknowledgments

Acknowledgment: Gratitude is extended to the Research and Community Service Institute of National University of the Republic of Indonesia (LPPM UKRI) for their generous funding support.

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